## **Supporting Information**

## **Modulation of electronic properties from stacking orders and spin-orbit coupling for 3R-type MoS<sup>2</sup>**

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## **Fig. S1**



Fig. S1 Band structures of 2-layer  $MoS<sub>2</sub>$  with ab-stacking near the bottom of conduction band at  $\Lambda$  (a) and K point (b), and schematic of conduction band splitting at  $\Lambda$  and K point due to spin-orbit coupling and layer's coupling (c)



Fig. S2 Isosurface of band-decomposed charge density of six states of 3-layer  $MoS<sub>2</sub>$  with aba stacking order, at the top of valance band of K point (in Fig. 5a) including the states  $-\vert 1+3\uparrow\rangle$ -1(a), ~ $\vert 1+3\uparrow\rangle$ -2(b), ~ $\vert 2\uparrow\rangle$ (c), ~ $\vert 1+3\downarrow\rangle$ -1(d), ~ $\vert 1+3\downarrow\rangle$ -2(e) and ~ $\vert 2\downarrow\rangle$  (f).

The spin-up states ~|1+3<sup> $\uparrow$ </sup>)-1and ~|1+3 $\uparrow$ <sup>2</sup>)-2 are almost degenerate. The spin-up states  $\sim|1+3\sqrt{2}$  and  $\sim|1+3\sqrt{2}$  are almost degenerate. This may due to the being of mirror symmetry of aba stacking.



Fig. S3 Isosurface of band-decomposed charge density of six states of 3-layer  $MoS<sub>2</sub>$  with aba stacking order, at the top of valance band of K point (in Fig. 5a) including the states  $\langle -|1 \uparrow \rangle$  (a),  $\sim |2 \uparrow \rangle$  (b),  $\sim |3 \uparrow \rangle$  (c),  $\sim |1 \downarrow \rangle$  (d),  $\sim |2 \downarrow \rangle$  (e) and  $\sim |3 \downarrow \rangle$  (f).





Fig. S4 Band structures of bulk  $2H-MoS<sub>2</sub>$  (a) and  $3R-MoS<sub>2</sub>$  (b) with spin-orbit coupling.

The band splitting of  $2H-MoS_2$  at CB-K is  $208 \text{ meV}$  and that of  $3R-MoS_2$  at CB-K is 149 meV.

## **Model Hamiltonian**

The model Hamiltonian for tree-layer  $MoS<sub>2</sub>$ 

$$
H(k) = \begin{pmatrix} E_0(k) & M_1(k) & M_2(k) \\ M_1^*(k) & E_0(k) & M_1(k) \\ M_2^*(k) & M_1^*(k) & E_0(k) \end{pmatrix}
$$

The model Hamiltonian for four-layer  $MoS<sub>2</sub>$ 

$$
H(k) = \begin{pmatrix} E_0(k) & M_1(k) & M_2(k) & M_3(k) \\ M_1^*(k) & E_0(k) & M_1(k) & M_2(k) \\ M_2^*(k) & M_1^*(k) & E_0(k) & M_1(k) \\ M_3^*(k) & M_2^*(k) & M_1^*(k) & E_0(k) \end{pmatrix}
$$

The model Hamiltonian is used to analyze the splitting of band splitting near band gap and understand the layer's coupling.  $M_1(k)$ ,  $M_2(k)$  and  $M_3(k)$  represent the coupling parameters of nearest-neighbor, second near-neighbor and third near-neighbor layer-coupling with same k point and same energy level , respectively.

Table S1. Band splitting values of the top of valance bands at  $\Gamma$  and K points (VB- $\Gamma$  and VB-K) and the bottom of conduction bands at  $\Lambda$  and K points (CB- $\Lambda$  and CB-K) for multi-layer 3R-type MoS<sub>2</sub> including 2L-ab, 3L-aba, 3L-abc, 4L-abab and 4L-abca with the different layers and stacking orders, calculated by DFT and fitted by model Hamiltonian (TM). Note that  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$  represent the splitting values between the nearest-neighbor energy levels and  $\Delta$  is for total splitting value.



It is obvious that the values of band splitting from model Hamiltonian are consistent with that from DFT.