# **Correcting for Measurement Error in Time-Varying Covariates in Marginal Structural Models**

# Web Table 1: Description of indirect/direct approaches for applying SIMEX to MSMs in single time interval setting

 $\lambda$  = A vector of values used to control artificial error, e.g. {0.5, 1.0, 1.5, 2.0}

- X = A binary indicator variable of exposure status
- Y = A continuous response variable
- L = An error-free ("true") continuous time-varying covariate
- $L^*$  = An error-prone ("observed") continuous time-varying covariate, i.e.  $L^* = L + error$
- $\epsilon$  = Random error
- $\sigma_{\delta}^2$  = Variance of measurement error

Treatment model: logit  $\left(P\left(X=1|L^*\right)\right) = \alpha_0 + \alpha_1 L^*$  Outcome model:  $Y = \beta_0 + \beta_1 X + \varepsilon$ 

Note: We describe a single time interval setting; the same general SIMEX approach is appropriate given two or more intervals.

i. Direct Approach

- 1) Add additional (artificial) error to time-varying covariate  $L^*$  from normal distribution with mean of zero and variance  $\lambda \times \sigma_{\delta}^2$ . Compute IPT-weighted estimates using  $L^*$  with additional error. Estimate the MSM parameter  $\hat{\beta}$ .
- 2) Repeat step (1) a total of *B* times, and then average all *B* estimates to obtain  $\hat{\beta}(\lambda)$ .
- 3) Repeat steps (1) and (2) for all values of  $\lambda$ .
- 4) Fit chosen regression function  $f(\hat{\beta}(\lambda))$ , then extrapolate back to  $\lambda = -1$  to obtain corrected  $\overline{\hat{\beta}}^{SIMEX}(\lambda = -1)$ .

## ii. Indirect Approach

- 1) Add additional error to L\* from normal distribution with  $\mu = 0$  and variance =  $\lambda \times \sigma_{\delta}^2$ . Estimate treatment model parameter  $\hat{\alpha}$ .
- 2) Repeat step (1) *B* times, and average all *B* estimates to obtain  $\overline{\hat{\alpha}}(\lambda)$ .
- 3) Repeat steps (1) and (2) for all values of  $\lambda$ . Compute  $\overline{\hat{\alpha}}(\lambda)$  and obtain  $\overline{\hat{\alpha}}^{SIMEX}(\lambda = -1)$ .
- 4) Fit chosen regression function  $f(\hat{\alpha}(\lambda))$ , then extrapolate back to  $\lambda = -1$  to obtain corrected  $\overline{\hat{\alpha}}^{SIMEX}(\lambda = -1)$ .
- 5) Use the corrected treatment parameter estimates obtained in step 4, to compute  $\hat{w}^{SIMEX}$  as  $logit^{-1}(\overline{\hat{\alpha}_0}^{SIMEX} + \overline{\hat{\alpha}_1}^{SIMEX}L^*)$  for the

treated, and  $\left(1-\text{logit}\left(\overline{\hat{\alpha}_0}^{SIMEX} + \overline{\hat{\alpha}_1}^{SIMEX}L^*\right)\right)^{-1}$  for the untreated (stabilizing if desired).

6) Use  $\hat{w}^{SIMEX}$  as the weights in a regression model to obtain MSM parameter estimates.

