

1 Supplementary Material

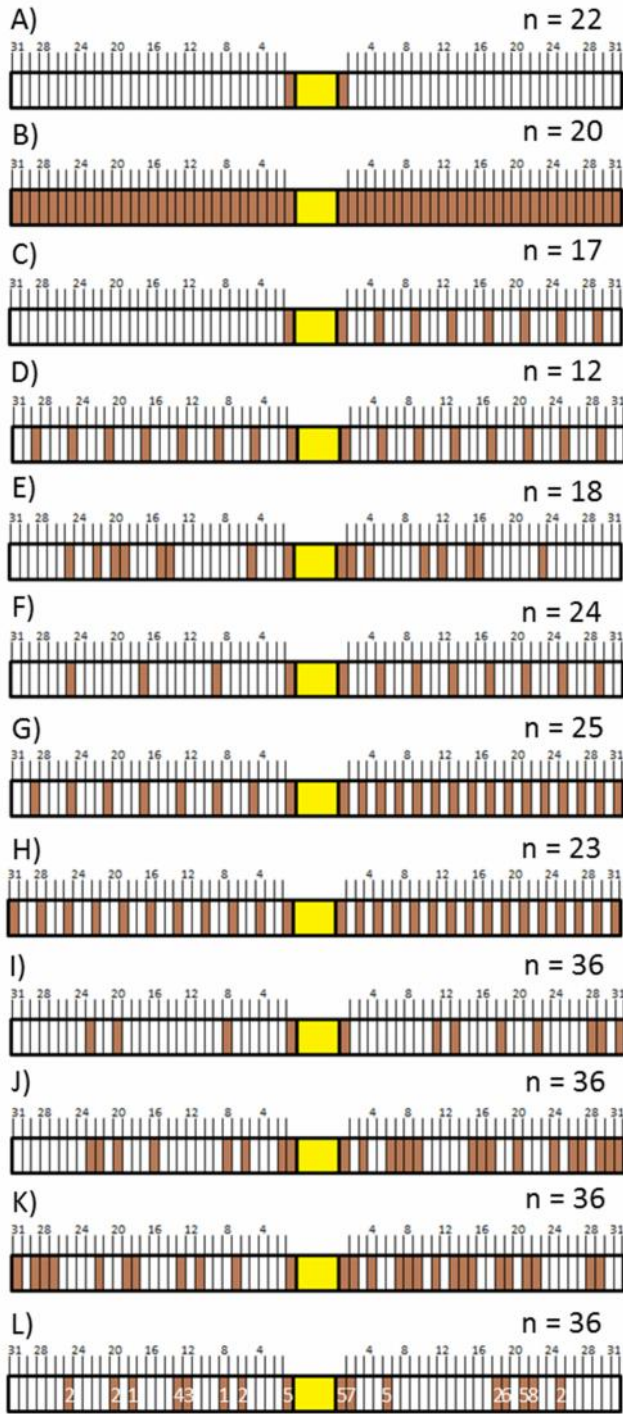


Fig. S1: Experimental setups for the different treatments. *Physarum polycephalum* biomass was placed in the center (yellow box). White boxes indicate blank agar sites

5 (non-rewarding), brown boxes indicate oat-agar food sites (rewarding). The first site on  
6 either arm was always a 5% oat-agar food site, to ensure the cell initialized exploration  
7 on both arms. A) 1 vs 1, B) 31 vs 31, C) 1 vs 8e, D) 8e vs 8e, E) 8r vs 8r (single  
8 example), F) 4e vs 8e, G) 8e vs 16e, H) 11e vs 16e, I) 4r vs 8r (single example), J) 8r vs  
9 16r (single example), K) 11r vs 16r (single example), L) ‘non-binary’ bandit (single  
10 example). Food sites were 5% oat-agar except in L) (see text), where the numbers give  
11 the percentage oat-agar in each reward site for a single example. See Fig. S2 for full  
12 details of the random distributions used in the experiments.

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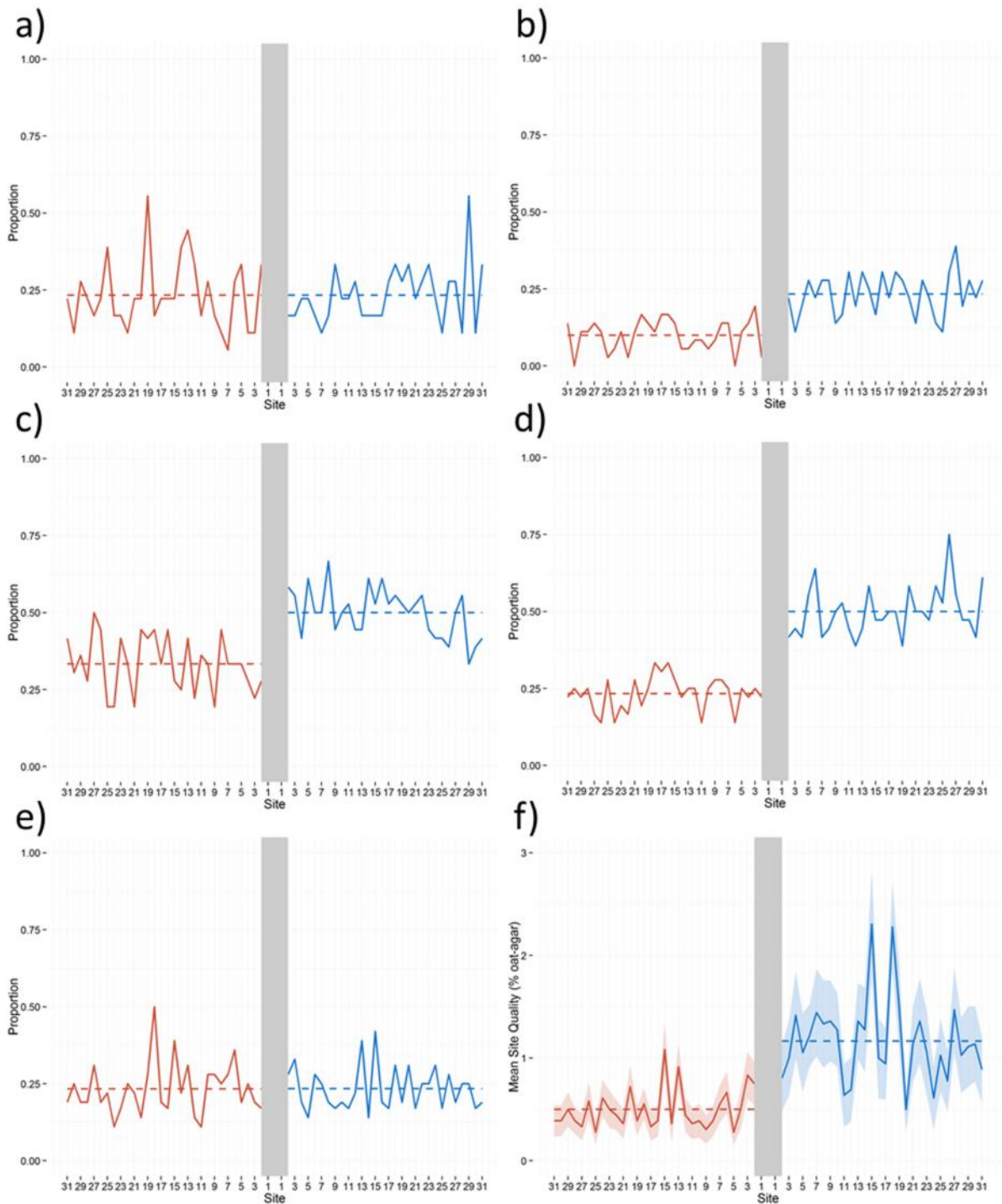
#### 14 **Random distributions**

15 The locations of reward sites were selected by random sampling without  
16 replacement (using the sample() function in R 3.2.0) from a list of all available sites. We  
17 repeated this procedure independently for each arm.

