Supplementary Note

A method to decipher pleiotropy by detecting underlying heterogeneity driven by hidden subgroups applied to autoimmune and neuropsychiatric diseases

Buhm Han^{1-4,25}, Jennie G Pouget^{1,5-7,25}, Kamil Slowikowski^{1,3,4,8}, Eli Stahl⁹, Cue Hyunkyu Lee¹⁰, Dorothee Diogo¹⁻³, Xinli Hu^{1,3,4,11}, Yu Rang Park^{10,12}, Eunji Kim^{10,13}, Peter K Gregersen¹⁴, Solbritt Rantapää Dahlqvist¹⁵, Jane Worthington^{16,17}, Javier Martin¹⁸, Steve Eyre^{16,17}, Lars Klareskog¹⁹, Tom Huizinga²⁰, Wei-Min Chen²¹, Suna Onengut-Gumuscu²¹, Stephen S Rich²¹, Major Depressive Disorder Working Group of the Psychiatric Genomics Consortium²², Naomi R Wray²³, Soumya Raychaudhuri^{1,3,4,19,24}

¹Department of Medicine, Brigham and Women's Hospital and Harvard Medical School, Boston, USA
²Department of Convergence Medicine, University of Ulsen College of Medicine, 8, Asso Institute for L ²Department of Convergence Medicine, University of Ulsan College of Medicine & Asan Institute for Life Sciences, Asan Medical Center, Seoul, Republic of Korea

³ Partners Center for Personalized Genetic Medicine, Boston, USA
⁴ Pregram in Medical and Penulation Canation, Preed Institute of M

⁴Program in Medical and Population Genetics, Broad Institute of MIT and Harvard, Cambridge, USA
⁵Campbell Family Mental Health Research Institute, Centre for Addiction and Mental Health, Toronto, Canada

⁶ Institute of Medical Sciences, University of Toronto, Toronto, Canada
⁷ Department of Ravabiata: University of Taranta, Taranta, Canada

Department of Psychiatry, University of Toronto, Toronto, Canada ⁸

BIOINFORM CONSTRUCTS and Integrative Genomics, Harvard University, Cambridge, USA

Bountermative and integrative current spinal School of Medicine, New York, USA
¹⁰Asan Institute for Life Sciences, University of Ulsan College of Medicine, Asan Medical Center, Seoul,

Republic of Korea
¹¹ Harvard-MIT Division of Health Sciences and Technology, Boston, USA

¹² Department of Biomedical Informatics, Asan Medical Center, Seoul, Republic of Korea
¹³ Department of Chemistry, Seoul National University, Seoul, Republic of Korea
¹⁴ Robert S. Boas Center for Genomics and Human G

Manhasset, USA
¹⁵Department of Public Health and Clinical Medicine, Rheumatology, Umeå University, Umeå, Sweden 16
Arthritis Research UK Centre for Genetics and Genomics, Musculoskeletal Research Centre, Institute for Inflammation and Repair, Manchester Academic Health Science Centre, University of Manchester, Manchester, UK

¹⁷National Institute for Health Research, Manchester Musculoskeletal Biomedical Research Unit, Central Manchester University Hospitals National Health Service Foundation Trust, Manchester Academic Health
Sciences Centre, Manchester, UK
¹⁸lastitute de Racculture

⁸Instituto de Parasitologíay Biomedicina López-Neyra, Consejo Superior de Investigaciones Científicas, Granada, Spain

¹⁹Rheumatology Unit, Department of Medicine, Karolinska Institutet and Karolinska University Hospital Solna, Stockholm, Sweden
²⁰Department of Rheumatology, Leiden University Medical Centre, Leiden, the Netherlands

Department of Nicamatology, Estate Sinceres, International Charlottesville, USA

²¹Center for Public Health Genomics, University of Virginia, Charlottesville, USA

²²A full list of members and affiliations appears in t

 25 These authors contributed equally to this work

Correspondence to:

Soumya Raychaudhuri

77 Avenue Louis Pasteur, Harvard New Research Building, Suite 250D, Boston, MA 02446, USA. soumya@broadinstitute.org; 617-525-4484 (tel); 617-525-4488 (fax)

Buhm Han

Asan Institute for Life Sciences, Asan Medical Center, 88, Olympic-ro 43-gil, Songpa-gu, Seoul 138-736, Korea. buhm.han@amc.seoul.kr; 82-2-3010-2648 (tel); 82-2-3010-2619 (fax)

TABLE OF CONTENTS

MOTIVATING INTUITION FOR BUHMBOX

Allele dosages are uncorrelated if there is no subgroup heterogeneity

Our method is built upon the intuition that if there is no subgroup heterogeneity in case individuals, then for loci that are independent in control individuals, under the additive model assumption, the risk alleles at those loci will be independent in the case individuals as well. Although this seems to be intuitive, analytically proving this will set up the reasoning for our null hypothesis of no correlations.