# Data-generating model for primary studies

i. One	-interval study (corresponds to Fi	gure 1a)	ii. Two	p-interval study (corresponds to Figure 1b) <sup><math>\ddagger</math></sup>					
L	~ Uniform[-2, 3]		$L_1$	~ Uniform[-2, 3]					
L*	~ L + Norm(0, $\sigma_{\delta}^2$ )		L <sub>1</sub> *	~ $L_1$ + Norm(0, $\sigma_{\delta}^2$ )					
X	~ Bern < logit $(0.25 + \alpha_1 L)$		$X_1$	~ Bern < logit $(0.25 + \alpha_1 L_1)$					
	where $\alpha_1 \in \{0, 25\}$ .	0.50. 0.75}		where $\alpha_1 \in \{0.50, 0.75\}$					
Y	$\sim 0 + 1.0X + 1.25L + Norm(0, 1)$	, <b>,</b>	$L_2$	$\sim 0.3L_1 + 1.25X_1 + Norm(0, 1)$					
	( · · )		$L_2^* \sim L_2 + Norm(0, \sigma_s^2)$						
			$X_2$	~ Bern < logit <sup>-1</sup> (0.25 + $\alpha_1 L_2$ -1.25X <sub>1</sub> )					
				where $\alpha_1 \in \{0.50, 0.75\}$					
			Y	$\sim 0 + 1.0X_1 + 1.25X_2 + 0.8L_1 + 0.3L_2 + Norm(0, 1)$					
Sumn	nary of fitted models								
i. One	-interval study	First interval model	for X						
	Treatment (denominator)	$logit(P(X = 1 L^*)) = c$	$\alpha_0 + \alpha_1 L$	*					
	Treatment (numerator)	1	<b>o</b> .						
	Outcome	$Y = \beta_0 + \beta_1 X + \varepsilon, \varepsilon \sim$	N(0,1)						
		· ·							
ii. Two	o-interval study <sup>‡</sup>	First interval model	for $X_1$	Second interval model for $X_2$					
	Treatment (denominator)	$logit(P(X_1 = 1 L_1^*)) =$	$= \alpha_0 + \alpha_1$	$_{1}L_{1}^{*}$ logit(P(X <sub>2</sub> = 1 X <sub>1</sub> + L <sub>2</sub> <sup>*</sup> )) =					
				$\alpha_0 + \alpha_1 X_1 + \alpha_2 L_2^*$					
	Treatment (numerator)	$P(X_1 = 1) = Mean(X)$	1)	logit(P( $X_2 = 1   X_1$ )) = $\alpha_0 + \alpha_1 X_1^*$					
	Outcome	$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$	, <u>2</u> + ղ, ղ	~ N(0,1)					

<sup>&</sup>lt;sup>+</sup> For the secondary study (scenario corresponding to Figure 1c) in which exposure depends upon the error-prone covariate,  $X_1$  was generated using  $L_1^*$  (instead of  $L_1$ ), and  $X_2$  depended on  $L_2^*$  (instead of  $L_2$ ).

#### Web Appendix 1: Reasoning for extrapolation to $\lambda = -1$

In the presence of additive error, the variance of the observed covariate  $L^*$  is equivalent to the sum of the variances of the true covariate L and its measurement error. The expression below illustrates this relationship (measurement error denoted by  $\delta$ ):

$$Var(L^*) = \sigma_L^2 + \sigma_\delta^2 + (\lambda \times \sigma_\delta^2)$$

It can be seen that this introduces bias most plainly in the linear regression setting, where the estimator is available in closed form.

Consider, for example, a simple linear regression of Y on  $L^*$ ; the unbiased regression parameter estimates are given by  $\theta = Cov(Y,L)/Var(L)$  but, in the presence of measurement error, estimated by the empirical covariance of Y and  $L^*$  (which is consistent for Cov(Y,L)) divided by the empirical variance of  $L^*$ , which is larger than the variance of L (by exactly  $\sigma_{\delta}^2$ ). Thus, substituting -1 for the value of  $\lambda$  will cancel out the two measurement error variance terms, and the variance of the error-prone covariate is now equivalent to that of the true (unobserved) covariate.

Consequently, extrapolating  $f(\lambda)$  to -1 yields the SIMEX estimator for the covariate in the absence of measurement error. While this result cannot be shown in closed form for estimators outside a linear regression setting, SIMEX has been used extensively in a variety of other applications (1-5).