18 When the magnitude of the reward sites was randomized as well, we followed the  
19 following procedure (independently for each arm) to determine the value of each reward  
20 site:

- 21 1. As for randomly-distributed, equally-rewarding sites, the locations of randomly-  
22 distributed, unequally-rewarding sites were selected by random sampling without  
23 replacement.
- 24 2. All reward sites were allocated a minimum reward magnitude (1% oat-agar). All  
25 non-reward sites were kept empty (blank agar).
- 26 3. We selected one of the reward sites by random sampling with replacement (using  
27 the sample() function in R 3.2.0). This site was given an extra 1% oat-agar, unless

28           it had already reached the maximum reward magnitude allowed for a single food  
29           site (8% oat-agar).  
30        4. We repeated step 2 until we reached the total amount of food reward allocated to  
31           the arm.



32

33 Fig. S2: Proportion of times each site contained reward for treatments; a) 8r vs 8r; b) 4r

34 (red) vs 8r (blue); c) 11r (red) vs 16r (blue); d) 8r (red) vs 16r (blue); e) non-binary

35 bandit, 2.5% arm (red) vs 5% arm (blue). The x axes of the graphs are laid out as if the

36 reader were looking at a bandit replicate, with the start block in the center (grey bar) and  
 37 2 opposing arms of 31 agar blocks, each beginning at site 1 and expanding out to site 31.  
 38 The start block at site 0 and the first sites on either arm (which were always 5% oat-agar)  
 39 were excluded from the graphs (greyed bar). The dashed lines show the mean proportion  
 40 over the entire arm. Mean site quality over all replicates for the non-binary bandit, 2.5%  
 41 arm (red) vs 5% arm (blue), is shown in f). The solid line shows the mean oat-agar  
 42 percentage for each site over all replicates. The shaded regions indicate standard error  
 43 about the mean. The overall mean site quality for each arm is less than 2.5% and 5%  
 44 because this calculation of the mean includes the blank agar sites on each arm.  
 45  
 46 Table S1: Binomial test for proportion of replicates reaching the end of the HQ arm first,  
 47 with the alternative hypothesis that the proportion will be due to random chance (0.5).

Treatment	Proportion reaching end of HQ arm first	p-value
1 vs 8e	0.94	<b>2.75 x10<sup>-4</sup></b>
4e vs 8e	0.92	<b>3.59x10<sup>-5</sup></b>
4r vs 8r	0.75	<b>3.93x10<sup>-3</sup></b>
11e vs 16e	0.96	<b>5.72x10<sup>-6</sup></b>
11r vs 16r	0.78	<b>1.19 x10<sup>-3</sup></b>
8e vs 16e	0.88	<b>1.57 x10<sup>-4</sup></b>
8r vs 16r	0.83	<b>6.96x10<sup>-5</sup></b>
Non-binary	0.81	<b>3.13 x10<sup>-4</sup></b>

48  
 49  
 50  
 51

## 52 **Rational beliefs about food density**

53           Models 7 and 8 (see main text) depend on the cell's current rational 'belief' about  
54 which arm contains the highest density of food, based on the observed successes and  
55 failures, along with the prior beliefs. Focusing on a single arm of the experiment, we  
56 specify as  $Q(x | A, B)$  the probability (from the perspective of the cell) that the food  
57 density on that arm is  $x$ , conditioned on the cell's number of previous encounters with  
58 food-filled or empty blocks on that arm. We condition on two parameters,  $A$  and  $B$ , to  
59 denote how the cell 'learns' from experience. To determine the rational belief  $Q(x | A, B)$   
60 we used a simple Bayesian update rule, based on a binomial likelihood function for  
61 receiving reward and a uniform flat prior on reward densities. We assign a uniform prior  
62 probability over all possible food densities before the cell has any experimental  
63 experience.

64

$$P(\text{Food density} = x) \equiv Q(x | A=1, B=1) = 1 \quad \forall 0 \leq x \leq 1$$

65

66 This flat prior can be expressed as a Beta distribution with parameters  $A = 1, B=1$  (hence  
67 our use of these parameters in  $Q$ ). The Beta distribution is a conjugate prior to the  
68 binomial likelihood, which specifies the likelihood (again from the cells perspective) of  
69 each new encounter with either food blocks or empty blocks. By the rules of conjugate  
70 updating, via Bayes rule, when the cell encounters blocks with or without food it will  
71 update its probability distribution over  $x$  to a Beta distribution with different parameters,  
72 adding one to  $A$  for each 'success' (encountering food) and adding one to  $B$  for each  
73 'failure' (encountering an empty block). Using this prior distribution therefore means

74 that once the cell has experienced (A-1) food blocks and (B-1) empty blocks on the focal  
75 arm, its rational belief about the food density will be:

76

$$Q(x|A, B) = \beta(x|A, B) = \frac{x^{A-1}(1-x)^{B-1}}{\int_0^1 x^{A-1}(1-x)^{B-1} dx}$$

77

78 To determine, for instance, the rational belief that the food density,  $y$ , on the right arm is  
79 greater than the food density,  $x$ , on the left, we simply calculate all the ways this could be  
80 true based on the current  $Q$ 's for each arm.

81

$$P(y > x) = \int_0^1 \int_x^1 Q(y|A_R, B_R) Q(x|A_L, B_L) dy dx$$

82

### 83 **Noise**

84 We did not expect that the cell would follow any of these rules absolutely  
85 faithfully at all times. To account for this variability we introduced a 'noise' parameter,  $\epsilon$ ,  
86 such that the rule is followed with probability  $(1 - \epsilon)$ , otherwise with probability  $\epsilon$  the cell  
87 chooses a direction at random;

88

$$89 P(m_t = R | \epsilon) = (1 - \epsilon) P(m_t = R) + \epsilon / 2$$

90

### 91 **Bayesian performance evaluation via marginal-likelihood**

92 The performance of each model was evaluated by calculating the marginal  
93 likelihood of all observed moves, across all experiments and all treatments, accounting

94 for the ‘noise’,  $\theta$  - the proportion of times that the cell makes a random choice rather than  
 95 following the rule. Summing over possible values of  $\theta$ , the marginal likelihood of a  
 96 model  $M$  is thus given as;

$$P(D|M) = \int_0^1 \prod_{\text{Experiments}} \left[ \prod_{\text{moves left}} ((1 - \theta)P(L|M, X) \right. \\ \left. + 0.5\theta) \prod_{\text{moves right}} ((1 - \theta)P(R|M, X) + 0.5\theta) \right] d\theta$$

98  
 99 where  $X$  indicates all the information the cell has from its previous movements, and  $D$   
 100 represents the data of all observed movements.