We will assume haploid model and consider two loci that are biallelic. If we consider two loci together, there will be 4 possible pairs of alleles. Thus, we can define a new "virtual locus" that consists of the two loci, which has 4 alleles. To deal with 4 alleles, in the following, it will be useful to define the multiallelic odds ratio. For a single biallelic locus, if we let p^+ and p^- be the risk allele frequencies (RAFs) in cases and controls respectively, the biallelic odds ratio (OR) is

$$
\gamma = \frac{p^+/(1-p^+)}{p^-/(1-p^-)}
$$

Thus, case RAF is a function of OR and control RAF,

$$
p^{+} = \frac{\gamma p^{-}}{(\gamma - 1)p^{-} + 1} \tag{1}
$$

Then we generalize biallelic OR to multiple alleles. Suppose that we have M different risk alleles in addition to a reference allele (total M+1 alleles). Let p_R^+ and p_R^- be the case and control reference allele frequencies. Let $p_{\rm i}^+$ and $p_{\rm i}^-$ be the case and control RAFs of allele *i* (*i*=1, …,*M*). The constraints are, $p_1^+ + p_2^+ + \cdots + p_M^+ + p_R^+ = 1$ and $p_1^- + p_2^- + \cdots + p_M^+$ $p_M^- + p_R^- = 1$. The multiallelic odds ratios are defined as

$$
\begin{array}{rcl}\n\gamma_1 &= \frac{p_1^+/p_R^+}{p_1^-/p_R^-} \\
\cdots & \cdots \\
\gamma_M &= \frac{p_M^+/p_R^+}{p_M^-/p_R^-}\n\end{array}
$$

Again, case RAF is a function of OR and control RAF. For reference allele,

$$
p_R^+ = \frac{1}{1 + \frac{1}{p_R^-} \sum_{i=1}^M p_i^- \gamma_i} \tag{2}
$$

and for allele $i=1,...,M$,

$$
p_i^+ = p_i^- \gamma_i \frac{p_R^+}{p_R^-}
$$
 (3)

Now recall that we consider two independent biallelic risk loci. Let γ_1 and γ_2 be their respective ORs and p_1^- and p_2^- be their control RAFs. Let p_1^+ and p_2^+ be the case RAFs, which can be derived from equation (1). We assume the standard additive model that, if an individual has risk alleles at both loci, the resulting OR is $\gamma_1 \times \gamma_2$. If we consider "virtual locus" spanning the two loci, there will be 4 possible alleles. We can build the following multi-allelic frequency table,

Note that the control allele frequency is simply multplication of the frequencies at the two loci ("Control" column in the table above), because these loci are uncorrelated in control individuals. Our goal is to prove that the case allele frequency can be similarly decomposed into the multiplication of the frequencies of the two loci. If that is the case, that will show that these loci are independent in the case individuals as well. Consider the reference allele frequency, p_{12}^+ . By equation (2), we have

$$
p_{12}^{+} = \frac{1}{1 + \frac{p_{12}^{-}}{p_{12}^{-}} \gamma_1 + \frac{p_{12}^{-}}{p_{12}^{-}} \gamma_2 + \frac{p_{12}^{-}}{p_{12}^{-}} \gamma_1 \gamma_2}
$$
\n
$$
= \frac{1}{1 + \frac{p_1^{-}(1 - p_2^{-})}{(1 - p_1^{-})(1 - p_2^{-})} \gamma_1 + \frac{(1 - p_1^{-})p_2^{-}}{(1 - p_1^{-})(1 - p_2^{-})} \gamma_2 + \frac{p_1^{-}}{p_2^{-}} \gamma_1 \gamma_2}
$$
\n
$$
= \frac{(1 - p_1^{-})(1 - p_2^{-})}{(1 - p_1^{-})(1 - p_2^{-}) + p_1^{-}(1 - p_2^{-}) \gamma_1 + (1 - p_1^{-}) p_2^{-} \gamma_2 + p_1^{-} p_2^{-} \gamma_1 \gamma_2}
$$
\n
$$
= \frac{(1 - p_1^{-})(1 - p_2^{-})}{((1 - p_1^{-}) + \gamma_1 p_1^{-})((1 - p_2^{-}) + \gamma_2 p_2^{-})}
$$
\n
$$
= (1 - p_1^{+})(1 - p_2^{+}) \tag{4}
$$

Thus, the case allele frequency is also a multiplication of the frequencies of the two loci. The similar decomposition can be done for the other three alleles. For example, by equations (3) and (4),

$$
p_{12}^{+} = p_{12}^{-} \gamma_1 \gamma_2 \frac{p_{12}^{+}}{p_{12}^{-}}
$$

= $p_1^{-} p_2^{-} \gamma_1 \gamma_2 \frac{(1-p_1^{+})(1-p_2^{+})}{(1-p_1^{-})(1-p_2^{-})}$
= $\gamma_1 p_1^{-} \frac{(1-p_1^{+})}{(1-p_1^{-})} \gamma_2 p_2^{-} \frac{(1-p_2^{+})}{(1-p_2^{-})}$
= $p_1^{+} p_2^{+}$

Since the frequency is the product of frequencies of each allele, the two loci are independent. Thus, we conclude that the loci that are independent in controls will also be independent in case individuals under the standard additive model.

