Supplementary materials for

Origins of the brain networks for advanced mathematics in expert mathematicians

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Contents:

Visual stimuli details

All stimuli were black on a white background. Faces, tools, houses and bodies were highly contrasted gray-level photographs matched for overall number of gray level. Faces were front or slightly lateral views of non-famous people. Houses consisted in outside views of houses or buildings. Tools were common hand-held household object such as a hair-dryer. Bodies were front pictures of headless standing bodies. Numbers, words and formulas were strings of 5 or 6 characters. All numbers were decimal forms of famous constants (e.g. 3.14159 = π). Formulas were extracted from classical mathematical equations or expressions (e.g. binomial coefficients or the Zeta function). Words were written either with upper or lower case letters and were of high lexical frequency (mean = 28.3 per million; http://lexique.org).

Although numbers, words and formulas were inevitably arranged horizontally relative to other images, the mean width of horizontal images was not significantly different from the mean length of vertical images or the mean side of the square ones, so that they were all inscribed in a circle of 310 pixels diameter, equivalent to a visual angle of 5°.

The stimuli were presented in short mini-blocks of eight stimuli belonging to the same category. Within each block, the subject's task was to click a button whenever he/she detected an image repetition (one-back task). Each of the seven categories of images comprised twelve items, among which eight items were randomly picked on a given mini-block. Each image was flashed for 300 ms and followed by a 300 ms fixation point, for a total duration of 4.8 s. The category blocks were separated by a brief resting period with a fixation point only, whose duration was randomly picked among 2.4 s, 3.6 s or 4.8 s.

Supplementary results

Behavioral results in auditory runs

Results are presented in figure 1B in the main text. With mathematical statements, mathematicians performed way above chance level (63.6 ± 2.8 % [mean ± standard error]; chance = 33.3%; Student's t test, $t = 11.3$ $p < 0.001$), while control subjects were just above chance (37.4 ± 1.6 %, $t = 2.6$, $p = 0.02$; difference between groups: $t = 8.5$, $p< 0.001$). With non-mathematical statements, both groups performed equally well (mathematicians: 65.4 ± 3.1 %, $t = 10.6$, $p < 0.001$; controls: 63.7 ± 3.8 %, $t = 8.3$, $p < 0.001$; no difference between groups: $t = 0.4$, $p = 0.7$). Importantly, mathematicians performed identically with math and non-math statements ($t = 0.5$, $p = 0.6$).

Above-chance performance could arise from a discrimination of meaningful and meaningless statements, from a discrimination of true versus false statements, or both. To separate these effects, we applied signal detection theory (SDT). First, we quantified subjects' ability to discriminate whether the statements were meaningful (pooling across true and false statements) or meaningless. We considered hits as "meaningful" responses to statements that were indeed meaningful, and false alarms as "meaningful" responses to meaningless statements. For both mathematics and non-mathematics, mathematicians' judgments of meaningfulness were highly above chance ($d'_{math} = 2.68 \pm 0.18$, $t = 15.9$, $p < 0.001$; $d'_{non\text{-}math} = 3.56 \pm 0.28$, $t = 13.0$, $p < 0.001$). On the contrary, controls' judgments of meaningfulness dropped nearly to 0 for mathematics ($d'_{math} = 0.67 \pm 0.17$, $t = 3.9$, $p = 0.002$), but were highly above chance for general knowledge ($d'_{non\text{-}math} = 3.16 \pm 0.47$, $t = 6.99$, p <

0.001). There was no significant difference comparing mathematicians and controls' capacity to discriminate meaningful non mathematical sentences (t = 0.76, p = 0.45). However, mathematicians were significantly better than controls at discriminating meaningful mathematical statements (t = 8.44, p < 0.001) (figure 1C).

We also applied SDT to evaluate the subjects' capacity to discriminate true and false statements. This analysis was restricted to meaningful statements that were judged meaningful. We considered hits as true statements correctly classified as true, and false alarms as false statements incorrectly classified as true. Mathematicians showed weak but significantly positive d-primes for mathematics ($d'_{math} = 0.78 \pm 0.16$, t = 5.0, p < 0.001), and for non-mathematics ($d'_{non\text{-}math} = 0.68 \pm 0.31$, t = 2.30, p = 0.04). Controls did not show a significantly positive d-prime for mathematics but they did for non-mathematics ($d'_{math} = 0.38 \pm 0.23$, t = 1.72, p = 0.11; $d'_{non\text{-}math} = 0.52 \pm 0.15$, t = 3.48, p = 0.004). The difference between mathematicians and controls failed to reach significance, either for mathematics (t = 1.46, p = 0.15) or for general knowledge (t = 0.49, p = 0.63) (figure 1D).

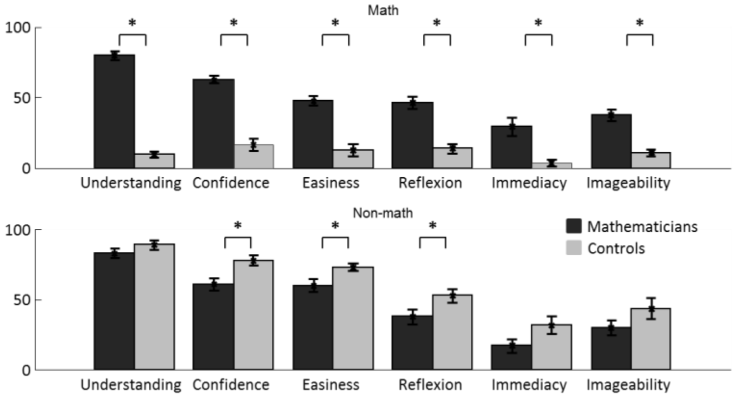
In summary, mathematicians performed equally well with both types of sentences. Within the allotted time period of 4 seconds, they managed to discriminate meaningful mathematical statements from meaningless ones, as well as to distinguish true statements from false ones. Controls only managed to understand and classify the non-mathematical sentences. Most importantly, the results indicate that mathematical statements and non-mathematical sentences were well matched in term of objective difficulty, as evaluated by percent success, and that mathematicians and control subjects were well matched in terms of their performance with non-mathematical statements.

Behavioral results in visual runs

SDT was also used to evaluate subjects' ability to perform the visual one-back task. Pooling across the groups, *d's* for each category were significantly greater than 0 (minimum d' averaged across subjects = 2.4, all $p<10^{-12}$), meaning that participants correctly detected repetitions within each visual category. An ANOVA on *d's*, with category as a within-subject factor and group as a between-subjects factor, indicated that neither mathematical expertise nor the category of pictures influenced the performance, and that both groups performed equally well in detecting repetitions regardless of the visual category (group: F(1)=0.18, p=0.67; category: F(6)=0.29, p=0.94; interaction group x category: F(6)=0.69, p=0.66). An ANOVA on reaction time showed equivalent results (group: F(1)=1.63, p=0.20; category: F(6)=0.67, p=0.67; interaction group x category: F(6)=0.54, p=0.78). Obviously, the one-back task was simple enough that, in spite of their mathematical expertise, mathematicians performed no better than controls in detecting repetitions, even with numbers (t = 0.83, p = 0.41) or formulas (t = 0.83, p = 0.41).

Subjective variables reported during the post-MRI questionnaire

For mathematical statements, mathematicians gave higher ratings than controls for all subjective variables (all p*s* < 0.001). For non-mathematical sentences, ratings of understanding, immediacy and imageability were equivalent for both groups, and controls responded with higher ratings than mathematicians for confidence, ease of responding, and reflection (p*s* <0.05). Those findings suggest that each group was more at ease with its respective domain of expertise.



To evaluate the reliability of subjective ratings, which were collected after the fMRI, we correlated them with objective performance to the same statements. Within the group of professional mathematicians, we observed that objective performance during fMRI was positively correlated with subsequent ratings of confidence (logistic regression, r = 0.36; p<0.001) and comprehension (r = 0.21; p<0.001) of the same statements, and negatively correlated with subjective difficulty (r = -0.28; p<0.001) and intuition (r = -0.11; p<0.001). Those relations indicate that subjective variables were reliable and that, unsurprisingly perhaps, mathematicians showed increasingly better performance on sentences that they understood better, rated as easier, were more confident about, and for which they deployed explicit reasoning rather than mere intuitive judgments.

Variation in brain activation across mathematical problems

Figure 3 shows that the majority of the mathematical expertise network was activated jointly by all four mathematical domains, as evidence by an intersection analysis (contrasts of algebra, analysis, geometry and topology, each relative to non-math, in mathematicians during the reflection period; each at p<0.001; cluster size >200 voxels). An F-test was used to identify the putative differences between those four contrasts at the whole-brain level. This test revealed significant differences in bilateral parietal posterior regions (peaks at 23, -72, 52; F=8.39, uncorrected p<0.001; and at -11, -75, 58; F=8.73, uncorrected p<0.001) and left inferior temporal regions (-50, -63, -5; F=12.01, uncorrected p<0.001) (figure 3A). Examination of the activation profiles, as well as further t-tests, revealed that this pattern was primarily due to a greater activation to geometry problems than to the other three domains combined (at -50, -63, -5, t=6.39, p<0.001; at 23, -72, 52, t=4.39, p<0.001; at -11, -75, 58, T=4.28, p<0.001). This contrast also revealed regions showing more activation to geometry than to the other domains of math in bilateral IT, bilateral superior parietal, right intraoccipital sulcus, left supramarginal gyrus, and left inferior parietal cortex. In addition, statements in analysis also induced greater activation than other domains in a mesial frontal orbital region, and statements in topology in the left middle frontal gyrus (table S2, peaks at p<0.001; cluster size > 200 voxels, corresponding to clusterwise p<0.05 corrected).

We also evaluated whether the mathematicians' subjective ratings in the post-MRI questionnaire correlated with brain activity evoked by different mathematical statements. We tested this potential correlation, in mathematicians only, for meaningful math statements, with each of the 6 subjective variables that were rated (comprehension, confidence, difficulty, intuition, immediacy and imageability). Only a single contrast revealed a significant positive correlation between imageability and brain activation, at two sites in the left inferior temporal cortex (peak at -57, -52, -7, T=7.38, p<0.001) and in the left intra-occipital sulcus (peak at -29, -72, 36, T=6.06, p<0.001) (figure 3B).

Activation to mathematical statements in control subjects without mathematical expertise

In control subjects, the math > non-math contrast identified a set of cortical areas involving right pre-central and left postcentral sulci, bilateral mesial parietal, middle occipital gyri, lingual gyri, insula overlapping with BA13, different frontal sites in BA10, parts of orbitofrontal prefrontal cortex and middle frontal gyrus, and subcortical regions, especially bilateral putamen (Figure S2A, Table S1). Those activations partly resemble the activations evoked by meaningless general-knowledge statements. Indeed, the meaningless > meaningful non-math contrast revealed activations in the right supramarginal gyrus, bilateral mesial parietal, right lingual gyrus, left anterior superior temporal gyrus (aSTG), near temporal pole, right pre-central and left post-central sulci. Activation maps for these two contrasts overlapped in the right pre-central and left post-central sulci, bilateral mesial parietal and right lingual gyrus (figure S2B). In aSTG, we observed a strong deactivation for meaningless non-math and no activation for math (figure S2C).

