Supplementary information for : Classification of polyhedral shapes from individual incomplete cryo-EM reconstructions

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Table S3 (in separate Excel spreadsheet due to size): List of 123 polyhedra in the library \mathcal{P} , together with class and topological profiles (TP). Polyhedral pairs with identical TP are noted.

Table S4 (in separate Excel spreadsheet due to size): List of estimated polyhedral graphs for 30 micro-compartments of *E. coli*, as represented by their topological profiles.





Figure S1: Distribution of number of edges per face for Platonic solids



Figure S2: Distribution of vertex degree (number of edges meeting at a vertex) for Platonic solids



Figure S3: Distribution of vertex degree for Archimedean solids



Figure S4: Distribution of number of edges per face for Archimedean solids



Figure S5: Distribution of vertex degree for 6 Johnson solids



Figure S6: Distribution of number of edges per face for Johnson solids



Figure S7: The distribution of vertex degree in MCs









(e)



Figure S8: Simulated standard Polyhedron and their view after truncation - (a) A simulated icosahedron, (b) the icosahedron with missing top, (c) A simulated sphenocorona, (d) the sphenocorona with missing top and (e) the ball-stick diagram on (d)



Figure S9: Distribution of aspect ratios by identified structure of BMCs. SP2 = λ_2/λ_1 , SP3 = λ_3/λ_1 , where $\lambda_1 \ge \lambda_2 \ge \lambda_3$ are the principal axes of an ellipsoid fitted to each reconstructed BMC.

Table S1: Test set misclassification error for SVM classifier summarised by class of solid. This analysis is based on the set of 54 solids with 20 vertices or less. The highlighted section refers to the chance of misclassification of a symmetric Platonic solid as a Johnson solid.

		Actual shape class				
Predicted shape class	Misclassification error	Johnson	Platonic	Archimedean	Catalan	
		Solids	Solids	Solids	Solids	
Johnson Solids	Minimum	0.0120	<mark>0.0000</mark>	0.0200	0.0000	
	Maximum	0.5240	<mark>0.0540</mark>	0.4620	0.0020	
	Median	0.2680	<mark>0.0320</mark>	0.2410	0.0000	
	Mean	0.2763	<mark>0.0287</mark>	0.2410	0.0005	

	SD	0.1462	<mark>0.0272</mark>	0.3125	0.0010
	5th Percentile	0.0520	<mark>0.0000</mark>	0.0200	0.0000
	95th Percentile	0.4990	<mark>0.0540</mark>	0.4620	0.0020
Platonic Solids	Minimum	0.0000	0.0000	0.0000	0.0000
	Maximum	0.0320	0.3200	0.2900	0.0000
	Median	0.0000	0.0320	0.1450	0.0000
	Mean	0.0014	0.1173	0.1450	0.0000
	SD	0.0063	0.1762	0.2051	0.0000
	5th Percentile	0.0000	0.0000	0.0000	0.0000
	95th Percentile	0.0085	0.3200	0.2900	0.0000
Archimedean Solids	Minimum	0.0000	0.0000	0.0040	0.0000
	Maximum	0.3960	0.0020	0.5400	0.0000
	Median	0.0000	0.0000	0.2720	0.0000
	Mean	0.0120	0.0007	0.2720	0.0000
	SD	0.0603	0.0012	0.3790	0.0000
	5th Percentile	0.0000	0.0000	0.0040	0.0000
	95th Percentile	0.0520	0.0020	0.5400	0.0000
Catalan Solids	Minimum	0.0000	0.0000	0.0000	0.0000
	Maximum	0.0380	0.0000	0.0000	0.0340
	Median	0.0000	0.0000	0.0000	0.0160
	Mean	0.0015	0.0000	0.0000	0.0165
	SD	0.0059	0.0000	0.0000	0.0154
	5th Percentile	0.0000	0.0000	0.0000	0.0000
	95th Percentile	0.0080	0.0000	0.0000	0.0340

Table S2: Predicted polyhedral shapes for 30 E. coli microcompartments using the SVM classifier. The names of the solids corresponding the serial numbers are given in Table S3. The positive predictive value (PPV) is the chance that the correct solid was identified, based on estimated misclassification errors obtained using a mis-specified polyhedral graph test set.

Microcompartment	Predicted	Positive	
No	Shape (Serial	Predictive	
	No.)	Value	

		(PPV)
1	17	0.70
2	11	0.69
3	16	0.75
4	11	0.69
5	16	0.75
6	86	0.69
7	86	0.69
8	86	0.69
9	16	0.75
10	86	0.69
11	88	0.59
12	86	0.69
13	54	0.62
14	16	0.75
15	86	0.69
16	86	0.69
17	16	0.75
18	88	0.59
19	50	0.76
20	87	0.60
21	86	0.69
22	87	0.60
23	16	0.75
24	17	0.70
25	16	0.75
26	16	0.75
27	16	0.75
28	16	0.75
29	62	0.81
30	11	0.69
Mean PPV		0.70

Table S5: Categorization of features in the topological profile (TP) of a polyhedral graph (PG)

		Feature type			
Topological profile component	Dimension	Complete	Incomplete	Global	Local
V,E,F	3	х		х	
Face type distribution	6	х			х

Vertex degree distribution	6	х		х
At least face type distribution	8		х	х
At least vertex type distribution	8		х	х
Edge adjacency matrix	10x10 = 100	х		х
Face adjacency matrix	10x10 = 100	х		х
Total	231			