#### Web Appendix 2: R code for direct and indirect SIMEX correction

```
# Generate B datasets for SIMEX adjustment in apply simex function
# (requires foreach package)
simex data <- foreach(i = sqrt(lambda)) %do% replicate(B,</pre>
            transform(indata, Lstar1 = Lstar1 + rnorm(n = nrow(indata),
                 mean = 0, sd = i * specerr), Lstar2 = Lstar2 +
                 rnorm(n = nrow(indata), mean = 0, sd = i * specerr)))
# Apply both direct and indirect SIMEX methods to address measurement error
# N.B. Implementation of the indirect SIMEX using R's simex package is far
#
       more convenient; however, we wished to obtain direct SIMEX results,
#
       while reducing the execution time. For reference, our code is included
       below.
#
apply simex <- function (simex data, lambda, B)
{
    numlam <- length(lambda)</pre>
    outcome model params <- replicate (numlam, list (matrix (data = NA,
        ncol = B, nrow = 3)))
    treatment t1 params <- replicate(numlam, list(matrix(data = NA,</pre>
        ncol = B, nrow = 2)))
    treatment t2 params <- replicate(numlam, list(matrix(data = NA,</pre>
        ncol = B, nrow = 3)))
    y x1mat <- simex data[[1]][["X1", 1]]</pre>
    y x2mat <- simex data[[1]][["X2", 1]]</pre>
    y ymat <- simex data[[1]][["Y", 1]]</pre>
    x_x <- cbind(1, X1 = simex_data[[1]][["X1", 1]], X2 = simex_data[[1]][["X2", 1]])</pre>
    for (i in 1:numlam) {
        for (j in 1:B) {
            x lstar1 <- cbind(1, Lstar1 = simex data[[i]][[c("Lstar1"),</pre>
                 i]])
            x lstar2n <- cbind(1, X1 = simex data[[i]][["X1",</pre>
                 j]))
            x lstar2 <- cbind(1, X1 = simex data[[i]][["X1",</pre>
                 j]], Lstar2 = simex data[[i]][["Lstar2", j]])
            model err1 <- glm.fit(x = x lstar1, y = y x1mat,</pre>
```

```
family = binomial())
        pred.model err1 <- ifelse(y x1mat == 1, model err1$fitted.values,</pre>
             1 - model err1$fitted.values)
        mod werr1 <- (ifelse(simex data[[i]][["X1", j]] ==</pre>
             1, (mean(simex data[[i]][["X1", j]] == 1))/(pred.model err1),
             (1 - mean(simex data[[i]][["X1", j]] == 1))/(pred.model err1)))
        model err2n <- glm.fit(x = x lstar2n, y = y x2mat,</pre>
             family = binomial())
        model err2 <- glm.fit(x = x lstar2, y = y x2mat,</pre>
             family = binomial())
        pred.model err2n <- ifelse(y x2mat == 1, model err2n$fitted.values,</pre>
             1 - model err2n$fitted.values)
        pred.model err2 <- ifelse(y x2mat == 1, model err2$fitted.values,</pre>
             1 - model err2$fitted.values)
        mod werr2 <- (pred.model err2n/(pred.model err2))</pre>
        comb werr <- mod werr1 * mod werr2</pre>
        model werr <- lm.wfit(x = x x, y = y ymat, w = comb werr)</pre>
        outcome model params[[i]][, j] <- model werr$coeff</pre>
        treatment t1 params[[i]][, j] <- model err1$coeff</pre>
        treatment t2 params[[i]][, j] <- model err2$coeff</pre>
    }
lambda matrix <- cbind(1, lambda + 1, ((lambda + 1)^2))
simex dir coefs list <- lapply(outcome model params, rowMeans)</pre>
simex dir coefs mat <- do.call(rbind, simex dir coefs list)</pre>
simex dir int <- simex dir coefs mat[, 1]</pre>
simex dir x1 <- simex dir coefs mat[, 2]</pre>
simex dir x2 <- simex dir coefs mat[, 3]</pre>
simex dir int adj <- lm.fit(x = lambda matrix, y = simex dir int)
simex dir x1 adj <- lm.fit(x = lambda matrix, y = simex dir x1)
simex dir x2 adj <- lm.fit(x = lambda matrix, y = simex dir x2)
simex ind t1 coefs list <- lapply(treatment t1 params, rowMeans)
simex ind t1 coefs mat <- do.call(rbind, simex ind t1 coefs list)
simex ind t1 int <- simex ind t1 coefs mat[, 1]</pre>
simex ind t1 Lstar1 <- simex ind t1 coefs mat[, 2]</pre>
simex ind t2 coefs list <- lapply(treatment t2 params, rowMeans)</pre>
simex ind t2 coefs mat <- do.call(rbind, simex ind t2 coefs list)</pre>
simex ind t2 int <- simex ind t2 coefs mat[, 1]</pre>
```