These results suggest that control subjects, when listening to mathematical statements (1) do not activate the same bilateral intraparietal and inferior temporal regions as professional mathematicians; and (2) process both meaningful and meaningless mathematical statements in a manner similar to meaningless non-mathematical statements.

ROI analyses in language-related areas

Additional analyses were performed in seven regions of interest (ROIs) that had been previous identified as related to language processing. They included the six cortical left-hemispheric ROIs previously reported by Pallier et al. (1) as involved in the constituent structure of sentences: temporal pole (TP), anterior superior temporal sulcus (aSTS), posterior superior temporal sulcus (pSTS), temporo-parietal junction (TPj), inferior frontal gyrus pars orbitalis (IFGorb), and inferior frontal gyrus pars triangularis (IFGtri) (the left putamen, present in Pallier et al, was not included here because we could not identify active voxels during language processing in this region in every subject). We added the cyto-architectonically defined left Brodmann area 44 (2).

Within each region, for each subject, we first used a separate functional localizer (3) to identify voxels activated by sentences (spoken or written) relative to rest (voxel p<0.001 uncorrected). We then averaged the responses in these subject-specific voxels across participants, and performed statistical t-tests across conditions. Figure S7 shows the temporal profile of activation, averaged across participants, at the peak subject-specific voxel, and table S5 presents the corresponding statistics. At this single-voxel level, none of these language regions showed evidence of a contribution to mathematical reflection. In fact, during the reflection period, in mathematicians, TP, pSTS, and IFGOrb responded significantly more to non-math than math. In controls, only aSTS and IFGtri responded more to non-math than to math. We also looked for differences between groups, but the only trends were in the direction of significantly greater activation in controls than in mathematicians (in aSTS and BA44 for non-math statements; and in TP for math statements; uncorrected p < 0.05). There was no interaction between group and category in any region. Furthermore, no significant activation was found in those regions for meaningful versus meaningless math statements, neither in mathematicians, nor in controls. However, for meaningful versus meaningless non-math, a significant activation was found in aSTS, and to a lesser extent in pSTS in mathematicians (table S5).

This sensitive ROI approach thus confirmed that language networks do not contribute to mathematical reflection. It could be, however, that these regions have a transient role during the processing of the mathematical statements themselves. We therefore replicated the above analyses with contrasts measuring activation during sentence presentation (table S5, lower part). None of the ROIs were engaged in math listening more than non-math listening, nor in meaningful > meaningless math listening, neither in mathematicians, nor in controls. The only effects were in the converse direction: there was more activation for non-math than for math in aSTS, pSTS, TPJ, IFGOrb, IFGtri and BA44 for mathematicians, and in TPJ and IFGOrb for control subjects. Only IFGOrb showed a group effect, activating less in mathematicians than in controls both during math listening and during non-math listening, without any significant interaction (table S5).

Overall, these results provide no indication that language areas contribute to mathematics, and in fact suggest that, if anything, they activated less for mathematics and/or less in mathematicians.

RSA analyses in math-related areas

First, thanks to independent localizer scans performed in a different cohort of 83 subjects, we defined 13 math-related regions in left and right Intraparietal sulci (IPS), infero-temporal cortex (IT), inferior, middle and superior frontal lobes (IFG, MFG, and SFG), mesial supplementary motor area (SMA) and bilateral foci in Cerebellum.

At subject level, within each of these 13 regions, we computed correlation coefficients between the activations evoked by our main experimental conditions: math and non-math statements, simple calculation and sentence processing, and formulas, numbers, words and non-symbolic pictures.

We then compared the correlation of math statements with other math-related condition to the correlation of math statements with the corresponding non-math control condition (figure 7). In all 13 regions, the activation evoked by mathematical reflection was more correlated to the activation evoked by simple calculation than to spoken or written sentence processing (all *p*s < 0.011 uncorrected, table S7). In inferior temporal regions, activation to mathematical reflection was significantly more correlated to activation to math-related visual conditions (formulas and numbers recognition) than to corresponding visual control conditions (non-symbolic pictures viewing or words recognition). Similar effects were also observed in other regions: e.g. left IPS, MFG and Cerebellum for formulas or all regions except right Cerebellum for numbers in the comparison with pictures (see table S7).

Moreover, left and right IPS and IT exhibited a strong correlation of activations to simple calculation and visual formula or number recognition, stronger than the correlation of activations to calculation and non-symbolic pictures or words (all *p*s < 0.027 uncorrected, table S7). Similar correlations with numbers were observed in the other regions except right cerebellum; and left frontal regions also exhibited a stronger correlation with formulas than with pictures (see table S7).

Activations during sentence presentation

We replicated the contrasts reported in the main text, but now analyzing the period of sentence presentation (with regressors proportional to sentence duration). In mathematicians, the contrast math>non-math indicated that a subset of the areas involved in math reflection already activated during the auditory presentation of the statements: bilateral IT (-57, -58, -10, t = 10.53; 59, -55, -17, t = 8.42); bilateral IPS (left: -59, -37, 46, t = 7.42 and -29, -73, 37, t = 8.08; right: 39, -61, 54, t = 4.17 and 29, -75, 42, t = 4.88); and bilateral PFC foci (left: -45, 37, 16, t = 7.09 and -48 8 25, t =6.92; right: 51, 7, 24, t = 6.40) (figure S10). Though activation was mostly bilateral, time courses of activation in bilateral intraparietal sulcus suggested that the math network activated early in the left hemisphere and then spread to the right hemisphere (Figure S1). Moreover, the bilateral and mesial superior frontal foci that we found activated during reflection were not present during sentence presentation. Conversely, we found an additional activation during sentence presentation in the right head of the caudate nucleus (12, 25, 1, t = 6.79).

For control subjects, the contrast of math > non-math during sentence presentation revealed again a completely different set of areas than the previously identified math network. Some of these areas were found during reflection and thus seemed to activate early, such as the bilateral middle occipital gyri and bilateral insula. Other regions seemed to activate only during sentence presentation. Notably, we found activation in different sub-cortical nuclei including bilateral thalamus (left: -18, -16, 4, t = 5.06; right: 18, -22, 6, t = 5.18), amygdala (left: -29, -6, -26, t = 5.48; right: 27, -1, -28, t = 4.99) and left hippocampus (-39, -30, -10, t = 5.67).

Concerning the non-math statements, the contrast of non-math > math in mathematicians revealed a network that we previously described for non-math > math during the reflection period. We found bilateral temporal activation: anterior MTG (left: -59, -7, -14, t = 10.8; right: 56, -6, -17, t = 9.68), posterior MTG (left: -59, -39, 1, t = 5.52; right: 60, -34, -2, t = 5.55), angular gyrus and temporo-parietal junction (left: -47, -61, 22, t = 10.1; right: 48, -63, 25, t = 6.59). We also found frontal activation: IFGOrb (left: -47, 25, -13, t = 9.28; right: 39, 35, -13, t = 8.11), IFGtri (left: -54, 20, 24, t = 7.79; right: 54, 23, 21, t = 6.06), and mesial frontal sites (superior frontal: -6, 56, 39, t = 8.07; orbitofrontal: -5, 55, -13, t = 5.76). In control subjects, we found additional sites around the calcarine sulcus (-3, -69, 22, t = 6.78), bilateral lingual gyri (left: -15, -57, 3, t = 7.30; right: 12, -49, 3, t = 6.03) and bilateral head of the caudate nucleus (left: -9, 17, -1, t = 5.19; right: 9, 13, -1, t = 5.41).

Remarkably, the head of the caudate nucleus activated for math > non-math in mathematicians and for non-math > math in control subjects, thus revealing a systematic engagement for the subject's main domain of predilection. This effect was confirmed by an examination of the SPM interaction of group and the math > non-math contrast, which was highly significant in the head of the caudate nucleus bilaterally (left: -11, 20, -1, t = 5.95; right 15, 25, -1, t = 7.39). Plots of temporal profiles of fMRI signals for math and non-math stimuli over the whole regions of interest, separately for the two groups, are shown in figure S11.

We then studied the contrast of meaningful > meaningless non-math during sentence presentation. The most important cluster was found in the left angular gyrus, extended to middle occipital gyrus and middle temporal gyrus (in mathematicians: -48, -60, 16, t = 5.28; in controls: -38, -75, 28, t = 4.75; in both groups together: -39, -76, 31, t = 6.12). In mathematicians, it was the only cluster revealed by this contrast. We found additional clusters in control subjects, including three sites exhibiting a significantly greater difference between meaningful and meaningless non-math in controls than in mathematicians: the bilateral middle temporal sulcus (left: -44, -23, -5, t = 5.85; right: 53, -19, 3, t = 4.85), and right Heschl's gyrus (36, -31, 9, t = 4.95).

Finally, in mathematicians, bilateral angular gyri (left: -48, -60, 16, t = 5.52; right: 44, -79, 22, t = 4.35), the head of the left caudate nucleus (-14, 19, -2, t = 5.28), some mesial frontal foci (superior frontal: -3, 68, 15, t = 4.95; orbitofrontal: 9, 44, -11, t = 4.28) and middle temporal region (-69, -18, -14, t = 4.74) revealed greater activation for meaningful than meaningless math. Those sites were essentially different from the ones observed during the reflection period, and interestingly, the left angular gyrus appeared in the intersection of meaningful > meaningless contrasts for math and for non-math (Figure S12A). In order to clarify the role of this region, we plotted the temporal profiles of the average fMRI signals within that intersection (Figure S12B & C). Such plots revealed that the observed differences occurred in the general context of a deactivation for all mathematical statements relative to baseline, particularly marked in the control subjects. Indeed, we found more deactivation for math in controls than in mathematicians within this region. Moreover, we observed a deactivation for both math and non-math meaningless statements in mathematicians and for all math and meaningless non-math statements in control subjects. In mathematicians, the only group able to distinguish meaningless from meaningful math statements, there was a small transient effect of greater activation to meaningful than to meaningless math. These results therefore suggest that this region is involved in semantic processing of sentences and distinguishes meaningful from meaningless sentences regardless of their mathematical or non-mathematical content. This interpretation fits with previous observations on this area (1, 4, 5), which demonstrate an increasing activation in this area in direct proportion to the amount of semantic information available in the stimulus and a systematic deactivation to meaningless materials (e.g. pseudowords or delexicalized “Jabberwocky” sentences), presumably reflecting the contribution of this region to semantic reflection in the resting state.

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Supplementary figures

Figure S1. Activation profiles in areas activated by mathematical reflection in professional mathematicians

Top, axial slices showing voxels where activation was higher during reflection on math statements relative to non-math statements (voxel p < 0.001, cluster p < 0.05 corrected for multiple comparisons at the whole-brain level). Plots show the fMRI signal (mean +/- one standard error) at the main peak of the main significant clusters. Time scale starts 3 seconds before the presentation of the sentence and lasts until the end of a trial. Black rectangles indicate the approximate time of sentence presentation.

