## **Supporting Information**

## Modulation of electronic properties from stacking orders and spin-orbit coupling for 3R-type MoS<sub>2</sub>

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## Fig. S1



Fig. S1 Band structures of 2-layer  $MoS_2$  with ab-stacking near the bottom of conduction band at  $\Lambda$  (a) and K point (b), and schematic of conduction band splitting at  $\Lambda$  and K point due to spin-orbit coupling and layer's coupling (c)



Fig. S2 Isosurface of band-decomposed charge density of six states of 3-layer MoS<sub>2</sub> with aba stacking order, at the top of valance band of K point (in Fig. 5a) including the states  $\sim |1+3\uparrow\rangle - 1(a), \sim |1+3\uparrow\rangle - 2(b), \sim |2\uparrow\rangle(c), \sim |1+3\downarrow\rangle - 1(d), \sim |1+3\downarrow\rangle - 2(e)$  and  $\sim |2\downarrow\rangle$  (f).

The spin-up states  $\sim |1+3\uparrow\rangle$ -1 and  $\sim |1+3\uparrow\rangle$ -2 are almost degenerate. The spin-up states  $\sim |1+3\downarrow\rangle$ -1 and  $\sim |1+3\downarrow\rangle$ -2 are almost degenerate. This may due to the being of mirror symmetry of aba stacking.



Fig. S3 Isosurface of band-decomposed charge density of six states of 3-layer MoS<sub>2</sub> with aba stacking order, at the top of valance band of K point (in Fig. 5a) including the states  $\sim |1\uparrow\rangle$  (a),  $\sim |2\uparrow\rangle$  (b),  $\sim |3\uparrow\rangle$  (c),  $\sim |1\downarrow\rangle$  (d),  $\sim |2\downarrow\rangle$  (e) and  $\sim |3\downarrow\rangle$  (f).





Fig. S4 Band structures of bulk  $2H-MoS_2$  (a) and  $3R-MoS_2$  (b) with spin-orbit coupling.

The band splitting of 2H-MoS<sub>2</sub> at CB-K is 208 meV and that of 3R-MoS<sub>2</sub> at CB-K is 149 meV.

## **Model Hamiltonian**

The model Hamiltonian for tree-layer MoS<sub>2</sub>

$$H(k) = \begin{pmatrix} E_0(k) & M_1(k) & M_2(k) \\ M_1^*(k) & E_0(k) & M_1(k) \\ M_2^*(k) & M_1^*(k) & E_0(k) \end{pmatrix}$$

The model Hamiltonian for four-layer MoS<sub>2</sub>

$$H(k) = \begin{pmatrix} E_0(k) & M_1(k) & M_2(k) & M_3(k) \\ M_1^*(k) & E_0(k) & M_1(k) & M_2(k) \\ M_2^*(k) & M_1^*(k) & E_0(k) & M_1(k) \\ M_3^*(k) & M_2^*(k) & M_1^*(k) & E_0(k) \end{pmatrix}$$

The model Hamiltonian is used to analyze the splitting of band splitting near band gap and understand the layer's coupling.  $M_1(k)$ ,  $M_2(k)$  and  $M_3(k)$  represent the coupling parameters of nearest-neighbor, second near-neighbor and third near-neighbor layer-coupling with same k point and same energy level, respectively.

Table S1. Band splitting values of the top of valance bands at  $\Gamma$  and K points (VB- $\Gamma$  and VB-K) and the bottom of conduction bands at  $\Lambda$  and K points (CB- $\Lambda$  and CB-K) for multi-layer 3R-type MoS<sub>2</sub> including 2L-ab, 3L-aba, 3L-abc, 4L-abab and 4L-abca with the different layers and stacking orders, calculated by DFT and fitted by model Hamiltonian (TM). Note that  $\Delta_1$ ,  $\Delta_2$  and  $\Delta_3$  represent the splitting values between the nearest-neighbor energy levels and  $\Delta$  is for total splitting value.

Stacking way	VB-Γ(eV)		CB-Λ(eV)		$CB-K(10^{-2} eV)$		$VB-K(10^{-2} eV)$	
2L-ab	DFT	TM	DFT	TM	DFT	TM	DFT	TM
Δ	0.6389	0.6389	0.3058	0.3058	5.43	5.43	6.11	6.11
3L-aba								
$\Delta_1$	0.2732	0.2757	0.2699	0.2698	5.5370	5.5340	0.7900	0.7920
$\Delta_2$	0.5324	0.5307	0.1377	0.1378	0.1100	0.1100	6.0420	6.0420
Δ	0.8057	0.8604	0.4076	0.4076	5.6470	5.6440	6.8320	6.8340
3L-abc								
$\Delta_1$	0.3052	0.3058	0.2278	0.2266	3.6910	3.6950	5.4900	5.4930
$\Delta_2$	0.5228	0.5218	0.1926	0.1936	5.3490	5.3450	4.2960	4.2930
Δ	0.8281	0.8276	0.4204	0.4202	9.0400	9.0400	9.7860	9.7860
4L-abab								
$\Delta_1$	0.1707	0.1708	0.1572	0.1580	0.9750	0.9750	0.8110	0.8040
$\Delta_2$	0.3067	0.3066	0.2001	0.2006	4.2560	4.2220	5.1840	5.1760
$\Delta_3$	0.4214	0.4204	0.1017	0.1004	0.7360	0.7750	1.1500	1.1640
Δ	0.8988	0.8978	0.4591	0.4590	5.9670	5.9720	7.1450	7.1440
4L-abca								
$\Delta_1$	0.1810	0.1809	0.1700	0.1698	4.0870	4.0730	5.6170	5.6640
$\Delta_2$	0.3037	0.3030	0.1798	0.1808	4.4640	4.4090	4.4890	4.4760
$\Delta_3$	0.4317	0.4319	0.1357	0.1357	5.4900	5.5470	4.6540	4.6480
Δ	0.9164	0.9159	0.4855	0.4862	14.0410	14.0290	14.7600	14.7880

It is obvious that the values of band splitting from model Hamiltonian are consistent with that from DFT.