}

```
simex_ind_t2_x1 <- simex_ind_t2_coefs_mat[, 2]</pre>
    simex ind t2 Lstar2 <- simex ind t2 coefs mat[, 3]</pre>
    simex ind t1 int adj <- lm.fit(x = lambda matrix, y = simex ind t1 int)
    simex ind t1 Lstar1 adj <- lm.fit(x = lambda matrix, y = simex ind t1 Lstar1)
    simex ind t2 int adj <- lm.fit(x = lambda matrix, y = simex ind t2 int)
    simex ind t2 x1 adj <- lm.fit(x = lambda matrix, y = simex ind t2 x1)
    simex ind t2 Lstar2 adj <- lm.fit(x = lambda matrix, y = simex ind t2 Lstar2)
    x lstar1 <- cbind(1, Lstar1 = simex data[[1]][[c("Lstar1"),</pre>
        1]])
    x lstar2 <- cbind(1, X1 = simex data[[1]][["X1", 1]], Lstar2 =</pre>
simex data[[1]][["Lstar2",
        1]])
    x lstar2n <- cbind(1, X1 = simex data[[1]][["X1", 1]])</pre>
    corrected.wts1 <- ifelse(y x1mat == 1, (mean(simex data[[1]][["X1",</pre>
        1]] == 1))/inv.logit(x lstar1 %*% rbind(simex ind t1 int adj$coeff[1],
        simex ind t1 Lstar1 adj$coeff[1])), (1 - mean(simex data[[1]][["X1",
        1]] == 1))/(1 - inv.logit(x lstar1 %*% rbind(simex ind t1 int adj$coeff[1],
        simex ind t1 Lstar1 adj$coeff[1]))))
    cw2.numerator <- ifelse(y x2mat == 1, model err2n$fitted.values,</pre>
        1 - model err2n$fitted.values)
    cw2.denominator <- ifelse(y x2mat == 1, inv.logit(x lstar2 %*%</pre>
        rbind(simex ind t2 int adj$coeff[1], simex ind t2 x1 adj$coeff[1],
            simex ind t2 Lstar2 adj$coeff[1])), 1 - (inv.logit(x lstar2 %*%
        rbind(simex ind t2 int adj$coeff[1], simex ind t2 x1 adj$coeff[1],
            simex ind t2 Lstar2 adj$coeff[1]))))
    corrected.wts2 <- cw2.numerator/cw2.denominator</pre>
    comb werr <- corrected.wts1 * corrected.wts2</pre>
    ind msm <- lm.wfit(x = x x, y = y ymat, w = comb werr)</pre>
    return(list(int dir = simex dir int adj$coeff[1], x1 dir =
simex dir x1 adj$coeff[1],
        x2 dir = simex dir x2 adj$coeff[1], int ind = ind msm$coeff[1],
        x1 ind = ind msm$coeff[2], x2 ind = ind msm$coeff[3]))
```

```
}
```

## Web Appendix 3: Additional analyses comparing direct and indirect SIMEX

Because there is no closed form solution for MSM estimators found using a SIMEX correction, whether direct or indirect, a theoretical proof of unbiasedness is infeasible. This is the case for adaptations of SIMEX to generalized linear models (3), non-linear regression models (2), quantile regression models (5), accelerated failure time models (9), and generalized linear mixed models (1). We describe additional analyses we conducted to explore and compare performance of both SIMEX approaches (Web Figures 1 and 2), and our choice of extrapolant function (Web Figure 3).

**Web Figure 1:** Simulations using Canadian Co-infection Cohort data (2003-2014). We explored the impact of increasing measurement error on the performance of the two approaches using simulations based on a real dataset, which we also used in the case study. Here we assume that the naïve model from that analysis is the "true" model, and contaminate the measures of GGT with random error drawn from an  $N(0, \sigma_{\delta})$  distribution, with standard deviations  $\sigma_{\delta}$  equal to 0.25x, 0.5x, 0.75x and 1 x 0.247, the measurement error of GGT applied in our previous analysis. Estimates are based on 300 datasets for each specified measurement error variance.



Given the figure above, the results for the two SIMEX approaches remain nearly unbiased when the ratio of the specified versus original measurement error variance is less than 1.0, while the naïve MSM exhibits up to 20% bias relative to the "true" model.