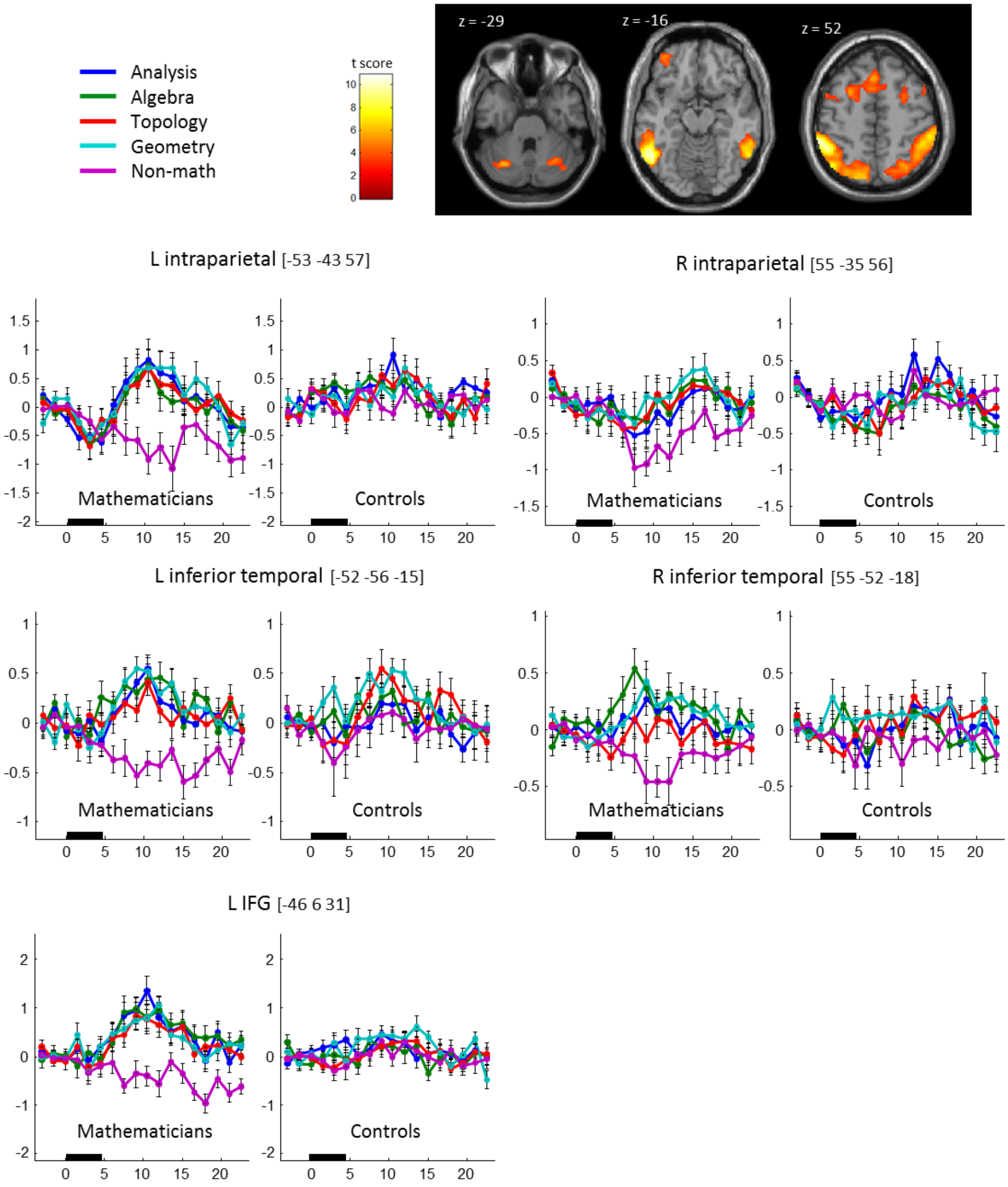


Figure S2. Brain areas showing a difference math > non-math in control subjects
(A) Axial slices showing voxels where activation was higher during reflection on math statements relatively to non-math sentences (voxel p < 0.001, cluster p < 0.05 corrected for multiple comparisons at the whole-brain level) in control subjects. (B) Slice showing commonalities between the math > non-math contrast and the meaningless > meaningful non-math contrast in control subjects. (C) Plots showing the temporal profile of activation at the main peak of each significantly activated region.

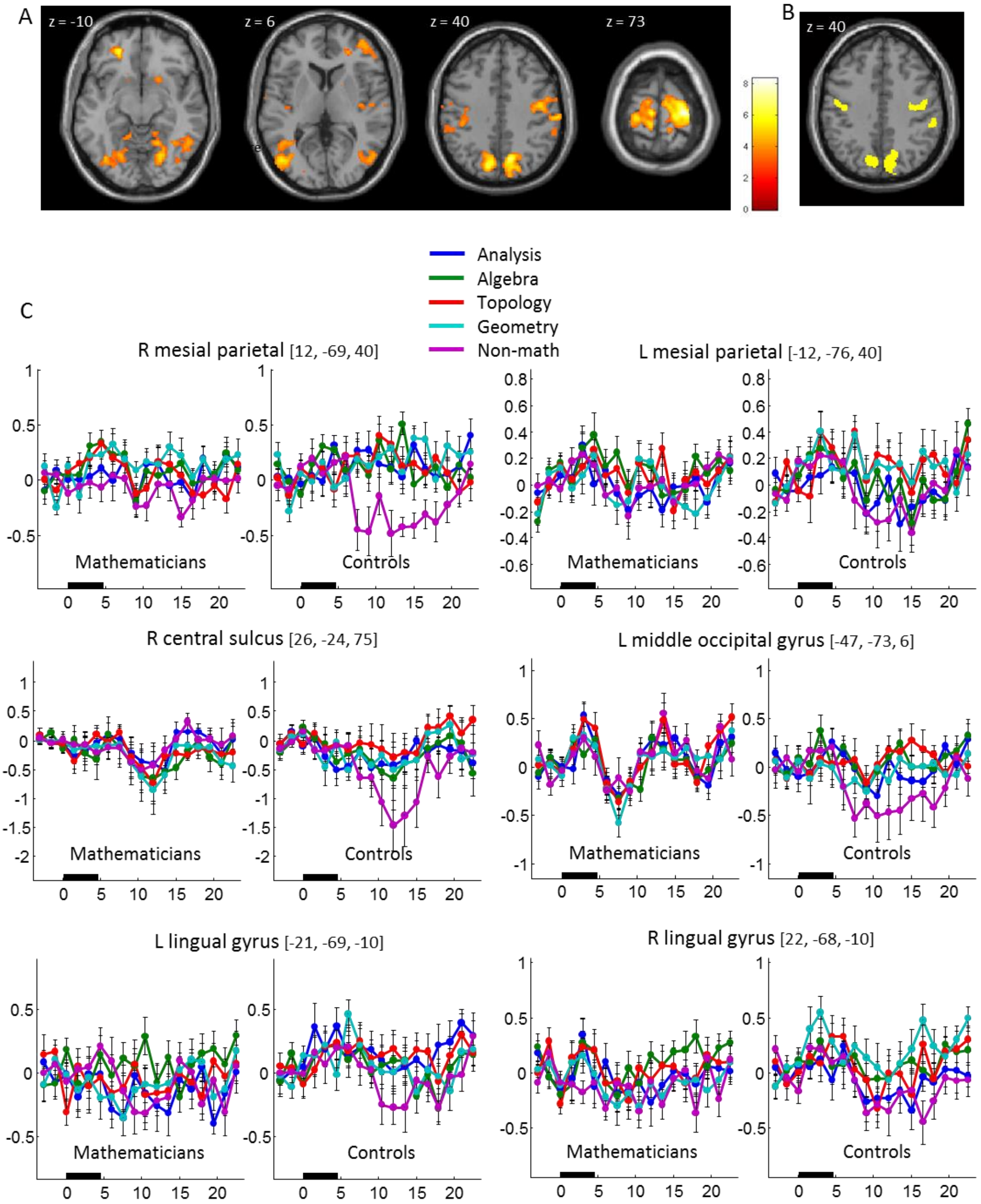


Figure S3. Activation profiles for meaningful and meaningless statements in brain areas responsive to mathematical statements.

For both groups, plots at the peaks of the 5 main regions identified in the contrast of math > non-math in mathematicians (same coordinates as figure S1).