Subgroup heterogeneity induces positive correlations

An equally important intuition is that if there is subgroup heterogeneity, the risk alleles at loci that are independent in control individuals will show positive correlations in case individuals. This fact sets the foundation stone for our alternative hypothesis of positive correlations.

Suppose that disease A (D_A) case individuals consist of two groups: one group genetically similar to a second trait (disease B, D_B) and the rest not similar to D_B . Say (π x 100)% of case individuals are in the D_B -similar group. We will call π the *heterogeneity proportion*. Consider two independent SNPs that are associated to the second trait. Let their risk allele frequencies be p_1^+ and p_2^+ in the D_B-similar group and p_1^- and p_2^- in the rest. Note that the two loci are uncorrelated within each subgroup (the D_B -similar group comprises individuals that are genetically cases for D_B , and we have already shown that independent risk loci will be uncorrelated in case individuals under the standard additive model).

If we consider frequencies of haplotypes spanning the two loci in the D_A case individuals consisting of both subgroups,

The expected value of Pearson correlation is therefore

$$
r_{12} = \frac{p_{12}p_{\overline{12}} - p_{\overline{12}}p_{1\overline{2}}}{\sqrt{p_1 \cdot p_{\overline{1}} p_{\cdot 2} p_{\cdot \overline{2}}}}\tag{5}
$$

where dot (⋅) in the subscript denotes marginal frequency, for example $p_1 = p_{12} + p_{12}$. A few interesting characteristics are, (1) r_{12} is always positive or zero because we considered risk allele dosage at both loci. (2) $r_{12} = 0$ if $\pi = 0$ or $\pi = 1$. (3) $r_{12} = 0$ if risk is zero ($p_1^+ = p_1$ and $p_2^+ = p_2$). (4) r_{12} is a function of RAF, OR, and π (but not of sample size). r_{12} is typically a very small value. **Supplementary Figure 2** shows the value of r_{12}

as a function of OR at the two loci (when we fix p_1 and p_2 to 0.5) and the value of r_{12} as a function of p_1 and p_2 (when we fix OR to 1.5 at both loci).

DERIVATION OF BUHMBOX

Null and alternative hypotheses

Building upon the aforementioned intuitions, we can build the BUHMBOX statistic to detect positive correlations between independent loci, which will be evidence of subgroup heterogeneity. Suppose that we examine D_A case individuals at M independent D_B -associated loci. Between these loci, we calculate correlations of risk allele dosages to obtain an $M \times M$ correlation matrix, R. The null hypothesis of our method is that the non-diagonal elements of **R** are zero. The alternative hypothesis of our method is that the non-diagonal elements of **R** are positive. We build our method in the following steps.

Combining correlations into one statistic

The first challenge is to combine $M(M - 1)/2$ non-diagonal elements of **R** into one statistic. To this end, we show that under the null hypothesis of no correlations, the nondiagonal elements of the observed correlation matrix will be independent of each other. We employ the framework of Jennrich¹. Jennrich describes a framework for testing deviance of a correlation matrix from a specified null matrix. To describe the framework briefly, let $P = (\rho_{ii})$ be a specific $M \times M$ correlation matrix that defines the null hypothesis. The goal is to test if the observed sample correlation matrix $\bf R$ deviates from . To define a statistic, what we need is the inverse of asymptotic covariance matrix for

the maximum likelihood estimates of ρ_{ij} , which we call Γ^{-1} . Note that Γ^{-1} is a $q \times q$ matrix where $q = M(M - 1)/2$.

Let $P^{-1} = (\rho^{ij})$. Let δ_{ij} be the Kronecker delta, that is, 1 if $i = j$ and zero otherwise. We define $\mathbf{T} = (t_{ij})$ as the following,

$$
t_{ij} = \delta_{ij} + \rho_{ij}\rho^{ij} \tag{6}
$$

Then, Γ^{-1} is given by

$$
\Gamma^{-1}(i,j;k,l) = \rho^{ik}\rho^{jl} + \rho^{il}\rho^{jk} - \rho^{ij}(t^{ik} + t^{jk} + t^{il} + t^{jl})\rho^{kl}
$$
 (7)

