Definition: In this note the region V in the (u, v)-plane is the region shown in Figure 6(a): bounded by the two null clines and to the left of $u = 1$.

Lemma 1 *Once the orbit enters the region V, then eventually, $v < 0$.*

Proof: Since $C(t) \to 0$ very quickly (cf. Figure 5), we assume throughout that $C = 0$.

Suppose that at $\tau = \tau_0$ the orbit has entered the region V and suppose to the contrary that $v(\tau) > 0$ for all $\tau > \tau_0$. Then, because orbits enter V from the two null clines: Γ_u above and Γ_v below, the orbit can not leave the region V^+ in V that lies in the upper half plane $\{v > 0\}$.

In V, and hence in V^+, the vector field point down and to the left. Therefore

(i) $v(\tau)$ is a decreasing function. Since it is bounded below it tends to a nonnegative limit, say v_∞, and $dv/d\tau \to 0$ as $\tau \to \infty$.

(ii) $u(\tau)$ is also a decreasing function: either it is bounded below, and tends to some limit $u_\infty < 1$ and $du/d\tau \to 0$ as $\tau \to \infty$, or it tends to $-\infty$.

If $u(\tau) \to u_\infty$, then the orbit converges to the point (u_∞, v_∞) as $\tau \to \infty$ and hence, (u_∞, v_∞) must be an equilibrium point in V and $u_\infty < 1$. However, the system has no such equilibrium points. This implies that $u(\tau) \to -\infty$ as $\tau \to \infty$.

It follows from equation (2.10) that, once $u < 0$, then, because $\psi(\tau) > 1$,

$$\frac{dv}{d\tau} = \psi(\tau) \cdot u - v < -u$$

Hence $dv/d\tau \to -\infty$ as $\tau \to \infty$, which contradicts the observation under (i) that $dv/d\tau \to 0$ as $\tau \to \infty$. Therefore the orbit must break though the u-axis, and v must become negative in finite time. $\square$