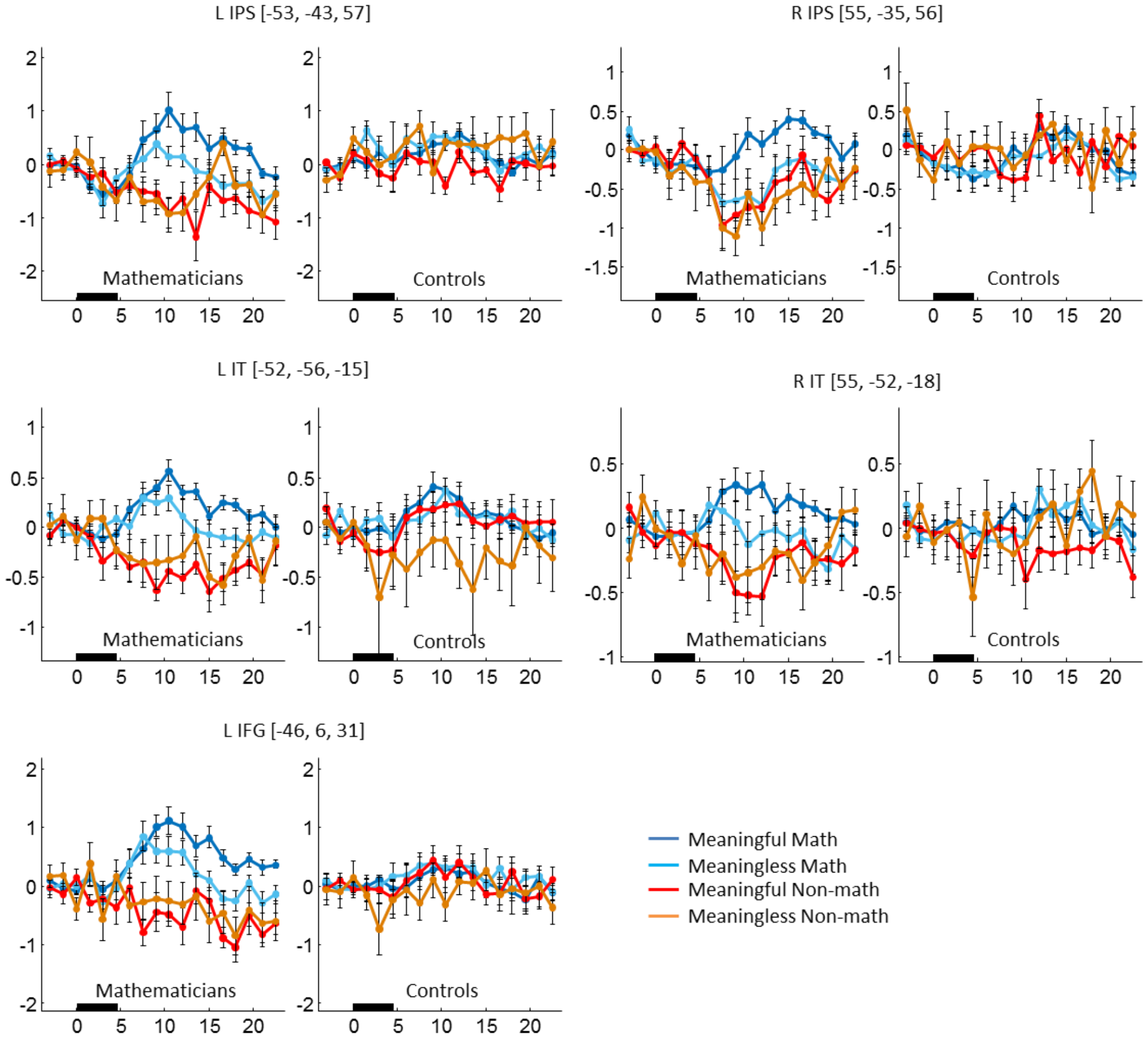


Figure S4. Control for task difficulty

For each subject, math and non-math statements were sorted into two levels of difficulty (easy versus difficult) depending on whether mean performance on a given statement was below or above the global percent correct. (A) Mean correct rates for easy and difficult math and non-math statements. The results again indicate that activation is organized according to domain (math versus non-math) rather than difficulty. (B) Axial slices showing the principal regions activated in the contrast "easy math > difficult non-math" in mathematicians across all meaningful problems (voxel p < 0.001, cluster p < 0.05 corrected for multiple comparisons at the whole-brain level). This contrast revealed virtually the same sites as those which were activated for the standard math > non-math contrast. (C) Plots report the temporal profile of activation at the principal peaks of the 5 main regions identified in the contrast of math > non-math in mathematicians (same coordinates as figure S1).

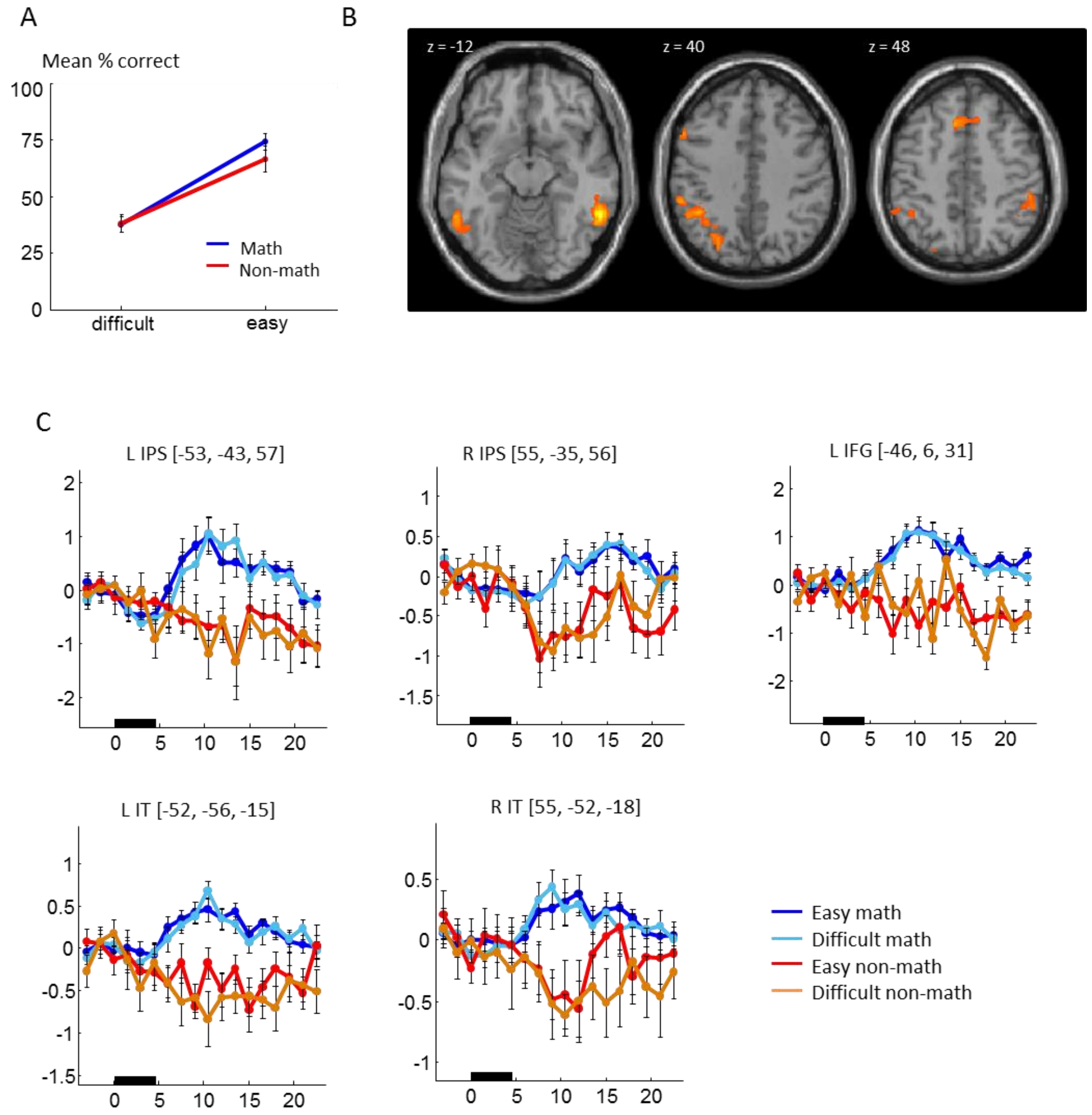


Figure S5. Activation profiles within areas of the general-knowledge network

Axial slices show voxels where activation was higher during reflection on non-math sentences relatively to math statements (voxel $p < 0.001$, cluster $p < 0.05$ corrected for multiple comparisons at the whole-brain level) in control subjects. Plots report the time course of activation at the principal peak of the activated areas.

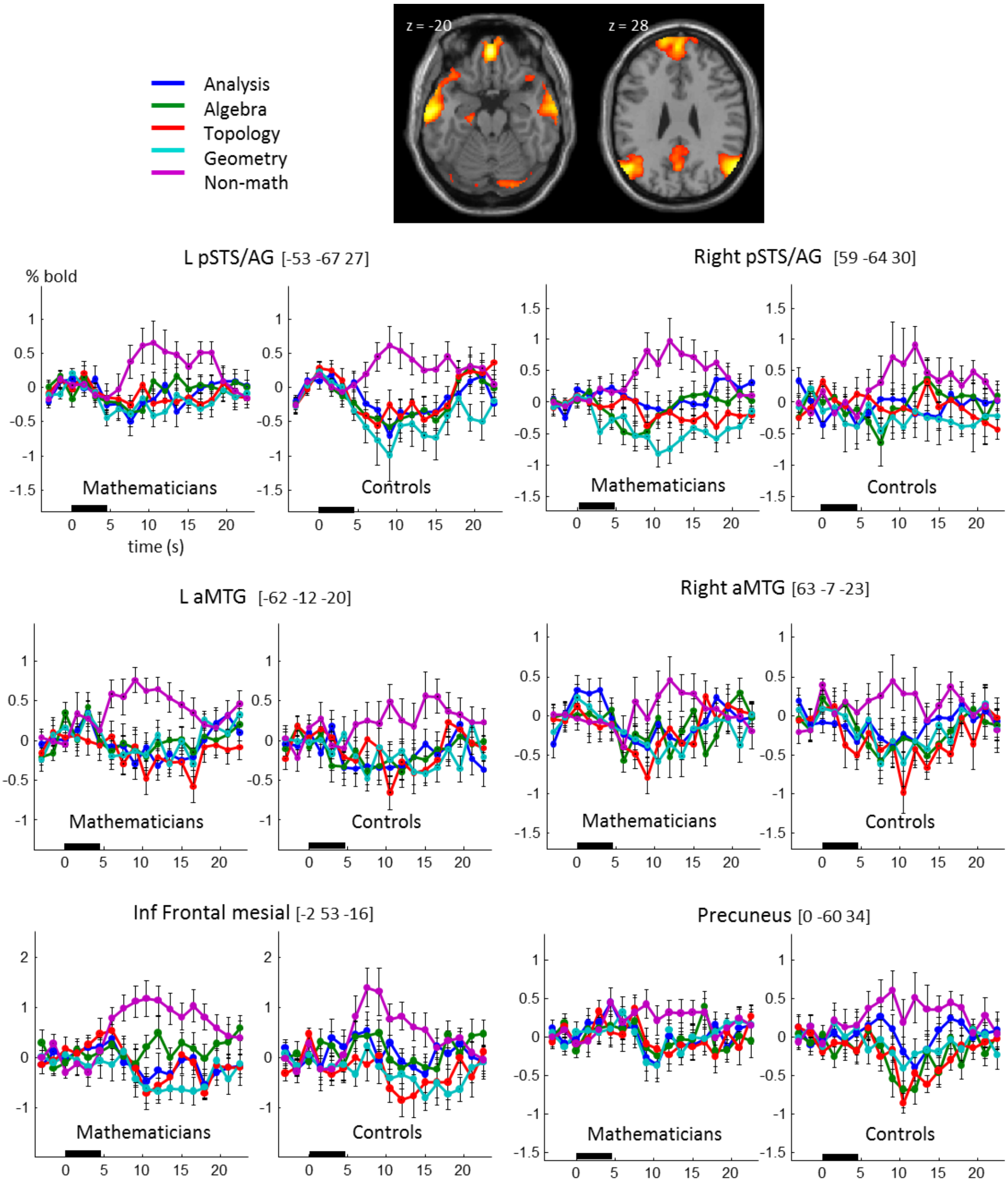


Figure S6. Activation profiles for meaningful and meaningless statements in brain areas mainly responsive to non-mathematical statements during the reflection period

Plots at the peaks of the 6 main regions identified in the contrast of non-math > math in both groups during the reflection period.