**Web Figure 2:** Pairwise correlations between SIMEX and naïve estimates. We computed pairwise correlations for the direct and indirect SIMEX, indirect SIMEX and naïve MSM, and direct SIMEX and naïve MSM across all 27 scenarios described in Table 1 and Supplementary Table 1c. As seen in the figure below, correlation between direct and indirect SIMEX estimates remained consistently high ( $\rho \ge 0.95$ ) regardless of the degree of specified measurement error.



**Web Figure 3:** Evaluating the accuracy of the quadratic extrapolation. To better understand the impact of measurement error on estimates from the naïve model (without any SIMEX correction), we modeled the functional relationship between  $\beta(\lambda)$  and  $\lambda$  for the direct SIMEX, and  $\alpha(\lambda)$  and  $\lambda$  for the indirect SIMEX. Given 21 gradually decreasing values of measurement error standard deviation (ranging from 1.50 to 0.00), we produced plots for  $\beta(\lambda)$  from the outcome model and  $\alpha(\lambda)$  from the treatment model versus  $\lambda$ . For each round of simulations, we generated 100 datasets containing 5000 observations each, with  $\alpha_0 = 0.25$ , and  $\alpha_1 = 0.50$ .

The dotted line on each plot shows the fit provided by a quadratic smoother. A comparison of the two fitted lines suggests that while the quadratic extrapolant may not provide a perfect fit, it offers a reasonable approximation to the observed relationship.



Scenario					Model	using erro	r-free L	Model using error-prone L*			
		Va	lues	Bias	MSE	MČSE	95% Boot Cov.	Bias	MSE	MCSE	95% Boot Cov.
	N	$\alpha_1$	$\sigma_\delta$	β <sub>1</sub>				$\beta_1$			
Α	100	0.25	0.25	-0.003	0.042	0.206	0.953	0.012	0.048	0.219	0.950
В			0.75					0.127	0.089	0.271	0.930
С			1.25					0.267	0.173	0.319	0.883
D	500		0.25	0.008	0.009	0.095	0.930	0.026	0.010	0.099	0.937
Е			0.75					0.145	0.036	0.123	0.800
F			1.25					0.286	0.104	0.149	0.453
G	1000		0.25	0.003	0.005	0.071	0.923	0.020	0.006	0.075	0.927
Н			0.75					0.135	0.027	0.092	0.623
I			1.25					0.272	0.085	0.106	0.223
J	100	0.50	0.25	-0.006	0.057	0.239	0.953	0.029	0.062	0.248	0.957
K			0.75					0.253	0.148	0.290	0.890
L			1.25					0.528	0.387	0.330	0.667
М	500		0.25	0.006	0.013	0.113	0.920	0.039	0.015	0.116	0.917
Ν			0.75					0.260	0.085	0.134	0.457
0			1.25					0.534	0.309	0.153	0.077
Р	1000		0.25	-0.001	0.006	0.08	0.920	0.031	0.008	0.084	0.917
Q			0.75					0.250	0.073	0.099	0.217
R			1.25					0.524	0.287	0.112	0.003
S	100	0.75	0.25	0.022	0.105	0.323	0.923	0.071	0.111	0.326	0.920
Т			0.75					0.383	0.274	0.357	0.763
U			1.25					0.778	0.741	0.370	0.443
V	500		0.25	0.002	0.020	0.14	0.930	0.044	0.022	0.143	0.887
W			0.75					0.355	0.151	0.156	0.343
Х			1.25					0.754	0.597	0.167	0.017
Y	1000		0.25	0.003	0.009	0.096	0.940	0.045	0.012	0.099	0.907
Z			0.75					0.354	0.137	0.111	0.113
AA			1.25					0.751	0.578	0.120	0.000

**Web Table 2:** Single time interval simulation results for error-free L and error-prone L\*

**Web Table 3:** Summary of two-interval simulation study results for four scenarios (rows), varying the coefficient for the confounder L ( $\alpha_1$ ), and the standard deviation of measurement error  $\sigma_{\delta}$  for models including an error-free L and uncorrected error-prone L\*. Bootstrap coverage was obtained using the percentiles approach. Mean squared error abbreviated as MSE, Monte Carlo standard error abbreviated as MCSE. Sample size of N = 1000 used for all simulations.