101 Fig. S3 shows the marginal likelihoods of each model both on the full data set of  
 102 all treatments, and for each treatment individually, normalized by the number of observed  
 103 movements to indicate the geometric mean probability assigned to each movement.  
 104 Values greater than 0.5 indicate that the model is predicting better than a null model  
 105 where each movement is left or right with equal probability. The ratio of the marginal  
 106 likelihoods of two models is termed the Bayes factor, Jeffreys (1) provides a scale for  
 107 qualitative interpretation of Bayes factors, for example considering a Bayes factor of 3 to  
 108 indicate ‘substantial’ evidence for model  $i$  over model  $j$ , and a Bayes factor of over 100 to  
 109 be ‘decisive’. Quantitatively, the Bayes factor requires no reinterpretation and simply  
 110 represents the relative probability that each model is correct. The Bayes factor between  
 111 our best performing model (‘Relative Successes’) and its nearest competitor exceeds  $10^{13}$ ,

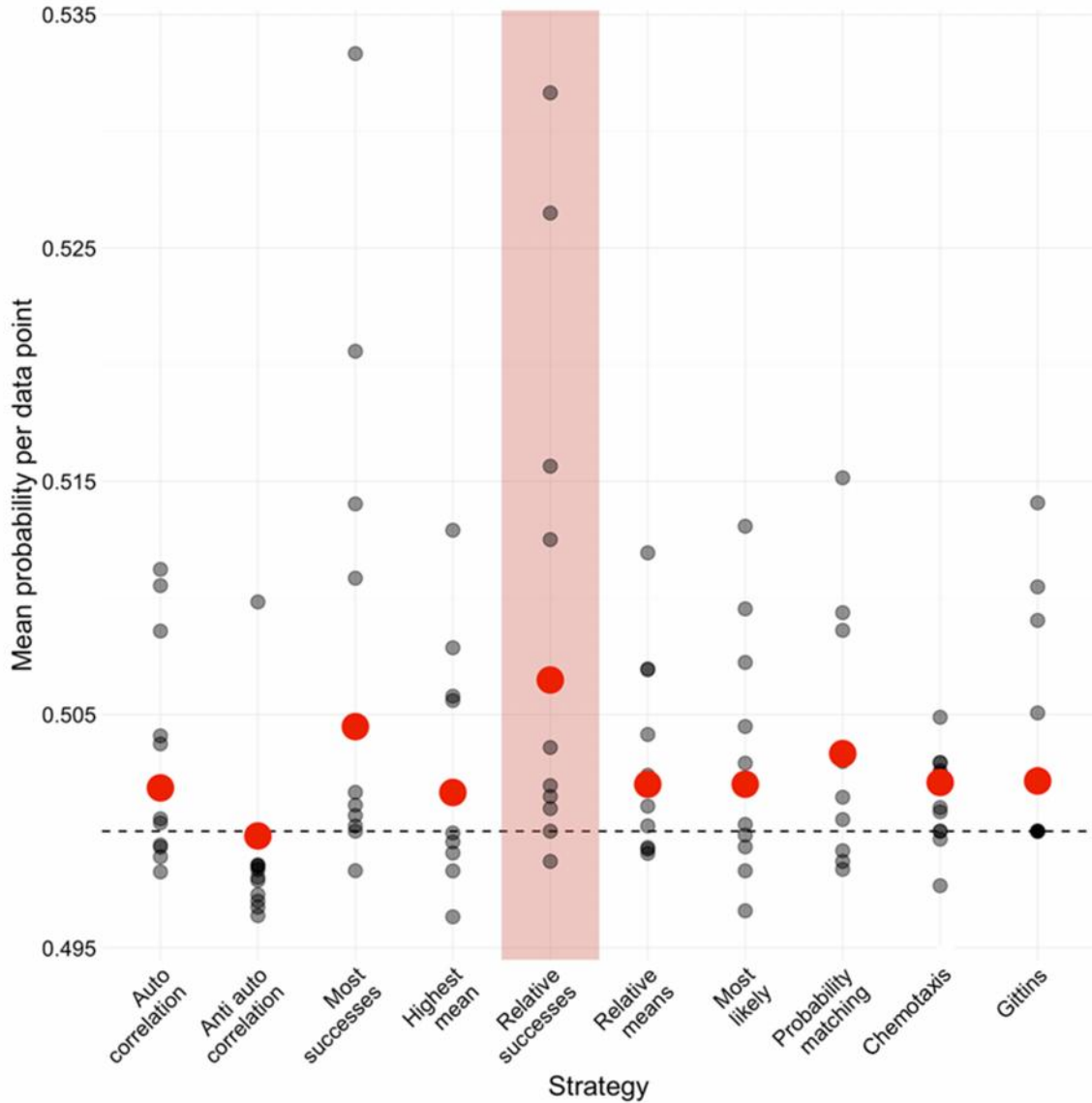


112 and therefore the evidence is clearly decisively in favor of this model from among those  
113 we have tested.

114         The Gittins algorithm contains a parameter, gamma, which controls the discount  
115 on possible future rewards. This is unique to the Gittins model and is not included in any  
116 of our heuristics. We wanted to be certain that the observed decisions of the cells could  
117 not be the result of applying the optimal Gittins algorithm, rather than one of our heuristic  
118 models. As such we gave the Gittins model (model 9) the freedom to select a discount  
119 parameter that best-fitted the observed decisions, rather than performing a Bayesian  
120 marginalization. Thus we were able to show that no set of parameters for the Gittins  
121 algorithm were able to predict the observed decisions better than our best-performing  
122 heuristic, 'Relative Successes'.

123         The results in Fig. S3 represent marginal likelihood evaluated on the basis of a  
124 uniform prior belief attributed to the cell; that is we assumed that before it observed any  
125 information about the food distribution the cell 'believes' that the food density is equally  
126 likely to be anywhere between 0 and 1. We relaxed this assumption by testing a wide  
127 range of other initial beliefs and found in all cases that the ordering of the models was  
128 unaffected. In particular 'Relative Successes' continued to outperform all other models  
129 by a large margin.

130



131

132 Fig. S3: Bayesian comparison between the proposed models. After calculating the  
 133 marginal-likelihood of the observed decisions for each model, we normalized by the  
 134 number of decisions to obtain the probability-per-decision. A value of 0.5 indicates the  
 135 equivalent of a random prediction, thus values above 0.5 perform better than the  
 136 'Random' model. Grey points indicate the results from the eleven different treatments for  
 137 each model, while the larger red points indicate the values from all of the treatments  
 138 combined. Overall the highlighted 'Relative Successes' model is strongly favored.