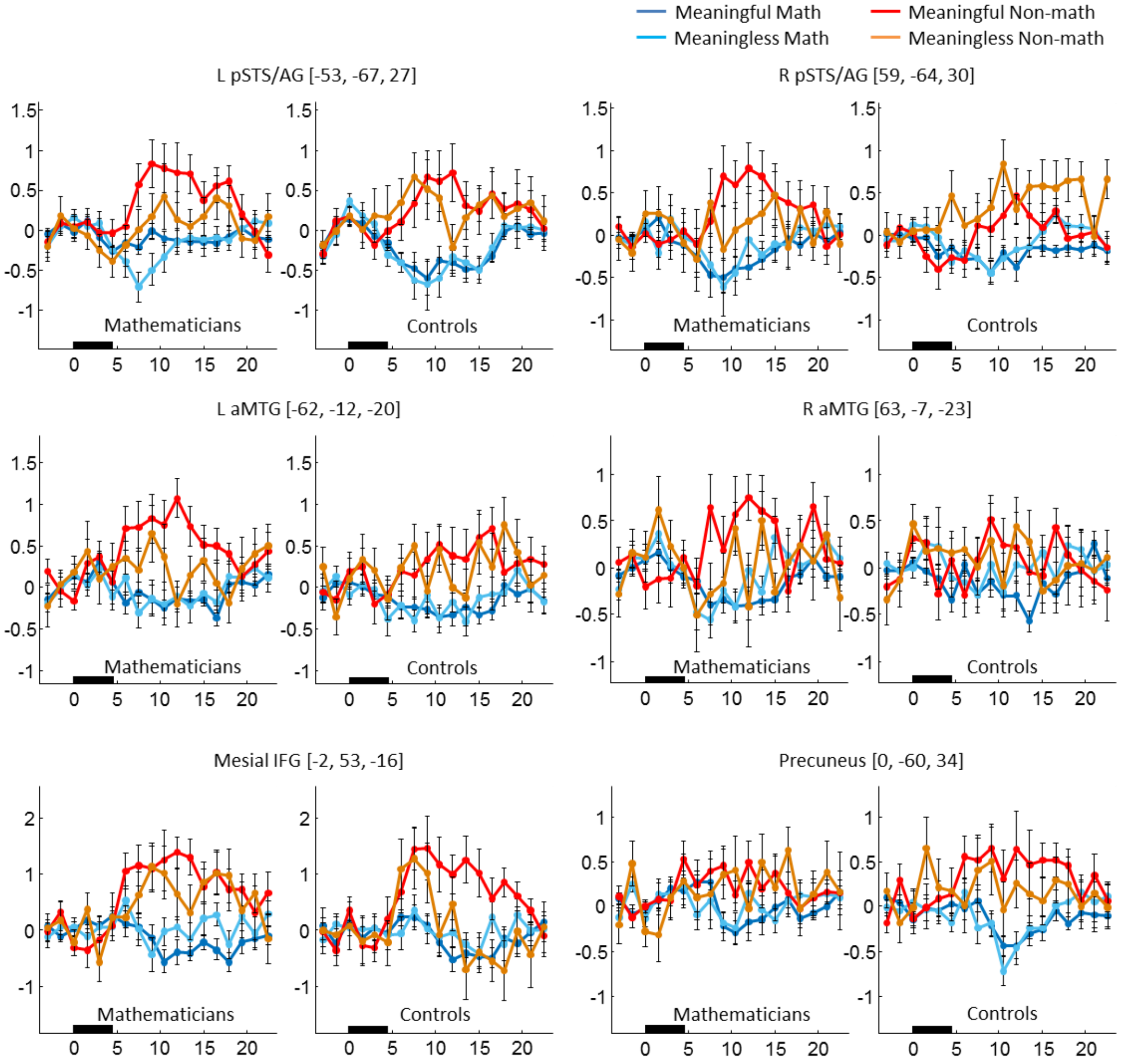


Figure S7. Activation evoked by mathematical and non-mathematical statements in classical language-related regions

The brain slice shows the localization of the seven cortical regions of interest: TP, aSTS, pSTS, TPJ, IFGorb, BA44 and IFGtri. Within each region, plots show the temporal profile of activation for the four domains of math and non-math, averaged across subjects, at the subject-specific peak of activity during an independent localizer for sentence processing. None of these regions appear to be specifically activated during mathematical reflection. On the contrary, several of them show greater activation by non-math than by math statements (see table S5 for statistics).

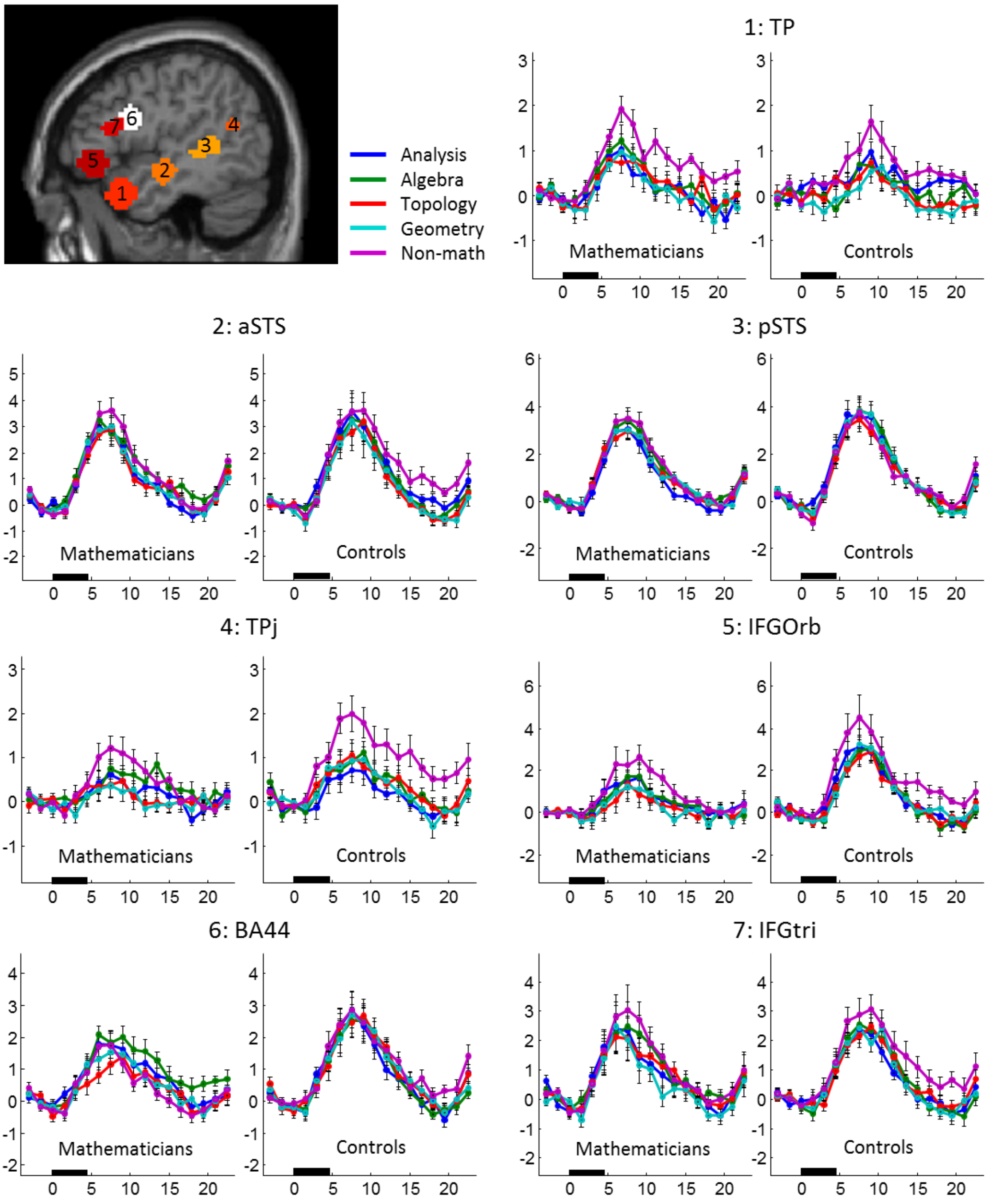


Figure S8. Spatial relationship between the math and language networks

The sagittal slices show, in red, the contrast of spoken and written sentences relatively to rest during an independent functional localizer scan and in yellow, (A) the contrast of math > non-math statements (during the reflection period) and (B) the contrast of meaningful > meaningless math statements (during the reflection period). A very small area of overlap appears in orange in superior frontal cortex mostly in A. The images show how the contours of the math network, in the frontal lobe, spare language-related areas in the left inferior frontal gyrus.

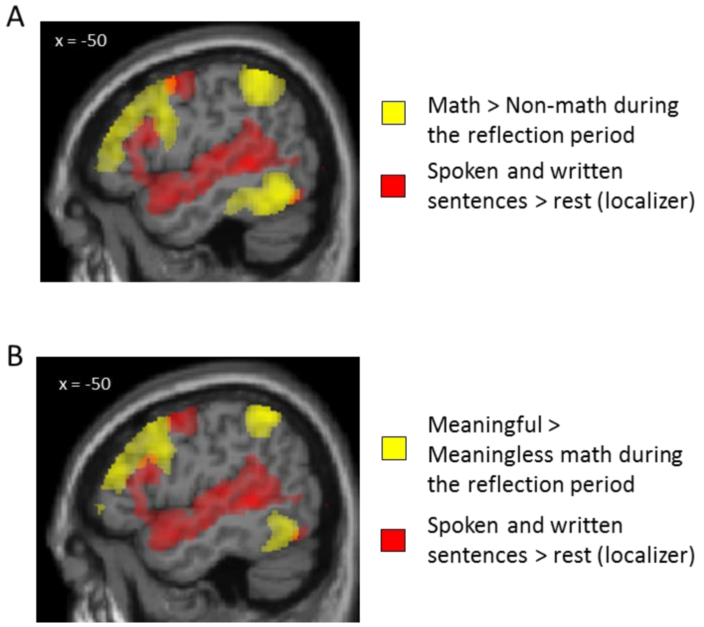


Figure S9. Activation for math > non-math in mathematicians, after removal of sentences containing occasional reference to numbers

Axial slices showing the principal regions activated in the math > non-math contrast in mathematicians, after having removed all statements that contained a reference to numbers. This analysis revealed virtually the same sites as those activated for the overall math > non-math contrast.

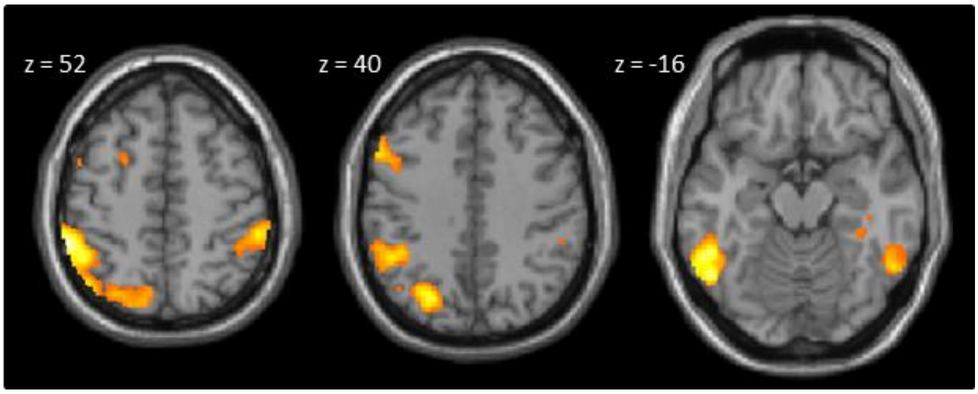


Figure S10. Superposition of the math > non-math contrasts in mathematicians during statement presentation and during the subsequent reflection period

Axial slices show the math > non-math contrasts in mathematicians, separately for activations evoked during sentence presentation in red, and during the reflection period in yellow. The intersection (in orange) reveals that most areas involved in mathematical reflection, particularly in the left hemisphere, were already activated when mathematicians listened to the statements.