Given these, if we define

$$
\mathbf{Y} = \sqrt{N}(\mathbf{R} - \mathbf{P}) = (y_{ij})
$$
\n(8)

where N is the number of samples used to calculate R , the test statistic is

$$
S_{Jennrich} = \sum_{i < j} \sum_{k < l} y_{ij} \Gamma^{-1}(i, j; k, l) y_{kl} \tag{9}
$$

which follows χ^2 distribution with q degrees of freedom under the null. The computation is challenging if p is large because of the time complexity $O(q^2) = O(p^4)$. Jennrich applies an optimization technique to simplify the formula to

$$
S_{Jennrich} = \frac{1}{2}tr(\mathbf{Y}\mathbf{P}^{-1}\mathbf{Y}\mathbf{P}^{-1}) - dg' \quad (\mathbf{P}^{-1}\mathbf{Y})\mathbf{T}^{-1}dg(\mathbf{P}^{-1}\mathbf{Y}) \tag{10}
$$

which only involves operations between $M \times M$ matrices requiring only $O(M^3)$.

In our situation, this statistic simplifies further. Our null hypothesis is no correlation. Thus, the identity matrix I is our null correlation matrix ($P = I$). Substituting P with I, the statistic simplifies to

$$
S_{Jennrich}|\mathbf{p}_{=I} = \frac{1}{2}tr(\mathbf{YY}) = \sum_{i < j} y_{ij}^2 \tag{11}
$$

Note that each y_{ij} asymptotically follows a normal distribution (thus, a z-score). The statistic can be interpreted as the following; under the specific situation $P = I$, the zscores become asymptotically independent. Thus, we can combine information by simply summing up their squares which will follow χ^2 distribution with q degrees of freedom.

OPTIMIZATION OF BUHMBOX

Accounting for directions to increase power

A straightforward application of Jennrich's approach in equation (11) is not optimal for our situation, because Jennrich's test is a general test that does not account for the direction of correlations. As we have described, subgroup heterogeneity only results in positive expected correlations between risk loci. Thus, accounting for this fact may give us better power.

To this end, we employ the meta-analytic framework Wei^{2,3} and Lin and Sullivan⁴. This framework combines multiple estimates whose asymptotic covariance is known while accounting for their directions. Applying this approach, we obtain a new statistic that is alternative to equation (9),

$$
S_{Directional} = \frac{\sum_{i < j} \sum_{k < l} y_{ij} \Gamma^{-1}(i, j; k, l)}{\sqrt{\sum_{i < j} \sum_{k < l} \Gamma^{-1}(i, j; k, l)}}
$$

which follows $N(0,1)$ under the null hypothesis of $R = P$. To reduce computational complexity, we can apply the same optimization technique of Jennrich to simplify the statistic to

$$
S_{Directional} = \frac{\frac{1}{2}tr(\text{YP}^{-1}\text{EP}^{-1}) - dg'}{\sqrt{\frac{1}{2}tr(\text{EP}^{-1}\text{EP}^{-1}) - dg'} \cdot (\text{P}^{-1}\text{E})T^{-1}dg(\text{P}^{-1}\text{E})}
$$

where E is an $M \times M$ matrix whose elements are all ones.

In our specific situation that $P = I$, this statistic further simplifies to

$$
S_{Directional}|\mathbf{p}_{=I} = \frac{\sum_{i < j} y_{ij}}{\sqrt{q}}\tag{12}
$$

which follows the standard normal distribution under the null hypothesis. We calculate significance of this statistic using the normal distribution in a positive one-sided test.

Optimizing weights based on effect sizes and allele frequencies

The directional test in equation (12) accounts for the directions of correlations, but does not account for the effect size and frequency differences between loci. As we have shown, expected correlation of two loci is a function of not only π , heterogeneity proportion, but also the effect sizes and RAFs of the two loci. Thus, each pair of loci will have different expected correlations. When we combine correlations to one statistic, we can have better power by giving higher weights to the pairs of high expected correlations.

Recall that the expected correlation is, as given in equation (5),

$$
r_{12} = \frac{p_{12}p_{\overline{12}} - p_{\overline{12}}p_{1\overline{2}}}{\sqrt{p_{1.}p_{\overline{1}}p_{.2}p_{.2}}}
$$

We examine what is the increase in r_{12} given an increase in π at the local region around the null hypothesis $\pi = 0$.

$$
w_{12} = \frac{\partial r_{12}}{\partial \pi} \big|_{\pi=0} = \frac{\sqrt{p_1(1-p_1)p_2(1-p_2)}(y_1-1)(y_2-1)}{((y_1-1)p_1+1)((y_2-1)p_2+1)}
$$

The value w_{12} can be thought of as the slope of the curves evaluated at $\pi = 0$. For any loci pair *i* and *j*, we can calculate w_{ij} .