139 **Relative Performance of Relative Successes Heuristic**

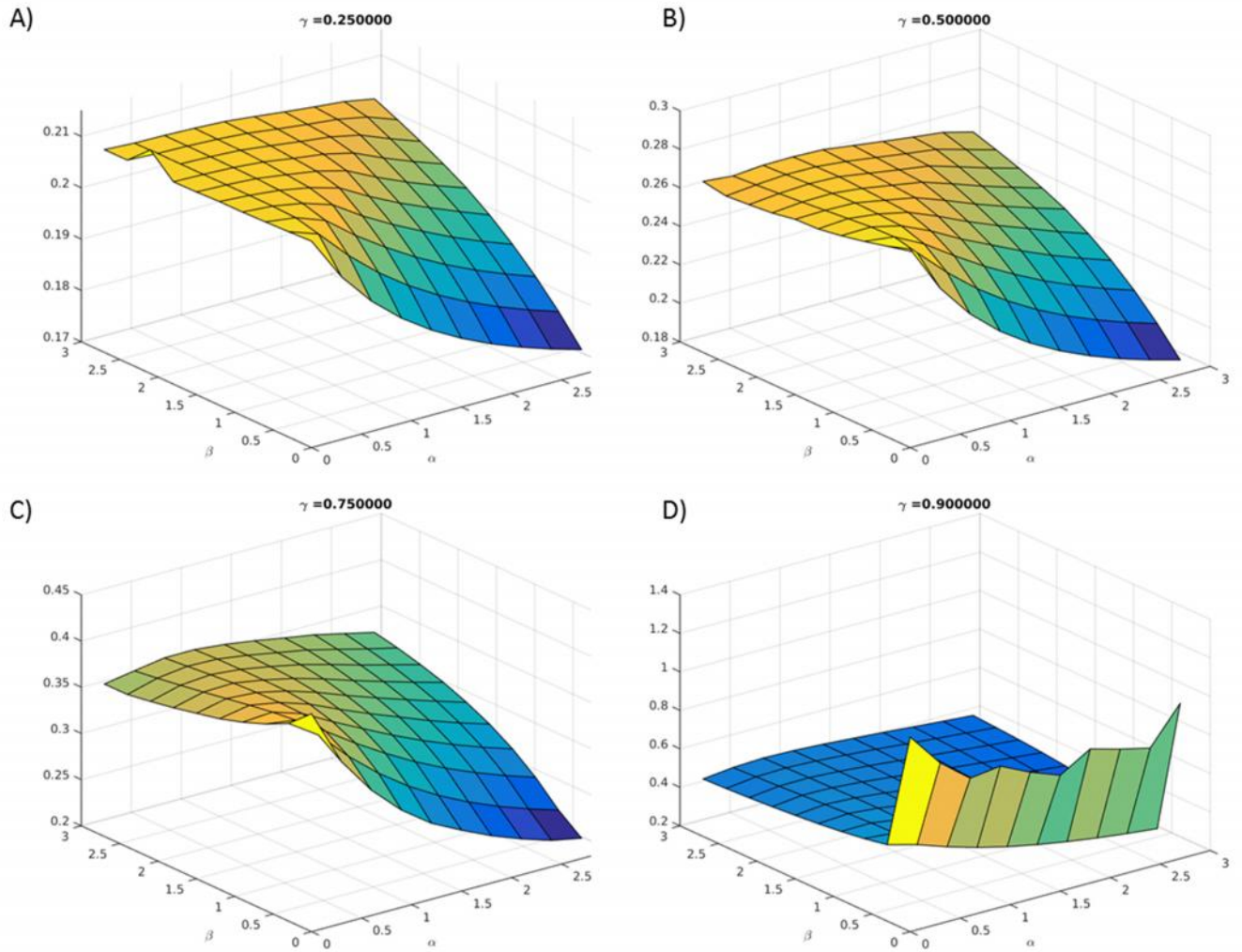
140 To benchmark the performance of the relative successes heuristic we compared its  
141 expected performance against the expected performance of random choice, and of the  
142 optimal Gittins index policy, for a variety of hyperparameters of the beta-distributed  
143 Bayesian prior (  $\alpha$  and  $\beta$  ), and discount rate over future rewards ( $1 - \gamma$  ), according to the  
144 equation;

145

146 
$$\text{relative performance} = (\text{heuristic performance} - \text{random performance}) / (\text{Gittins}$$
  
147 
$$\text{performance} - \text{random performance}),$$

148

149 where we assume that the decision maker is equipped with the correct prior for the  
150 environment it is situated in. Sample relative performances are plotted in Fig. S4, which  
151 shows that as discount rate decreases (  $\gamma$  increases and future rewards become more  
152 important) the relative successes heuristic becomes closer to optimal for a larger range of  
153 possible priors. At  $\gamma = 0.9$  the relative successes heuristic's performance is half way  
154 between random performance and the maximum possible performance as realized by the  
155 Gittins index strategy for almost all priors studied.