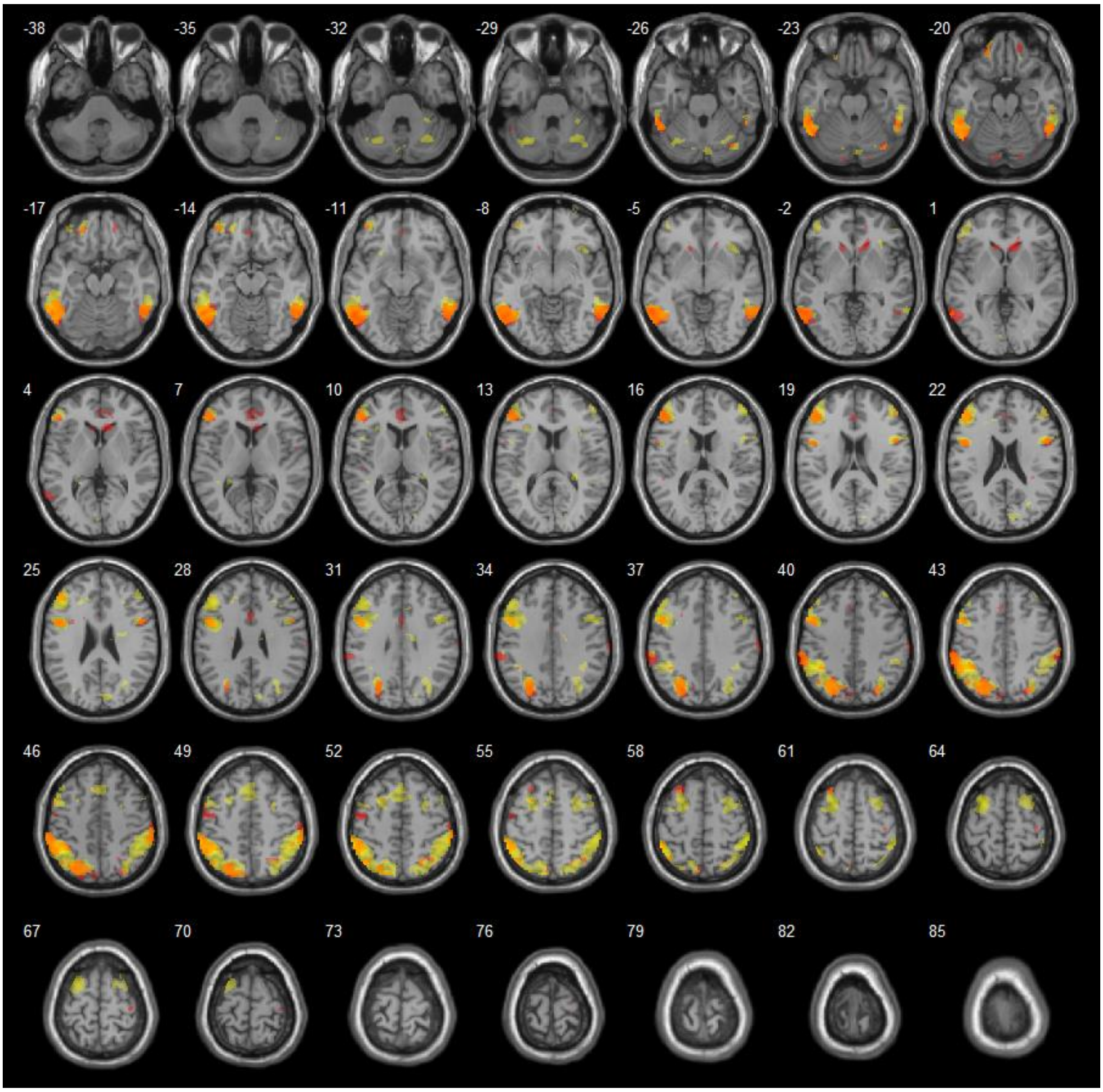


Figure S11. Interaction between group and problem type during statement presentation in the head of the caudate nucleus

The axial slice shows a bilateral activation during statement presentation in the head of the caudate nucleus in the interaction (math>non-math) X (mathematicians – controls) (voxel $p < 0.001$, cluster corrected $p < 0.05$). Plots show the corresponding temporal profile of fMRI signals for the four different domains of math and non-math, separately in mathematicians and control subjects. Signals were averaged across the entire caudate cluster.

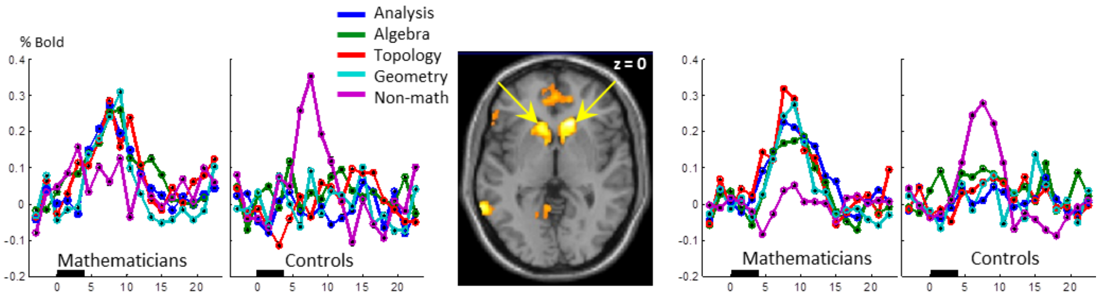
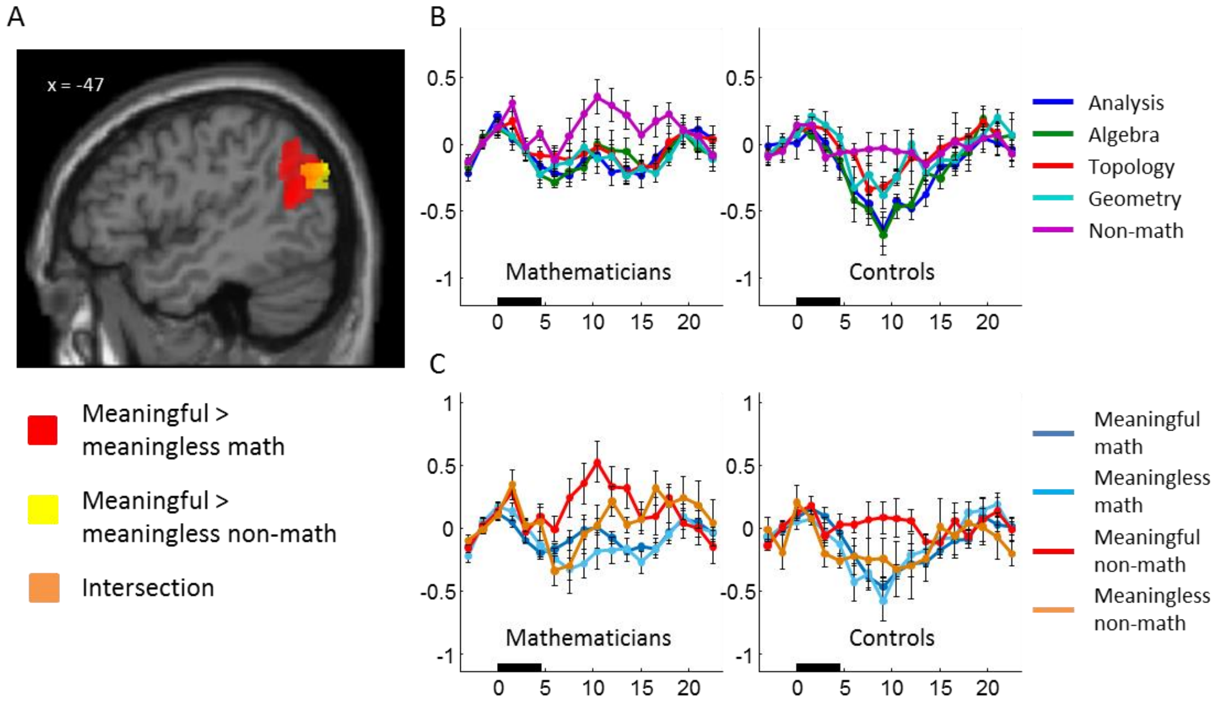


Figure S12. Transient effect of meaningful versus meaningless statements during sentence presentation in the angular gyrus.

(A) Sagittal slice centered on the left angular gyrus showing activations to meaningful > meaningless math (in red) and to meaningful > meaningless non-math (in yellow) during sentence presentation (voxel $p < 0.001$, cluster corrected $p < 0.05$). The intersection of both contrasts maps appears in orange. (B) Time course of the mean activation within the voxels belonging to the intersection presented in panel A, for the four domains of math and non-math statements in both groups. (C) Time course of the mean activation to meaningful and meaningless math and non-math statements. A transient difference between meaningful and meaningless math is seen only in mathematicians.



Supplementary tables

Table S1. Main activation peaks for the math > non-math and the meaningful > meaningless math contrasts.