Then what we need is an optimal strategy to incorporate w_{ij} into our testing, so that our method can have the local optimum property at around $\pi = 0$. That is, should we weight y_{ij} by w_{ij} or $\sqrt{w_{ij}}$? We note that our situation is analogous to a situation where in a meta-analysis, the effect sizes are the same for all participating studies but their units or scales are different, thus requiring different weights. We extended the traditional meta-analysis method, fixed effects model (FE), to a new model which can deal with the situation that the scales are different between studies. We present the details in the end of **Supplmentary Note**. Briefly speaking of the conclusion, the optimal strategy is multiplying the scaling parameters directly into the weights of the sum of weighted zscores.

Based on this reasoning, our statistic becomes

$$
S_{BUHMBOX} = \frac{\sum_{i < j} w_{ij} y_{ij}}{\sqrt{\sum_{i < j} w_{ij}^2}}\tag{13}
$$

which follows $N(0,1)$ under the null hypothesis. We calculate the significance of this statistic uising the normal distribution assuming a positive one-sided test.

Controlling for LD and utilizing control samples

Because linkage disequilibrium (LD) can induce unexpected correlations between loci, one should prune the loci before applying BUHMBOX. We suggest a harsh criterion (e.g. removing SNPs that are r^2 < 0.1 or that are nearby (within \pm 1Mb) to other SNPs). However, even after harsh pruning, there can be residual LD that can affect the results. To minimize the effect of residual LD, BUHMBOX uses control samples.

Recall that when we defined the z-scores based on correlations in case samples, we used the formula in equation (8),

$$
\mathbf{Y} = \sqrt{N}(\mathbf{R} - \mathbf{P}) = (y_{ij})
$$

which means that we should multiply the correlation elements by square root of sample size to obtain z-scores. Now if we use control samples, and let \mathbb{R}' be the correlation matrix of control individuals, we can define a new Y' as

$$
\mathbf{Y}' = \sqrt{\frac{NN'}{N+N'}} (\mathbf{R} - \mathbf{R}')
$$

where N' is the control sample size.

Although the basic form of our approach is case-only statistic, using only cases requires a strong assumption of no residual LD. By subtracting control correlations from case correlations to use "*delta-correlations*", we obtain a more robust statistic against the effect of LD.

Controlling for population stratification

We correct for population stratification by regressing out PCs from the vector of case/control allele dosage of each locus. Note that we regress out PCs from each locus one by one, not simultaneously from the whole dosage matrix including multiple loci. This way, our approach can be thought of as obtaining partial correlations.

Meta-analysis of BUHMBOX results

The BUHMBOX statistic is a z-score. Therefore, we can meta-analyze BUHMBOX results using the standard weighted sum of z-score approach, where z-scores are weighted by the square root of the total sample size.

POLYGENIC MODELING AND BUHMBOX

We assessed by simulations whether our method can benefit by taking advantage of a polygenic modeling approach. In GWAS, the use of a stringent threshold (P-value threshold *t*=5x10⁻⁸) minimizes false positives, but likely misses true positives due to imperfect power. Therefore, investigators often use a polygenic modeling approach that applies a more liberal threshold to define a larger set of variants⁵. We simulated GWASs

based on the Bayesian polygenic model⁶ and employed more liberal values of *t* ranging from $5x10^{-8}$ up to 0.01, to obtain larger sets of variants.

Specifically, We adapted Stahl et al.'s Bayesian polygenic model⁶, which predicted 2,231 causal loci among 84,000 independent genome-wide loci for RA. To simulate 2,231 causal variants, we combined 71 independent known loci of $RA⁷$ to an additional 2,160 loci sampled from the joint posterior distribution of RAF and OR presented in Stahl *et al.* For null loci, we also used the null RAF distribution presented in Stahl *et al.* Given this disease model, we simulated a GWAS with 3,964 cases and 12,052 controls (sample sizes from Stahl *et al.*), assuming prevalence of 0.01. Given GWAS results, we used only the top *k* GWAS loci defined by p-value threshold *t* and their observed odds ratios for BUHMBOX power simulations. We assumed *N=5,000* and *π=0.5* for power evaluation and tried different p-value threshold *t* from 5x10⁻⁸ to 0.01.

We observed that the statistical power of BUHMBOX increased when we included variants with moderately significant p-values (**Supplementary Figure 4**). In this simulation, 29.5% power at *t*=5.0x10⁻⁸ increased up to 88.1% at *t*=3.6x10⁻⁴ and then gradually dropped as we used even more liberal *t* values. This shows that BUHMBOX can benefit from polygenic modeling.

INTERPRETATION OF BUHMBOX RESULTS:

Here we sought to describe in detail what can specifically cause subgroup heterogeneity.

Misclassifications can cause subgroup heterogeneity

Phenotypic misclassifications can cause subgroup heterogeneity. Suppose that a proportion of diagnosed case individuals were actually case individuals for a different disease. In this situation, the heterogeneity proportion π corresponds to the misclassification proportion.

Molecular subtypes can cause subgroup heterogeneity

Suppose that disease A can occur because of multiple different molecular pathways, and one of the pathways is shared with disease B. Then, the subgroup that shares a molecular pathway with disease B will show genetic characteristics that are similar to disease B patients.