				l using e	Model using error-prone <i>L</i> <sub>1</sub> *, <i>L</i> <sub>2</sub> *										
Values		Bi	as	N	ISE	MC	SE	95% Bootstrap Coverage	Bias MS		SE	MCSE		95% Bootstrap Coverage	
$\alpha_1$	$\sigma_\delta$	β <sub>1</sub>	$\beta_2$	$\beta_1$	β2	$\beta_1$	$\beta_2$	β <sub>1</sub> , β <sub>2</sub>	β1	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	β2	β1, β2
0.50	0.25	-0.004	0.009	0.009	0.011	0.094	0.106	0.910, 0.973	0.025	0.03	0.009	0.012	0.094	0.106	0.917, 0.957
	1.25	-0.004	0.009	0.009	0.011	0.094	0.106	0.907, 0.970	0.411	0.238	0.179	0.067	0.097	0.103	0.010, 0.400
0.75	0.25 1.25	0.006 0.001	0.013 -0.001	0.014 0.014	0.018 0.019	0.12 0.118	0.135 0.138	0.930, 0.927 0.940, 0.947	0.045 0.561	0.044 0.319	0.016 0.325	0.02 0.115	0.121 0.098	0.133 0.116	0.903, 0.927 0.000, 0.233
	Valu α <sub>1</sub> 0.50	Values $\alpha_1$ $\sigma_{\delta}$ 0.50       0.25         1.25         0.75       0.25         1.25	Values         Bi $\alpha_1$ $\sigma_{\delta}$ $\beta_1$ 0.50         0.25         -0.004           1.25         -0.004           0.75         0.25         0.006           1.25         0.001	Values         Bias $\alpha_1$ $\sigma_{\delta}$ $\beta_1$ $\beta_2$ 0.50         0.25         -0.004         0.009           1.25         -0.004         0.009           0.75         0.25         0.006         0.013           1.25         0.001         -0.001	ValuesBiasMode $\alpha_1$ $\sigma_{\delta}$ $\beta_1$ $\beta_2$ $\beta_1$ 0.500.25-0.0040.0090.0091.25-0.0040.0090.0090.750.250.0060.0130.0141.250.001-0.0010.014	ValuesBiasMSE $\alpha_1$ $\sigma_{\delta}$ $\beta_1$ $\beta_2$ $\beta_1$ $\beta_2$ 0.500.25-0.0040.0090.0090.0111.25-0.0040.0090.0090.0110.750.250.0060.0130.0140.0181.250.001-0.0010.0140.019	ValuesBiasMSEMC $\alpha_1$ $\sigma_{\delta}$ $\beta_1$ $\beta_2$ 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# Web Appendix 4: Additional details related to case study analyses of CCC data

We consulted trace line plots to assess convergence of imputation models, given 10 iterations and 30 imputations per iteration; we specified these values based on advice provided by Bodner (6). We imputed missing values for continuous variables using predictive mean matching, and applied logistic regression to impute missing indicators of treatment status, as suggested by van Buuren et al. (7, 8). Censoring models included all time-varying covariates except for GGT, which did not strongly predict censoring.

#### Web Figure 4: Participant flow diagram for Case Study using Canadian Co-infection Cohort Study data



**Web Table 4:** Characteristics of 186 participants in the Canadian Co-infection Cohort Study (2003-2014) with at least one visit following hepatitis C virus (HCV) therapy and known sustained virologic response status at second visit

	SVR attaine		
Variable	Yes	No	<i>P</i> value <sup>†</sup>
	(n = 111)	(n = 75)	
Age in years (mean (SD))	45.85 (8.72)	47.27 (7.90)	0.25
Female (%)	18 (16)	10 (13)	0.59
Duration of HCV infection in years (mean (SD))	16.52 (11.10)	19.79 (10.52)	0.04
HCV genotype of 2/3/4 (%)	42 (38)	18 (24)	0.048

<sup>†</sup> Computed from *t*-test for continuous variables, and chi-square test for categorical variables.