	Mathematicians								Controls								Mathematicians > Controls							
	Math > Non-math				Meaningful > Meaningless math				Math > Non-math				Meaningful > Meaningless math				Math > Non-math				Meaningful > Meaningless math			
	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
L IPS	-53	-43	57	10.9	-50	-51	52	9.07	-	-	-	-	-	-	-	-	-27	-75	52	7.88	-51	-46	54	6.68
R IPS	50	-36	56	7.30	51	-40	51	7.90	-	-	-	-	-	-	-	-	33	-73	49	5.43	53	-40	51	5.45
L IT	-53	-57	-16	10.4	-56	-58	-16	7.88	-	-	-	-	-	-	-	-	-53	-60	-17	8.26	-62	-57	-10	4.64
R IT	52	-52	-19	7.50	60	-54	-13	9.46	-	-	-	-	-	-	-	-	56	-39	22	5.27	60	-54	-11	7.22
L MFG/ BA46	-44	31	27	7.81	-48	37	22	7.57	-	-	-	-	-	-	-	-	-45	-26	28	7.14	-47	13	36	4.88
L MFG/ BA9	-47	7	31	8.21	-50	10	33	7.33	-	-	-	-	-	-	-	-	-54	14	39	8.57	-53	37	22	5.11
L SFS	-24	8	64	7.11	-26	5	63	7.39	-	-	-	-	-	-	-	-	-27	11	66	7.45	-27	14	60	5.10
R SFS	32	5	56	4.97	30	8	57	9.88	-	-	-	-	-	-	-	-	-	-	-	-	30	8	57	5.79
R MFG/ BA46	50	47	16	6.74	48	38	22	7.60	-	-	-	-	-	-	-	-	-	-	-	-	48	37	22	5.14
R MFG/ BA9 - BA10	50	10	21	6.03	51	11	22	6.61	42	47	25	4.91	-	-	-	-	-	-	-	-	51	11	25	5.45
SMA	-2	23	51	6.12	0	26	49	7.24	-	-	-	-	-	-	-	-	-2	23	51	6.87	-	-	-	-
BA10	-20	47	-16	5.78	-42	55	-13	6.25	-22	44	-10	6.26	-	-	-	-	-	-	-	-	-	-	-	-
L Cereb. 6th lobule	-29	-66	-29	6.00	-3	-81	25	5.22	-	-	-	-	-	-	-	-	-5	-82	-26	6.28	3	-79	-25	4.61
R Cereb. 6th lobule	39	-73	-26	5.24	14	-82	-25	6.03	-	-	-	-	-	-	-	-	8	-81	-23	7.04	8	-78	-28	4.10
L mesial parietal	-	-	-	-	-	-	-	-	-12	76	40	6.50	-	-	-	-	-	-	-	-	-	-	-	-
R mesial parietal	-	-	-	-	-	-	-	-	12	-69	40	6.94	-	-	-	-	-	-	-	-	-	-	-	-
R pre-central sulcus	-	-	-	-	-	-	-	-	26	-24	75	8.34	-	-	-	-	-	-	-	-	-	-	-	-
L post-central sulcus	-	-	-	-	-	-	-	-	-63	0	28	5.85	-	-	-	-	-	-	-	-	-	-	-	-
L MOG	-	-	-	-	-	-	-	-	-47	-73	6	5.50	-	-	-	-	-	-	-	-	-	-	-	-
R MOG	-	-	-	-	-	-	-	-	53	-67	-4	5.56	-	-	-	-	-	-	-	-	-	-	-	-
L Lingual gyrus	-	-	-	-	-	-	-	-	-21	-69	-10	4.50	-	-	-	-	-	-	-	-	-	-	-	-
R Lingual gyrus	-	-	-	-	-	-	-	-	22	-68	-10	5.12	-	-	-	-	-	-	-	-	-	-	-	-
L insula/ BA13	-	-	-	-	-	-	-	-	-38	-19	12	5.47	-	-	-	-	-	-	-	-	-	-	-	-
R insula/ BA13	-	-	-	-	-	-	-	-	40	-14	2	4.96	-	-	-	-	-	-	-	-	-	-	-	-
L Putamen	-	-	-	-	-	-	-	-	-14	18	-2	4.86	-	-	-	-	-	-	-	-	-	-	-	-
R Putamen	-	-	-	-	-	-	-	-	18	16	-2	4.85	-	-	-	-	-	-	-	-	-	-	-	-

Table S2. Activation peaks unique to a mathematical domain in mathematicians

Mathematicians	Analysis > other domains				Algebra > other domains				Topology > other domains				Geometry > other domains			
	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
Mesial frontal orbital	-2	65	-1	4.49	-	-	-	-	-	-	-	-	-	-	-	-
L middle frontal gyrus	-	-	-	-	-	-	-	-	-50	13	27	4.23	-	-	-	-
L inferior temporal	-	-	-	-	-	-	-	-	-	-	-	-	-50	-63	-5	6.39
R inferior temporal	-	-	-	-	-	-	-	-	-	-	-	-	50	-58	-14	5.8
R superior parietal	-	-	-	-	-	-	-	-	-	-	-	-	18	-72	52	5.05
L superior parietal	-	-	-	-	-	-	-	-	-	-	-	-	-23	-66	52	4.94
L supra marginal gyrus	-	-	-	-	-	-	-	-	-	-	-	-	-65	-30	37	4.32
L inferior parietal	-	-	-	-	-	-	-	-	-	-	-	-	-42	-37	42	4.22
R intra occipital sulcus	-	-	-	-	-	-	-	-	-	-	-	-	42	-81	21	5.02

Table S3. Main activation peaks for the non-math > math and the meaningful > meaningless non-math contrasts

	Mathematicians								Controls								Mathematicians > Controls							
	Non-math > Math				Meaningful > Meaningless non-math				Non-math > Math				Meaningful > Meaningless non-math				Non-math > Math				Meaningful > Meaningless non-math			
	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t	x	y	z	t
L inferior AG/TP	-56	-70	25	8.30	-	-	-	-	-51	-66	27	8.53	-42	-69	28	4.58	-	-	-	-	-	-	-	-
R inferior AG/TP	60	-64	22	9.83	57	-67	27	4.79	50	-70	33	5.90	41	-66	34	4.01	56	-69	21	5.45	-	-	-	-
L aMTG/ STS	-59	-4	-19	9.16	56	-15	-23	4.69	-63	-7	-10	6.66	-63	-10	-8	5.19	-	-	-	-	-	-	-	-
R aMTG/ STS	60	-9	-25	8.95	-	-	-	-	63	4	-13	5.16	-	-	-	-	60	-7	-25	4.91	-	-	-	-
Precuneus	2	-60	42	6.90	-	-	-	-	-2	-60	34	6.35	-	-	-	-	-	-	-	-	-	-	-	-
L IFGOrb / BA47	-	-	-	-	-51	43	-11	4.95	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
R FGOrb / BA47	-	-	-	-	-	-	-	-	53	25	33	5.39	-	-	-	-	-	-	-	-	-	-	-	-
L SFG	-	-	-	-	-14	43	52	4.96	-18	58	34	7.88	-21	43	48	4.61	-	-	-	-	-	-	-	-
R SFG	-	-	-	-	26	31	57	4.19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Mesial BA 9, 10	0	55	34	7.70	-	-	-	-	2	53	16	5.26	-	-	-	-	-	-	-	-	-	-	-	-
Mesial frontal Orb/ BA 11	3	59	-7	9.52	-8	41	-16	5.20	-2	53	-16	8.46	-6	44	-17	5.37	-	-	-	-	-	-	-	-
L Cereb. Crus I	-18	-88	-29	6.78	-	-	-	-	-6	-84	-25	7.88	-	-	-	-	-	-	-	-	-	-	-	-
R Cereb. Crus I	27	-79	-34	6.11	-	-	-	-	23	-85	-26	9.08	-	-	-	-	-	-	-	-	-	-	-	-
L MOG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-47	-72	6	4.86	-	-	-	-
R MOG	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	56	-69	21	5.45	-	-	-	-
L para-central /BA4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-15	-31	70	5.04	-	-	-	-
R pre-central	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	26	-24	75	7.21	-	-	-	-
SMA	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-18	52	5.04	-	-	-	-
Heschl / Rolandic Oper	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-39	-18	12	4.99	-	-	-	-
Anterior cingulate	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	37	-7	4.39	-	-	-	-

Table S4. Interaction of meaningfulness by math vs. non-math in mathematicians

Mathematicians	Meaningful > Meaningless math - Meaningful > Meaningless non-math				Meaningful > Meaningless non-math - Meaningful > Meaningless math			
	x	y	z	t	x	y	z	t
L Intraparietal sulcus	-62	-34	42	7.78	-	-	-	-
R Intraparietal sulcus	65	-37	46	6.94	-	-	-	-
L inferior temporal	-60	-58	-8	5.00	-	-	-	-
R inferior temporal	59	-57	-10	5.22	-	-	-	-
L lateral IFG/MFG	-44	50	22	5.14	-	-	-	-
R SF sulcus	26	4	55	4.71	-	-	-	-
R pSTS/AG	-	-	-	-	59	-66	27	5.46
L aMTG	-	-	-	-	-57	-15	-11	4.34
R aMTG	-	-	-	-	57	-10	-19	4.64
Mesial frontal Orb	-	-	-	-	2	67	-13	5.4
Mesial superior frontal	-	-	-	-	-14	43	51	4.07

Table S5. Results of regions-of-interest (ROI) analysis in left-hemispheric language regions during reflection.

The table shows the results of contrasts applied to activation from either the reflection period (top) or the sentence presentation period (bottom) of the main task (math/non-math truth value judgment) in voxels isolated in a subject-specific manner, with each ROI, for their responsiveness to spoken or written sentences. A negative sign in the t test indicates an effect in the direction opposite to that indicated in the column title. Significant trends are highlighted in yellow (p<0.05, uncorrected) and in green (p<0.05 with Bonferroni correction for multiple comparisons across the 7 ROIs).

During reflection period																
	Non-math > Math				**Meaningful > Meaningless non-math**				**Meaningful > Meaningless math**				**Controls > Mathematicians**			
	Mathematicians		**Controls**		**Mathematicians**		**Controls**		**Mathematicians**		**Controls**		**During math**		**During non-math**	
	p	t	p	t	p	t	p	t	p	t	p	t	p	t	p	t
TP	0.039	2.29	0.119	1.67	0.272	1.15	0.248	1.21	0.080	-1.90	0.859	0.18	0.039	2.17	0.227	1.24
aSTS	0.082	1.89	0.003	3.53	0.009	3.09	0.669	0.44	0.289	1.10	0.931	0.09	0.114	1.64	0.031	2.27
pSTS	0.001	4.11	0.862	0.18	0.051	2.15	0.068	1.98	0.426	0.82	0.167	1.46	0.378	0.90	0.957	0.05
TPJ	0.080	1.91	0.083	1.95	0.169	1.46	0.458	0.78	0.993	-0.01	0.799	-0.26	0.468	0.74	0.380	0.90
IFGorb	0.024	2.65	0.380	0.91	0.544	0.63	0.442	-0.80	0.313	-1.06	0.578	-0.57	0.386	-0.88	0.254	-1.17
IFGtri	0.289	1.11	0.029	2.46	0.468	0.75	0.568	0.59	0.451	0.78	0.311	1.06	0.955	0.06	0.512	0.67
BA44	0.077	-1.97	0.492	0.71	0.219	1.31	0.807	-0.25	0.111	1.75	0.967	-0.04	0.442	0.78	0.014	2.64