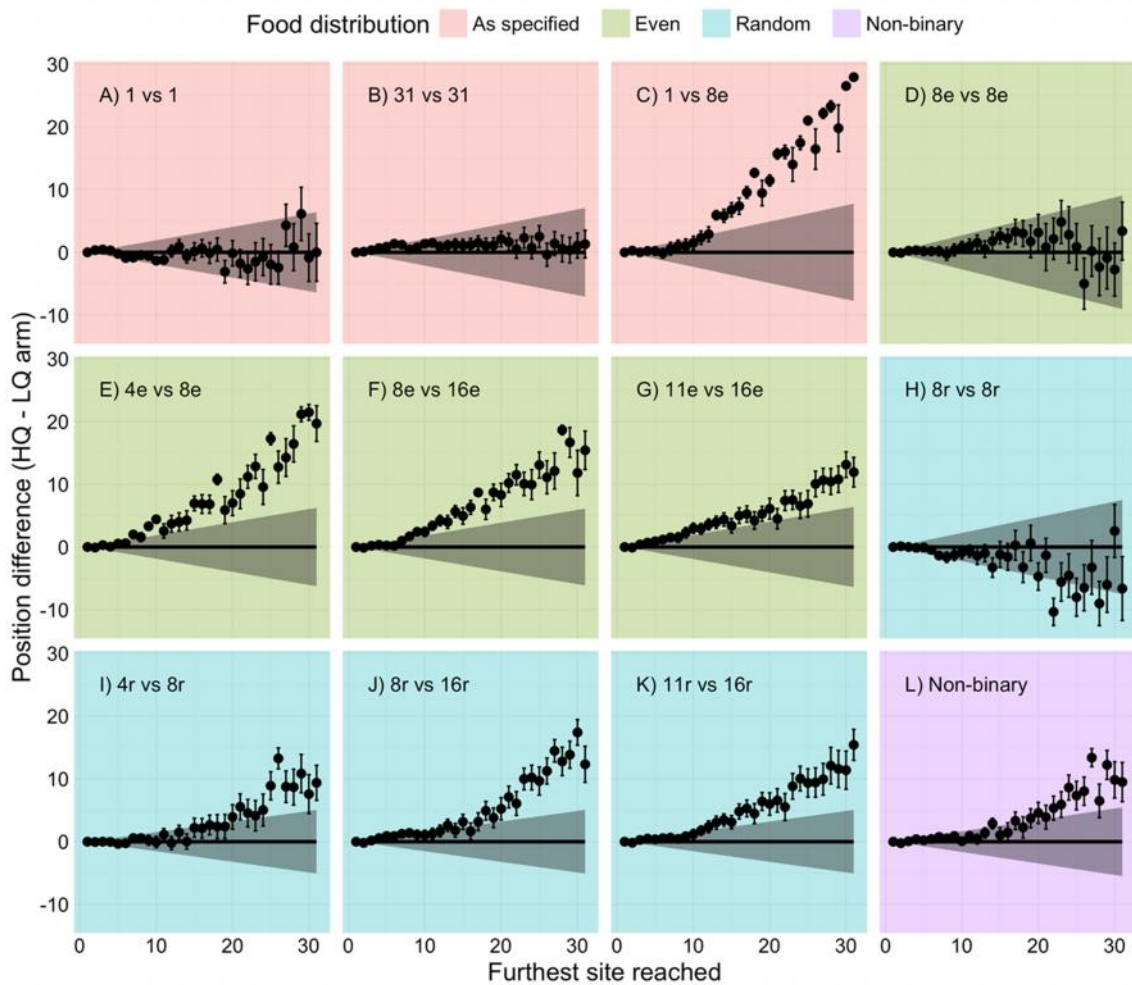


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157 Fig. S4: Sensitivity of best-fitting heuristic's relative performance (0 = random  
 158 performance, 1 = optimal performance calculated according to Gittins index policy) to  
 159 variation in hyperparameters of the beta-distribution describing food distribution ( $\alpha$  and  
 160  $\beta$ ) and discount rate applied to future rewards ( $\gamma$ ). As the future becomes more important  
 161 ( $\gamma$  increases) performance of the best-fitting heuristic relative to the theoretical optimum  
 162 increases. With the exception of  $\gamma=0.9$  highest relative performance is achieved when  
 163  $\alpha > \beta$ , which corresponds to food distributions in which each foraging step is more likely  
 164 to result in no food than in the discovery of food, suggesting the ancestral environments  
 165 in which the heuristic evolved may have had this feature.

166 **Relative Performance of Other Heuristics**

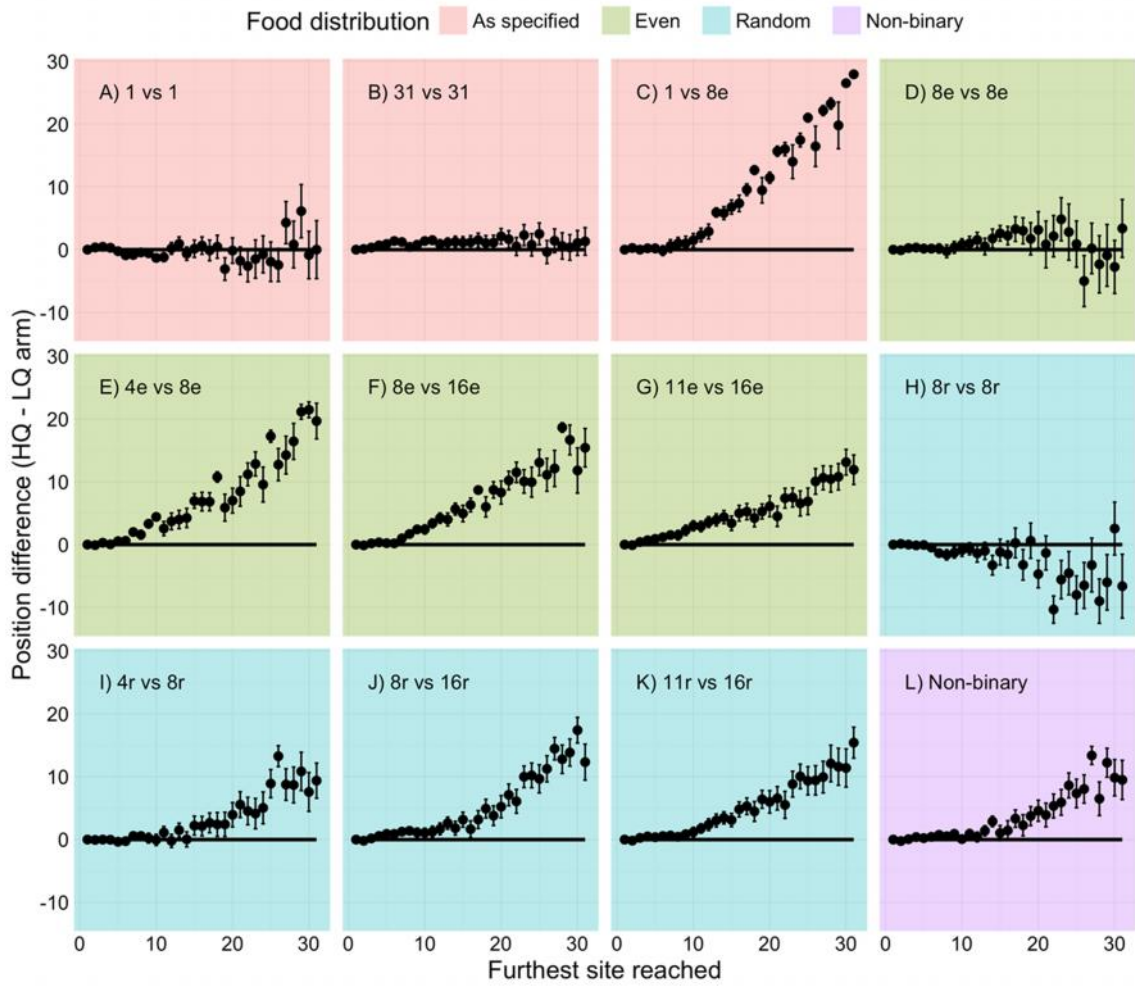
167 The following figures show difference in site discovery between HQ and LQ arms  
168 for the first time each site was discovered on either arm in each choice scenario (as in  
169 Fig. 2). Filled circles are experimental data means, error bars are the 95% confidence  
170 intervals. Solid lines are the pattern of site discovery predicted by particular heuristic,  
171 shaded regions are 1.96 standard errors.