Phenotypic causality can cause subgroup heterogeneity

This is a situation that is often called "mediated pleiotropy"⁸. Assume that there are two conditions A and B whose population prevalences are K_A and K_B . First, consider the null situation that A and B are not causal to each other. Obviously, B-associated loci will be uncorrelated within A cases. Within A cases, the prevalence of B will be K_B .

Now consider the situation that B causes A (having condition B increases the chance of acquiring A). Let $K_{A|B}$ and $K_{A|B}$ be the frequency of A among B patients and among non-B-patients. The population attributable risk of A to B is

$$
PAR = K_{A|B} - K_{A|\overline{B}} > 0
$$

The population prevalence of A can be written

$$
K_A = K_B K_{A|B} + (1 - K_B) K_{A|\overline{B}}
$$

Thus, within A cases, the proportion of individuals having B will be

$$
K_{B|A} = \frac{K_B K_{A|B}}{K_A} = \frac{K_B K_{A|B}}{K_B K_{A|B} + (1 - K_B)K_{A|\overline{B}}} > F_B
$$

In this situation, the heterogeneity proportion π corresponds to the excessive proportion of individuals having B among A cases, that would not have occurred without the causal relationship;

$$
\pi = K_{B|A} - K_B
$$

which has the following relationship to PAR

$$
\pi = \frac{K_B(1 - K_B)}{K_A} \text{ PAR}
$$

Whole-group pleiotropy cannot cause subgroup heterogeneity

Common genetic basis between all patients of A and all patients of B (wholegroup pleiotropy) does not cause subgroup heterogeneity. If disease A and B share risk alleles, within A cases, the frequencies of risk alleles for B may increase. However, there will not be a subgroup who has excessivie numbers of risk alleles. Instead, the risk alleles will occur independently and homogeneously across all A cases even if only a subset of variants have pleiotropic effects. Thus, there will not be correlations among Bassociated-risk alleles among A cases. Note that the prevalence of B may increase

 $(K_{B|A} > K_B)$, but we can think of A cases being sampled from a new homogeneous population which has a larger prevalence of B.

Inverse causal relationship cannot cause subgroup heterogeneity

We previously considered the causal relationship of disease B causing A, but now consider the inverse situation that A causes B, while B does not cause A. Again we examine the correlations between B-associated loci in A cases. This is a similar situation to pleiotropy in that we can also think of A cases being sampled from a new population which has a larger prevalence of B. Since there will not be a subgroup, there will not be correlations.

META-ANALYSIS WITH SCALE DIFFERENCES

In deriving the strategy to incorporate weights in equation (13), our BUHMBOX method utilized a meta-analytic framework that accounts for the scale or unit differences between studies. Because the description of the framework is long, we pushed the description back to here. We will first introduce the well-known meta-analytic method, fixed effects model, and extend it to account for scale differences.

Fixed effects model

We first review the fixed effects model meta-analysis method. Let X_i be the observed effect size of study *i* and V_i be the variance of it. By definition, z-score is defined $Z_i =$ X_i $\frac{U_i}{V_i}$. Let $W_i = V_i^{-1}$ be the inverse variance. Under the fixed effects model, we assume that X_i has mean μ that is constant (fixed) across the studies. There are two common

approaches under the fixed effects model: the inverse variance weighted average and the weighted sum of z-scores. Two approaches are related as shown below.

Inverse variance weighted average In the inverse variance weighted average approach, the goal is to obtain the best estimate of μ . A commonly used estimate is weighted average, $\bar{X}_i=\frac{\sum_i c_i X_i}{\sum_i c_i},$ which is an unbiased estimate of μ for any $c_i>0.$ The variance of \bar{X}_i is $\bar{V}_i=\frac{\sum_i c_i^2 V_i}{(\sum_i c_i)^2}$. To obtain the best estimate, we choose c_i that minimizes the variance. By the Cauchy-Schwarz inequality,

$$
\overline{V}_i = \frac{\sum_i c_i^2 V_i}{(\sum_i c_i)^2} \ge \left(\sum_i \frac{c_i \sqrt{V_i}}{\sum_j c_j} \times \frac{1}{\sqrt{V_i}}\right) / \sum_i \frac{1}{V_i} = 1 / \sum_i \frac{1}{V_i}
$$

