SUPPORTING INFORMATION

Multiscale Mechano-biological Finite Element Modelling of Oncoplastic Breast Surgery – Numerical Study Towards Surgical Planning and Cosmetic Outcome Prediction

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Implementation of the Multiscale Mechano-biological FE Framework

The proposed multiscale mechano-biological Finite Element (FE) framework has been incorporated into the existing in-house numerical analysis framework *FEB3* – pronounced *Phoebe* – which is freely available upon request from: `https://bitbucket.org/vasvav/feb3-finite-element-bioengineering-in-3d/wiki/Home`. The FE computational framework has been implemented in an object-oriented, scalable, C++ code that incorporates several open-source numerical libraries:

- *blitz++*[1] is an meta-template library in C++ which was utilised in *FEB3* for tensor algebra and multi-dimensional tensor manipulation [1].
- *GNU Scientific Library* is an ANSI-C library that contains an wide range of mathematical routines (over 1000 in total) such as random number generators, special functions, statistics, numerical differentiation, data fitting, etc. (see online documentation: `https://www.gnu.org/software/gsl/manual/html_node/`) [2].
- *METIS*[2] and *ParMETIS*[3] is a pair of libraries containing established algorithms for partitioning graphs, partitioning finite element meshes and producing fill-reducing orderings for sparse matrices, in serial and in parallel computing respectively [3].
- *MPICH*[4] is a standardized and portable message-passing system which is a communication protocol for programming parallel computers.
- *PETSc*[5] is a suite of data structures and routines for the solution of scientific applications, and is been used within *FEB3* in solving linear and nonlinear systems [4, 5]. *PETSc* is also integrated with *MPICH* to facilitate parallel computations, while it communicates with *METIS* and *ParMETIS* for sparse system partitioning.
- *libMesh*[6] [6] is the top-level library used by *FEB3*. *libMesh* is an object-oriented C++ framework for the numerical simulation of partial differential equations using arbitrary unstructured discretisations on serial and parallel platforms, while it integrates with high-performance computing libraries such as *PETSc*.

[1] `http://sourceforge.net/projects/blitz/`
[2] `http://glaros.dtc.umn.edu/gkhome/metis/metis/overview`
[3] `http://glaros.dtc.umn.edu/gkhome/metis/parmetis/overview`
[4] `www.mpich.org`
[5] `http://www.mcs.anl.gov/petsc/`
[6] `http://libmesh.github.io/`

References

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