172

173 Fig. S5: Relative performance of 'Autocorrelation' heuristic.

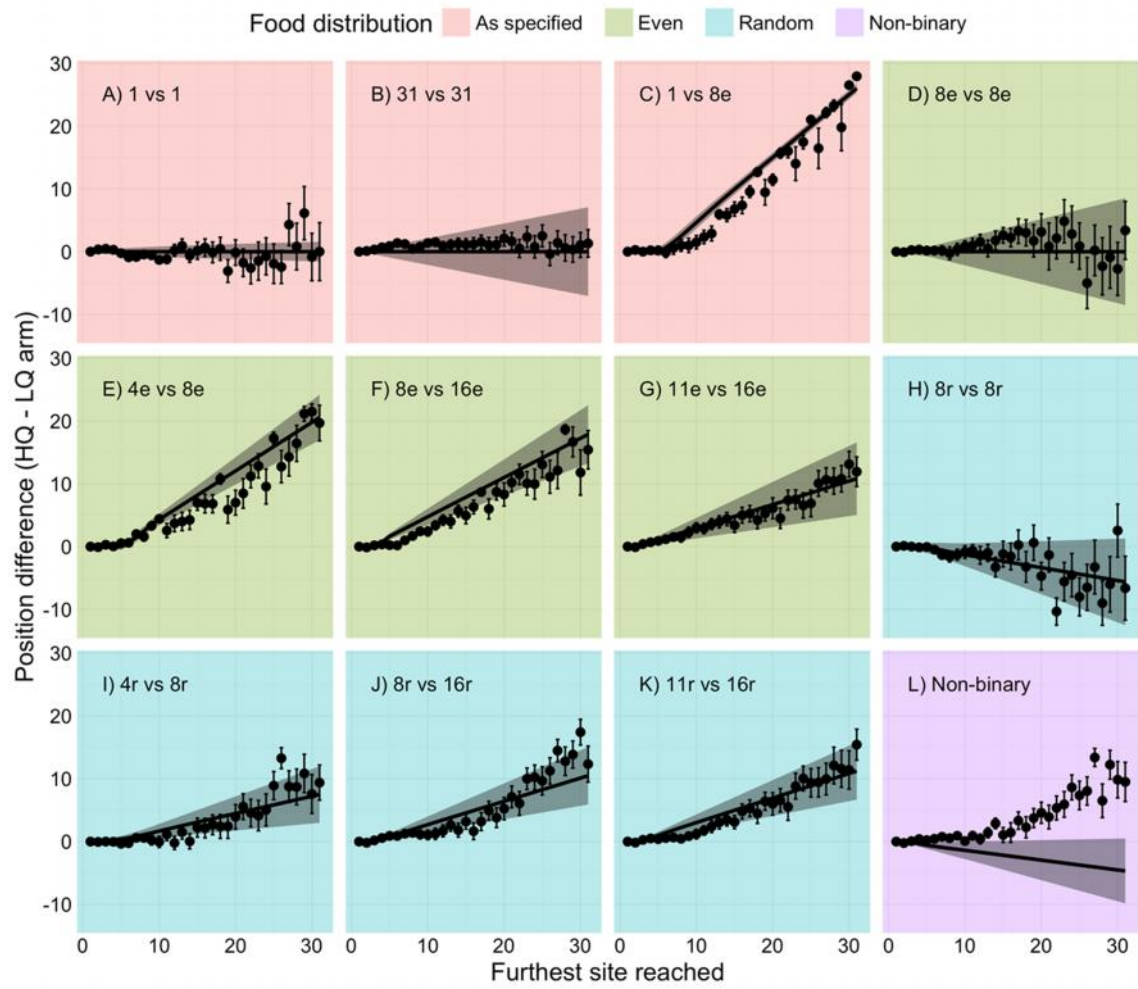
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176 Fig. S6: Relative performance of 'Anti-autocorrelation' heuristic.

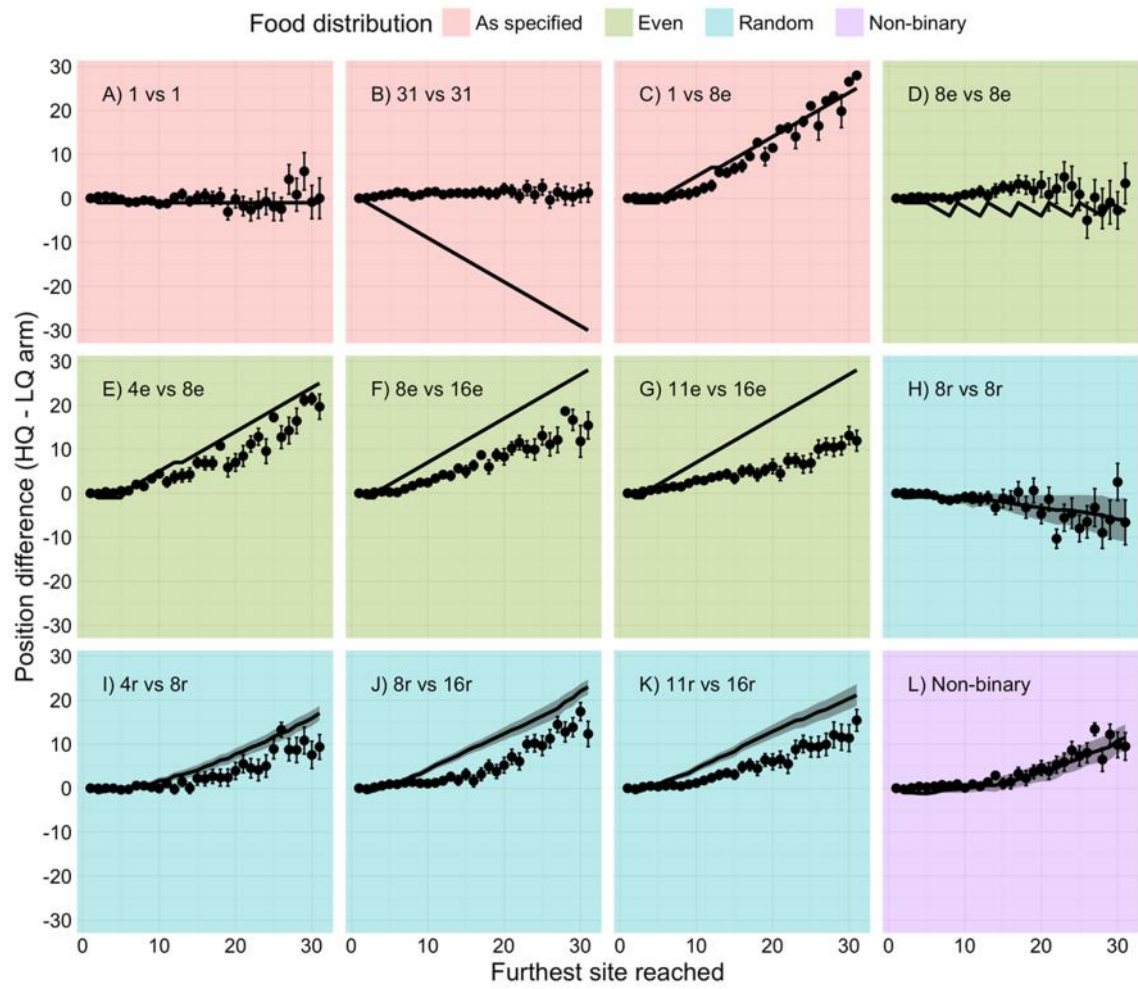
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179 Fig. S7: Relative performance of 'Most successes' heuristic.