The equality is achieved when

$$
\frac{c_i\sqrt{V_i}}{\sum_j c_j} = k \cdot \frac{1}{\sqrt{V_i}}
$$

for a constant $k > 0$. Without losing generality, we can assume $\sum_j c_j$ is a constant. Thus, we can achieve the equality by choosing $c_i = W_i$, which is why the method is called inverse variance weighted average. Given these weights, the average estimate and its variance are

$$
\overline{X} = \frac{\sum_{i} W_{i} X_{i}}{\sum_{i} W_{i}}
$$
\n
$$
(15)
$$

$$
\bar{V} = \frac{1}{\Sigma_i W_i} \tag{16}
$$

Weighted sum of z-scores In the weighted sum of z-scores approach, the goal is to maximize the power of statistic. Given z-scores $Z_i = \frac{X_i}{\sqrt{V_i}}$, we can define a weighted sum of z-scores statistic $\overline{Z} = \frac{\sum_i b_i Z_i}{\sqrt{2\pi}}$ $\frac{\partial Z_i}{\partial t}$. \overline{Z} is also a z-score (normally distributed and of

variance 1) for any weights $b_i > 0$. To maximize power, we want to maximize the noncentrality parameter of $\bar{Z},\,E[\bar{Z}].$ Since in each study $E[Z_i]=\frac{\mu}{\sqrt{V_i}}$, we have

$$
E[\bar{Z}] = \frac{\sum_{i} b_{i} \mu / \sqrt{V_{i}}}{\sqrt{\sum_{i} b_{i}^{2}}}
$$

Again, by the Cauchy-Schwarz inequality,

$$
E[\bar{Z}] = \mu \cdot \Sigma_i \frac{b_i}{\sqrt{\Sigma_j b_j^2}} \cdot \frac{1}{\sqrt{V_i}} \le \mu \sqrt{\left(\Sigma_i \frac{b_i^2}{\Sigma_j b_j^2}\right) \left(\Sigma_i \frac{1}{V_i}\right)}
$$

The equality is achieved when

$$
\frac{b_i}{\sqrt{\sum_j b_j^2}} = k \cdot \frac{1}{\sqrt{V_i}}
$$

for a constant $k > 0$. Without losing generality, we can assume $\sum_j b_j^2$ is a constant. Thus, we can achieve the equality by choosing $b_i = 1/\sqrt{V_i} = \sqrt{W_i}$. Given these weights, the weighted sum of z-scores statistic is

$$
\bar{Z} = \frac{\sum_{i} \sqrt{W_i} Z_i}{\sqrt{\sum_{i} W_i}} \tag{17}
$$

Note that in many applications, we can approximate $\sqrt{W_i} \propto \sqrt{N_i p_i(1-p_i)}$ where N_i is the sample size of study *i* and p_i is the allele frequency in study *i*. If we can assume the allele frequencies are the same for all studies, the weights b_i approximates to $\sqrt{N_i}$, which is the widely used sample-size-based weight for this approach.

Relation of two approaches The two approaches, the inverse weighted average and the weighted sum of z-scores, are closely related. Given the inverse variance weighted average and its variance in equations (15) and (16), one can construct a zscore for statistical testing. The z-score is

$$
\bar{Z}^* = \bar{X}/\sqrt{\bar{V}} = \frac{\sum_i w_i x_i}{\sum_i w_i} / \frac{1}{\sqrt{\sum_i w_i}} = \frac{\sum_i \sqrt{W_i} Z_i}{\sqrt{\sum_i W_i}} = \bar{Z}
$$

, exactly resulting in the same z-score in the weighted sum of z-scores approach in equation (17). Thus, although the goals of the two approaches were different (obtaining the best estimate and maximizing power), the results of statistical test will be exactly the same for the two approaches, or at least similar if we use the approximation $\sqrt{W_i} \propto$ $\sqrt{N_i p_i(1-p_i)}$ or $\sqrt{W_i} \propto \sqrt{N_i}$.

Fixed effects model with scale differences

We extend the fixed effects model (FE) to a new model accounting for scale differences. We use the similar notations; let X_i and $V_i = W_i^{-1}$ be the observed effect size and its variance in study i. In the new model, we assume that there is a baseline effect μ which is manifested in different scales for each study. We assume that in study i , X_i has mean $\mu_i = \rho_i \mu$, where ρ_i is a scaling factor that is known a priori. Under this model, we can also propose the inverse variance weighted average and the weighted sum of z-score approaches.

Inverse variance weighted average In this approach, our goal is to obtain the best estimate of μ . In each study, we can define $Y_{i} = \frac{X_i}{\alpha}$ $\frac{\lambda_l}{\rho_i}$, an estimator of μ . The variance of Y_i is $Var(Y_i) = V_i/\rho_i^2$. We can define the weighted average estimate for $\mu,$

$$
\overline{Y} = \frac{\sum_i c_i Y_i}{\sum_i c_i}
$$

Using Cauchy-Schwarz inequality, we can show that the variance of \bar{Y} is minimum when

 $c_i = \frac{1}{Var(Y_i)} = \frac{\rho_i^2}{V_i}$ $\frac{\rho_i}{v_i}$. Given these weights, the average estimate and its variance are

$$
\overline{Y} = \frac{\sum_{i} \rho_i W_i X_i}{\sum_{i} \rho_i^2 W_i}
$$
 (18)

$$
Var(\bar{Y}) = \frac{1}{\sum_{i} \rho_i^2 W_i}
$$
 (19)