# Baseline characteristics and standardized differences prior to and subsequent to weighting

**Web Table 5:** Comparison of baseline characteristics between those attaining or not attaining SVR12 prior to weighting, and subsequent to weighting using naïve model with quadratic terms for GGT and HIV RNA levels.

		Prior to weighting		Weighted using naïve weights, with quadratic log <sub>10</sub> (GGT), log <sub>10</sub> (HIV RNA) terms					
	SV	R <sub>12</sub>		SV					
	No	Yes	Standardized	No	Yes	Standardized			
Variable	N=75	N=111	difference	N=181.63	N=188.79	difference			
log <sub>10</sub> (GGT)	1.94 (0.36)	1.58 (0.35)	1.01	1.74 (0.39)	1.73 (0.40)	<0.01			
BMI	24.84 (3.63)	25.83 (6.85)	0.18	24.79 (3.41)	25.17 (5.83)	0.08			
log <sub>10</sub> (HIV RNA)	2.03 (0.99)	1.87 (0.80)	0.18	1.96 (1.40)	2.14 (1.43)	0.09			
Time since stopping HCV therapy	125.41 (146.57)	106.44 (123.56)	0.14	101.10 (10.06)	106.92 (10.34)	0.05			
IDU in past 6 months	14 (19%)	15 (20%)	0.14	24.41 (13%)	26.57 (14%)	0.02			
Any alcohol use in past 6 months	36 (48%)	62 (56%)	0.16	79.51 (44%)	97.79 (52%)	0.16			
Currently receiving HIV anti-retroviral therapy	67 (89%)	101 (91%)	0.06	157.72 (87%)	160.93 (85%)	0.05			

N.B. Mean (standard deviation) reported for continuous variables, while N (%) is reported for dichotomous variables.

	HCV genotype	γ-glutamyl transferase		-glutamyl BMI HIV RNA ransferase		Α	Any IDU in past 6 months		Any alcohol use in past 6		Currently on anti- retroviral therapy		APRI	
										months	6			
		Visit 1	Visit 2	Visit 1	Visit 2	Visit 1	Visit 2	Visit 1	Visit 2	Visit 1	Visit 2	Visit 1	Visit 2	Visit 3
Missing	22	22	57	25	60	11	43	2	35	6	43	0	34	7
(%)	(9%)	(9%)	(31%)	(10%)	(32%)	(4%)	(23%)	(1%)	(19%)	(2%)	(23%)	(0%)	(18%)	(4%)

Web Table 6: Distribution of missingness prior to imputation. Percentages based on N of uncensored individuals at each visit.

N.B. We did not impute SVR<sub>12</sub>; for reference, the number of participants with missing SVR status is available in Web Figure 4 above.

### Web References

- 1. Wang N, Lin X, Gutierrez RG, Carroll RJ. Bias Analysis and SIMEX Approach in Generalized Linear Mixed Measurement Error Models. J Am Stat Assoc. 1998;93(441):249-61.
- 2. Carroll RJ, Küchenhoff H, Lombard F, Stefanski LA. Asymptotics for the SIMEX Estimator in Nonlinear Measurement Error Models. J Am Stat Assoc. 1996;91(433):242-50.
- 3. Li Y, Lin X. Functional Inference in Frailty Measurement Error Models for Clustered Survival Data Using the SIMEX Approach. J Am Stat Assoc. 2003;98(461):191-203.
- 4. Kim J, Gleser LJ. SIMEX approaches to measurement error in ROC studies. Communications in Statistics: Theory and Methods. 2000;29(11):2473-91.
- 5. Shang Y. Measurement Error Adjustment Using the SIMEX Method: An Application to Student Growth Percentiles. Journal of Educational Measurement. 2012;49(4):446-65.
- 6. Bodner TE. What improves with increased missing data imputations? Structural Equation Modeling. 2008;15(4):651-75.
- 7. Buuren SV, Groothuis-Oudshoorn K. Multivariate imputation by chained equations in R. J Stat Softw. 2011;45(3):1-67.
- 8. van Buuren S. Flexible imputation of missing data. Boca Raton, Florida: Chapman & Hall/CRC; 2012.
- 9. He W, Yi GY, Xiong J. Accelerated failure time models with covariates subject to measurement error. Stat Med. 2007;26:4817-32.