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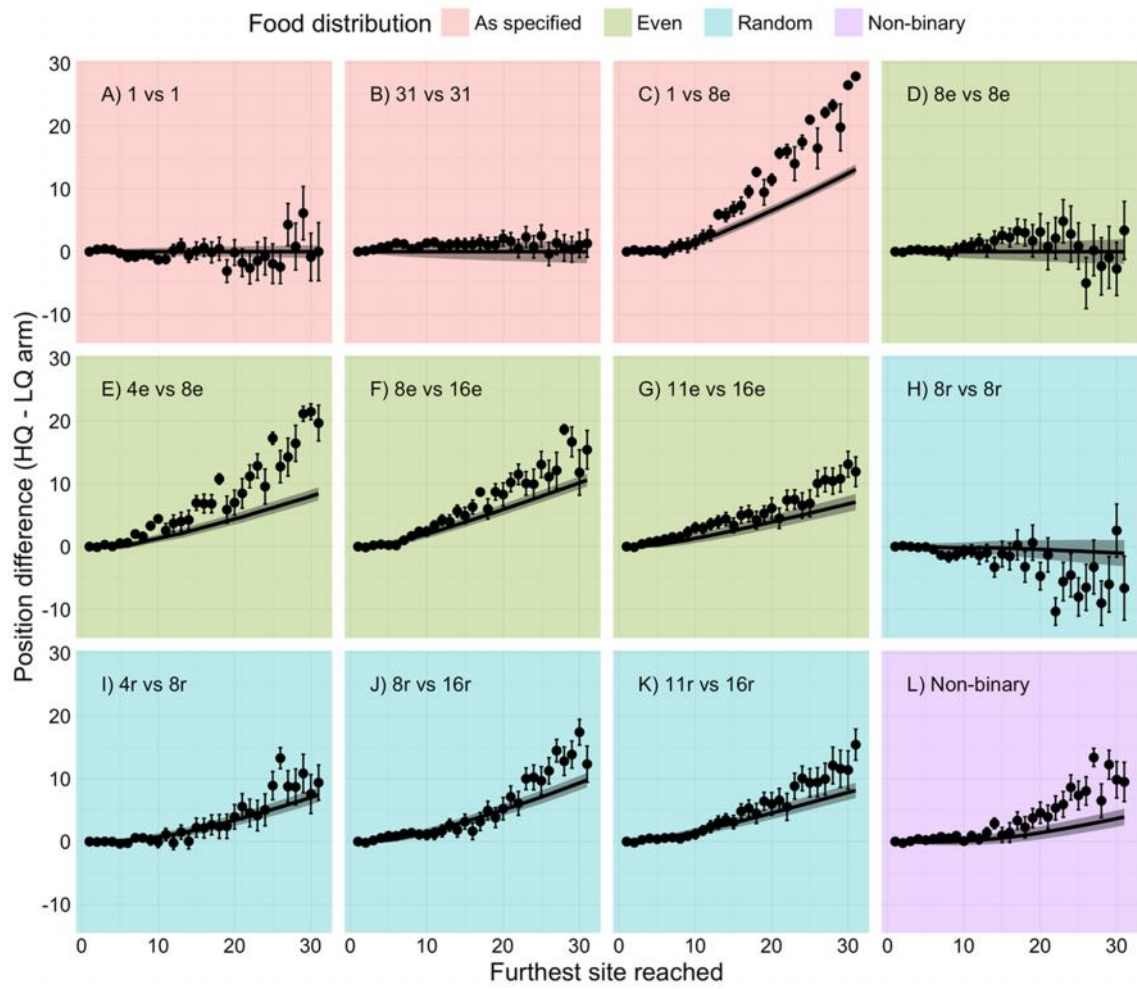


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182 Fig. S8: Relative performance of 'Highest mean' heuristic.

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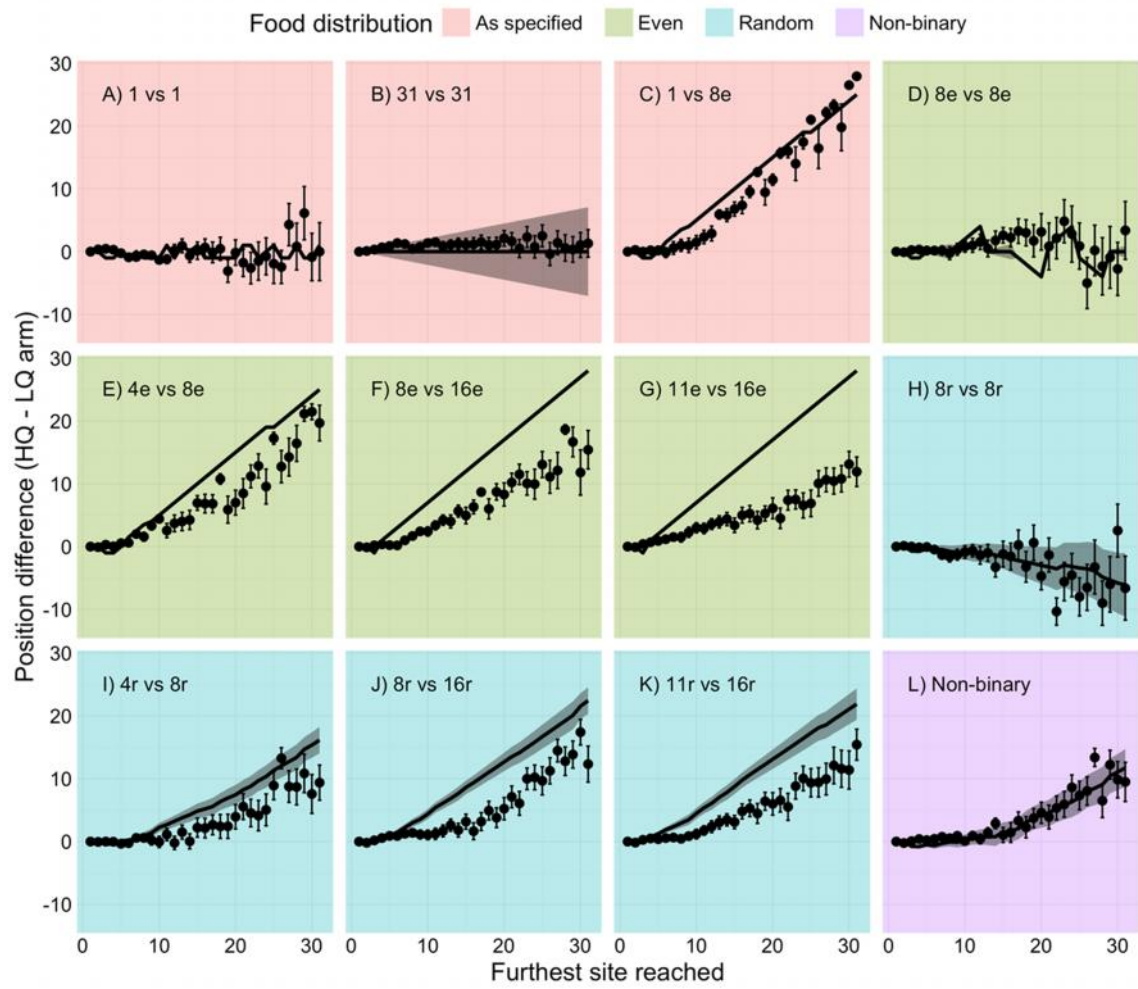




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185 Fig. S9: Relative performance of 'Relative means' heuristic.