Weighted sum of z-scores Given z-scores $Z_i = \frac{Y_i}{\sqrt{Var(Y_i)}} = \frac{X_i}{\sqrt{V_i}}$, we can construct a weighted sum of z-scores statistic, $\hat{Z} = \frac{\sum_i b_i Z_i}{\sqrt{2\pi}}$ $\frac{\partial_i Z_i}{\partial_i B_i^2}$. Since in each study $E[Z_i] = \frac{\rho_i \mu}{\sqrt{V_i}}$, we

have

$$
E[\hat{Z}] = \frac{\sum_{i} b_{i} \rho_{i} \mu / \sqrt{V_{i}}}{\sqrt{\sum_{i} b_{i}^{2}}}
$$

Again, by the Cauchy-Schwarz inequality,

$$
E[\hat{Z}] = \mu \cdot \Sigma_i \frac{b_i}{\sqrt{\Sigma_j b_j^2}} \cdot \frac{\rho_i}{\sqrt{V_i}} \le \mu \sqrt{\left(\Sigma_i \frac{b_i^2}{\Sigma_j b_j^2}\right) \left(\Sigma_i \frac{\rho_i^2}{V_i}\right)}
$$

The equality is achieved when

$$
\frac{b_i}{\sqrt{\sum_j b_j^2}} = k \cdot \frac{\rho_i}{\sqrt{V_i}}
$$

for a constant $k > 0$. Thus, we can achieve the equality by choosing $b_i = \rho_i / \sqrt{V_i} =$ $\rho_i\sqrt{W_i}$. In other words, the scaling factor ρ_i is directly multiplied to the original z-score weights used in FE. Given these weights, the weighted sum of z-scores statistic is

$$
\hat{Z} = \frac{\sum_{i} \rho_i \sqrt{W_i} Z_i}{\sqrt{\sum_{i} \rho_i^2 W_i}}
$$
(20)

Relation of two approaches The two approaches, the inverse weighted average and the weighted sum of z-scores, have close relation in the new model, as they have close relation in FE. Given the inverse variance weighted average and its variance in equations (18) and (19), one can construct a z-score for statistical testing. The z-score is

$$
\hat{Z}^* = \overline{Y}/\sqrt{Var(\overline{Y})} = \frac{\sum_i \rho_i w_i x_i}{\sum_i \rho_i^2 w_i} / \frac{1}{\sqrt{\sum_i \rho_i^2 w_i}} = \frac{\sum_i \rho_i \sqrt{w_i z_i}}{\sqrt{\sum_i \rho_i^2 w_i}} = \hat{Z}
$$

, exactly resulting in the same z-score in the weighted sum of z-scores approach in equation (20). Thus, the results of statistical test will be the same for the two approaches, or similar if we use the approximation $\sqrt{W_i} \propto \sqrt{N_i p_i(1-p_i)}$ or $\sqrt{W_i} \propto \sqrt{N_i}$.

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Supplementary Tables

Supplementary Table 1. Null and alternative hypotheses of GRS and BUHMBOX

Supplementary Table 2. False positive rate of BUHMBOX

We simulated a million null studies assuming sample size *N*=2,000 and number of risk loci *M*=50. Then given threshold α, the false positive rate was estimated as the proportion of simulated studies with p-value ≤ α.

Supplementary Table 3. Detailed SNP information used for GRS and BUHMBOX

analyses

Please refer to separate Excel file.

Supplementary Table 4. GRS and BUHMBOX results

Please refer to separate Excel file.

Supplementary Table 5. MDD sample description

Full dataset (used for BUHMBOX analysis)

Schizophrenia-GWAS-independent dataset (used for GRS analysis)

Study	Tag	N Controls	N Cases
Genetic Association Information Network (GAIN)-MDD ¹	gain	1,682	1,693
MDD2000 ⁴	mdd2kb	751	433
	mdd2ks	960	1,016
Max Planck Institute of Psychiatry, Munich ⁵	munich	537	375
RADIANT-UK ⁶	raduk	1,583	1,624
Sequenced Treatment Alternatives to Relieve	stardfull	101	1.241
Depression (STARD) ⁸			
	Total	5,614	6,382

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Supplementary Table 6. Summary of selected previous estimates of genetic overlap between MDD and SCZ

1. Cross-Disorder Group of the Psychiatric Genomics Consortium. Identification of risk loci with shared effects on five major psychiatric disorders: a genome-wide analysis. *Lancet* **381**, 1371-1379 (2013).

2. Cross-Disorder Group of the Psychiatric Genomics Consortium. Genetic relationship between five psychiatric disorders estimated from genome-wide SNPs. *Nat Genet* **45**, 984-994 (2013).

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Members of the Major Depressive Disorder Working Group of the Psychiatric Genomics Consortium