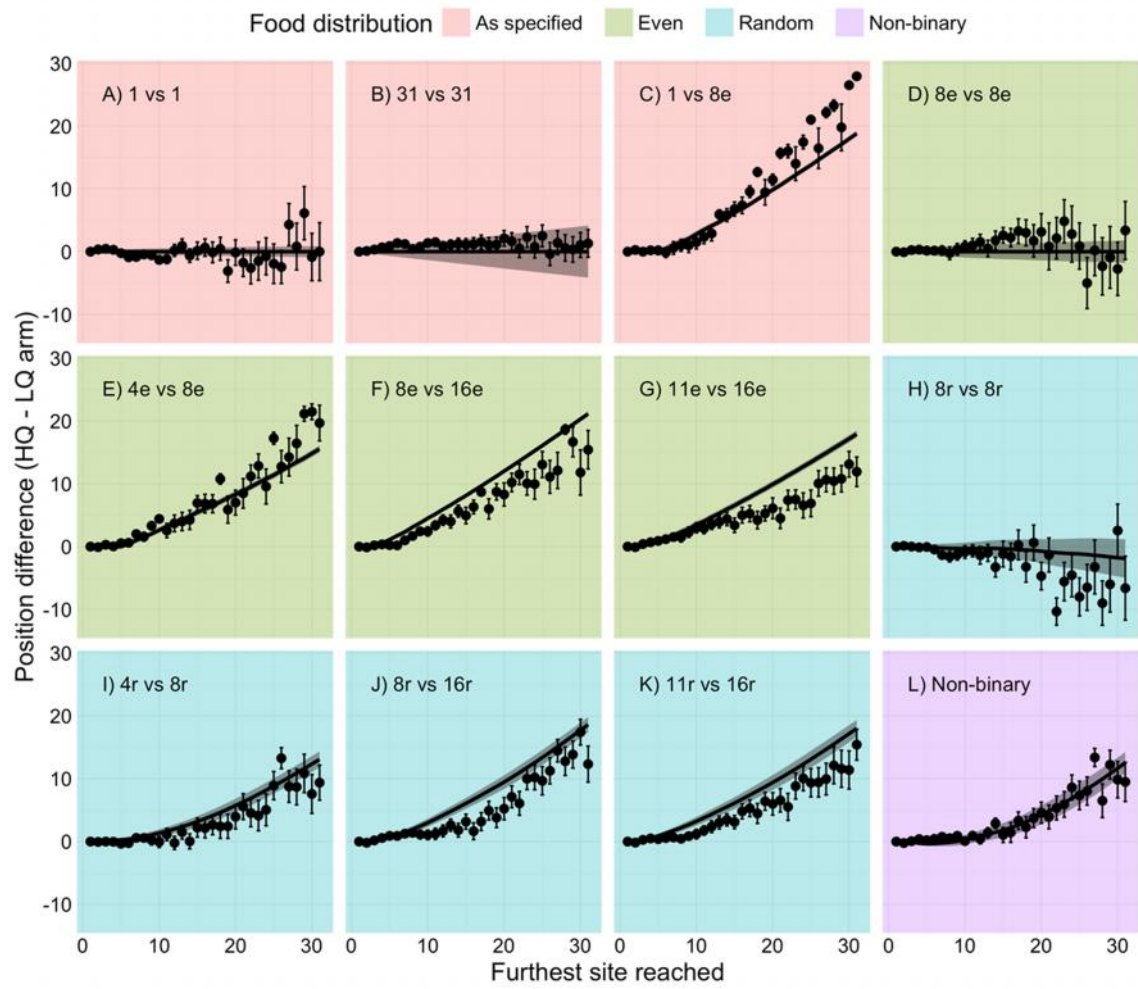
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188 Fig. S10: Relative performance of 'Most likely' heuristic.

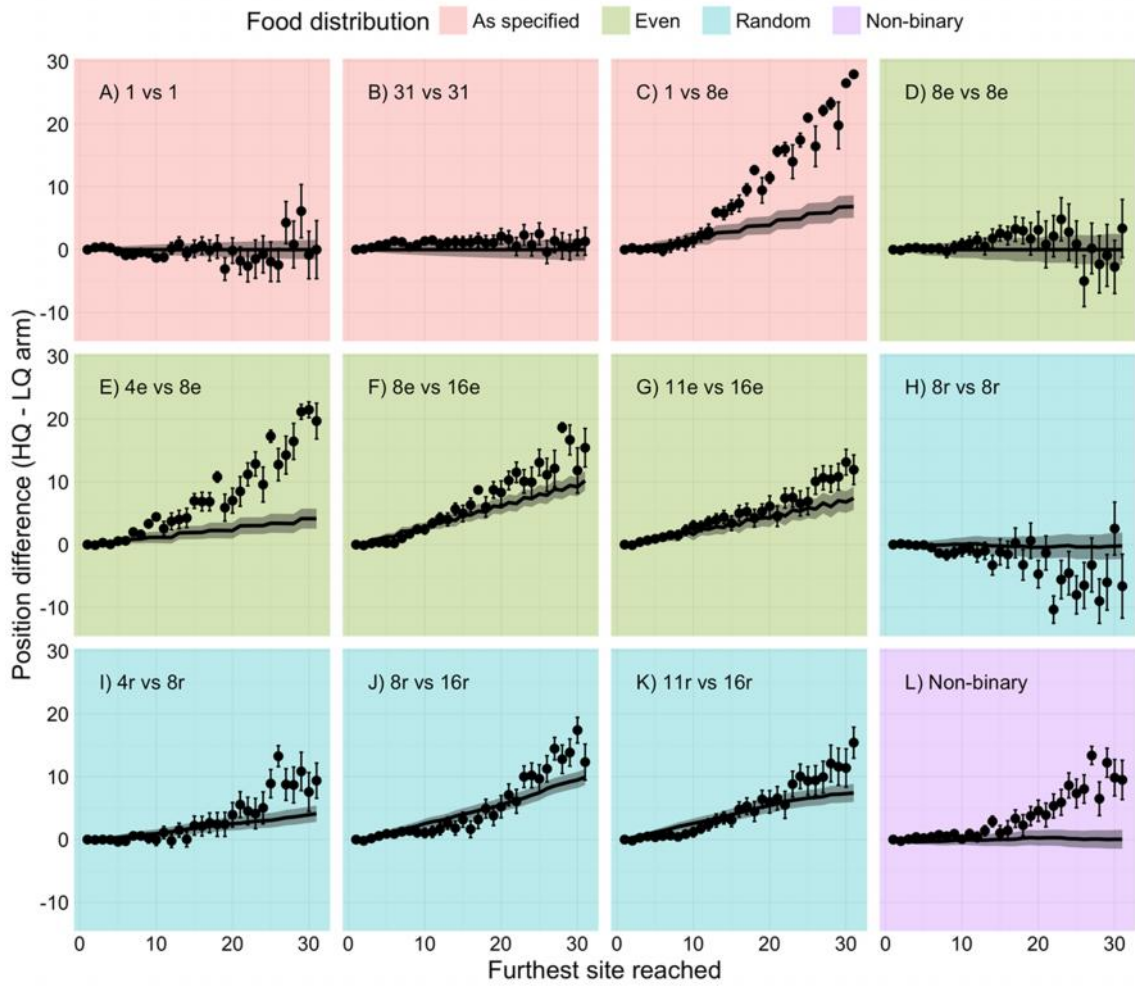
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191 Fig. S11: Relative performance of 'Probability matching' heuristic.

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193

194 Fig. S12: Relative performance of ‘‘Chemotaxis’ heuristic.

195

196 References

197 1. Jeffreys H. Theory of Probability. Oxford: Oxford University Press; 1939.