During sentence presentation																
	Non-math > Math				**Meaningful > Meaningless non-math**				**Meaningful > Meaningless math**				**Controls > Mathematicians**			
	Mathematicians		**Controls**		**Mathematicians**		**Controls**		**Mathematicians**		**Controls**		**During math**		**During non-math**	
	p	t	p	t	p	t	p	t	p	t	p	t	p	t	p	t
TP	0.169	1.46	0.141	1.57	0.888	0.14	0.304	-1.07	0.192	-1.38	0.309	1.06	0.090	-1.76	0.286	-1.09
aSTS	0.002	3.98	0.257	1.18	0.087	-1.85	0.671	0.43	0.029	-2.46	0.540	-0.63	0.647	0.46	0.956	0.06
pSTS	0.033	2.38	0.123	1.64	0.123	-1.65	0.096	-1.78	0.354	-0.96	0.693	-0.40	0.486	0.71	0.507	0.67
TPJ	0.013	2.91	0.002	4.21	0.460	0.76	0.267	-1.18	0.071	1.98	0.179	1.46	0.132	1.57	0.173	1.41
IFGorb	0.001	4.79	0.042	2.27	0.439	-0.81	0.092	-1.83	0.325	-1.04	0.898	-0.13	0.045	2.12	0.033	2.27
IFGtri	0.026	2.57	0.568	0.59	0.109	-1.75	0.220	-1.29	0.634	-0.49	0.545	-0.62	0.947	-0.07	0.794	-0.26
BA44	0.046	2.28	0.960	-0.05	0.052	-2.20	0.357	0.95	0.034	-2.45	0.143	1.55	0.185	1.36	0.399	0.86

Table S6. Main peaks for math > non-math and meaningful > meaningless math, after removal of occasional references to numbers, in mathematicians

Mathematicians	Math > Non-math				Meaningful > Meaningless math			
	x	y	z	t	x	y	z	t
L Intraparietal sulcus	-53	-43	57	8	-50	-51	52	7
R Intraparietal sulcus	50	-42	58	5.4	51	-40	52	5.8
L inferior temporal	-56	-49	-19	6.9	-57	-57	-16	7.1
R inferior temporal	53	-51	-19	5.2	60	-58	-13	7.1
L MFG/BA46	-48	39	23	5.6	-49	34	21	5.8
L MFG/BA9	-47	7	31	5.6	-47	18	50	6.3
L SF sulcus	-24	4	64	4.8	-24	4	61	5
R MFG/BA46	-	-	-	-	51	38	21	5.7
R MFG/BA9 - BA10	-	-	-	-	53	11	21	4.4
R SF sulcus	-	-	-	-	30	8	58	7.2
SMA/Frontal Sup mesial	-	-	-	-	-2	28	51	4.8
BA10	-	-	-	-	-41	50	-14	5.3

Table S7. Subject-specific analyses of the relationships between advanced mathematics, simple arithmetic, and number and formula recognition in mathematicians

The top part of the table shows the activations evoked by mathematical reflection, numbers, and mathematical formulas, in subject-specific voxels isolated by their activation during simple arithmetic, within specified regions of interest (ROIs). The bottom part shows, in the same ROIs, comparisons of activation patterns similarity in several math-related stimuli and tasks, versus math and non-math-related stimuli and tasks. Significant trends are highlighted in yellow (p<0.05, uncorrected) and in green (p<0.05 with Bonferroni correction for multiple comparisons across the 13 ROIs). All approaches indicates that advanced mathematics evokes very similar patterns of activity as simple arithmetic, number recognition, and the recognition of mathematical formulas, particularly in bilateral IPS and IT cortex.

			L Intraparietal sulcus	R Intraparietal sulcus	L inferior temporal	R inferior temporal	L MFG/BA46	L MFG/BA9	L SF sulcus	R MFG/BA46	R MFG/BA9 - BA10	R SF sulcus	SMA/Frontal Sup mesial	L Cerebellum 6th lobule	R Cerebellum 6th lobule
Activation in mathematicians at best localizer peaks for calculation	Math > Non-math reflection	p	0.001	3E-04	0.002	0.009	0.003	1E-04	3E-04	0.006	0.001	0.016	0.063	0.784	0.655
		t	4.10	4.72	3.92	3.04	3.51	5.17	4.75	3.27	3.98	2.73	2.02	0.28	0.46
	Numbers > others	p	0.001	4E-05	0.007	7E-05	0.013	4E-04	0.001	3E-04	0.047	0.004	0.011	0.006	0.115
		t	4.40	5.91	3.14	5.57	2.85	4.64	4.35	4.79	2.18	3.43	2.92	3.28	1.68
	Formulas > others	p	0.018	0.029	0.011	4E-04	0.146	0.026	0.203	0.249	0.469	0.821	0.075	0.919	0.914
		t	2.67	2.43	2.97	4.76	1.55	2.49	1.34	-1.21	-0.75	-0.23	1.95	-0.10	0.11
Statistics on similarity patterns	math*math > math*non-math	p	1.4E-11	3.9E-11	7.0E-10	3.1E-09	3.0E-10	2.6E-08	1.8E-09	9.2E-13	3.3E-10	1.8E-10	5.3E-10	4.5E-13	8.2E-10
		t	19.59	18.19	14.64	13.07	15.59	11.07	13.63	23.96	15.51	16.25	14.95	25.24	14.47
	math*calculation > math*sentence	p	2E-05	1E-04	3.4E-04	0.001	7E-05	7E-06	0.001	0.002	0.001	0.001	4E-04	1E-04	0.011
		t	6.46	5.19	4.71	4.04	5.57	6.92	4.15	3.88	4.05	4.04	4.57	5.23	2.92
	math * formulas > math * non-symbolic pictures	p	0.014	0.301	0.003	0.001	0.003	0.011	0.074	0.651	0.058	0.085	0.077	0.025	0.842
		t	2.82	1.07	3.53	4.18	3.66	2.91	1.93	0.46	-2.06	1.85	1.91	2.50	-0.20
	math * numbers > math * non-symbolic pictures	p	5E-04	0.002	0.001	2E-04	0.002	0.002	4E-04	0.002	0.013	0.029	0.003	0.034	0.072
		t	4.51	3.88	4.06	5.02	3.75	3.81	4.65	3.72	2.84	2.44	3.65	2.34	1.95
	math * formulas > math * words	p	0.807	0.910	0.033	0.179	0.083	0.147	0.292	0.541	0.095	0.273	0.645	0.109	0.228
		t	0.25	-0.11	2.36	1.41	1.87	1.53	1.09	0.63	-1.79	1.14	0.47	1.71	-1.26
	math * numbers > math * words	p	0.062	0.094	0.011	0.021	0.058	0.015	0.006	0.017	0.085	0.129	0.036	0.110	0.669
		t	2.03	1.80	2.91	2.61	2.06	2.77	3.24	2.71	1.85	1.62	2.32	1.71	0.44
	calculation * formulas > calculation * non-symbolic pictures	p	0.001	0.001	2E-06	9E-05	0.006	2E-05	0.006	0.374	0.020	0.930	0.059	0.116	0.427
		t	4.34	4.29	7.88	5.41	3.23	6.20	3.21	0.92	2.62	-0.09	2.06	1.67	0.82
	calculation * numbers > calculation * non-symbolic pictures	p	6E-06	5E-07	3E-07	4E-06	0.002	6E-05	3E-05	0.001	0.001	0.010	0.001	0.014	0.067
		t	6.98	8.70	9.14	7.27	3.86	5.66	6.02	4.23	4.15	3.00	4.37	2.82	1.99
	formulas * (numbers – non-symbolic pictures)	p	5E-06	4E-05	7E-05	1E-04	0.002	6E-05	0.010	0.001	0.003	5E-05	8E-07	0.072	0.513
		t	7.14	5.93	5.57	5.25	3.73	5.67	2.98	4.10	3.52	5.76	8.36	1.95	0.67
	calculation * formulas > calculation * words	p	0.029	0.027	0.006	0.041	0.079	0.222	0.236	0.425	0.454	0.074	0.828	0.063	0.298
		t	2.43	2.48	3.22	2.25	1.90	1.28	1.24	0.82	0.77	1.93	-0.22	2.02	1.08
	calculation * numbers > calculation * words	p	0.003	0.001	0.003	0.002	0.031	0.102	0.015	0.018	0.041	0.002	0.026	0.003	0.091
		t	3.66	4.07	3.55	3.91	2.39	1.75	2.77	2.67	2.25	3.77	2.49	3.62	1.82

Table S8. Volume of activation to different visual stimuli in mathematicians and control subjects

	Principal peaks in both groups				Mathematicians		Controls		Mathematicians > Controls	
	x	y	z	t	volume (mm^3)	Standard error	volume (mm^3)	Standard error	p	t
L EBA	-50	-76	7	19.1	2846	46	2785	63	0.843	0.20
R EBA	54	-67	3	16.8	2961	45	3055	68	0.768	-0.30
L FFA	-38	-49	-20	10.3	261	14	295	15	0.685	-0.41
R FFA	42	-48	-22	13.4	509	16	521	26	0.918	-0.10
L formulas	-51	-61	-11	11.6	2276	90	1334	63	0.035	2.21
R formulas	55	-55	-17	9.36	803	30	394	22	0.008	2.85
L LOC	-48	-73	-5	9.98	3719	120	2401	141	0.076	1.84
R LOC	50	-70	-7	6.33	1125	62	955	50	0.587	0.55
L PPA	-29	-49	-7	12.4	2739	121	1347	86	0.022	2.42
R PPA	29	-49	-8	13.1	2594	130	2393	132	0.781	0.28
L VNFA	-56	-51	-19	7.94	812	46	591	28	0.303	1.05
R VNFA	62	-39	-17	8.44	643	35	341	19	0.060	1.96
VWFA	-42	-45	-17	4.76	82	6	99	7	0.645	-0.47

Appendix. List of mathematical and non-mathematical statements

List of mathematical and non-mathematical statements

1 Analysis

1.1 True :

Statement 1. The Fourier series expansion of a continuous and piecewise $\mathcal{C}^1$ function f converges pointwise to f.

Statement 2. Any locally polynomial function from $\mathbb{R}$ to $\mathbb{R}$ is polynomial.

Statement 3. The function $\frac{1}{\Gamma(z)}$ admits an analytic continuation to the whole complex plane.

Statement 4. Any compact topological group admits a unique probability measure invariant under left-translations.

Statement 5. The set of test functions is dense in every space L^p, for $p \geqslant 1$.

Statement 6. A smooth function whose derivatives are all non-negative is analytic.

1.2 False :

Statement 7. The spaces L^p are separable.

Statement 8. The Fourier transform is an isometry from $L^1(\mathbb{R}^n)$ onto itself.

Statement 9. The topological dual of $\mathrm{L}^\infty(\mathbb{R})$ is $\mathrm{L}^1(\mathbb{R})$.

Statement 10. An inequality between two functions remains valid for their primitives.

Statement 11. There exists a continuous map from the unit ball into itself without any fixed point.

Statement 12. The distributional derivative of the Heaviside step function is the Heaviside step function.

1.3 Meaningless :

Statement 13. Any Dirac Heaviside function admits a Taylor expansion in L^p.

Statement 14. The space $\mathrm{L}^1(\mathbb{R}^n)$ admits a locally polynomial, separable and analytic measure.

Statement 15. In finite measure, the series expansion of the roots of a holomorphic map is reflexive.

Statement 16. The topological dual of a Fourier series admits an analytic continuation.

Statement 17. The trace of the unit ball diverges for some $p \notin \{1, \infty\}$.

Statement 18. Any compact polynomial space is isometric to a unique space L^p.

2 Algebra

2.1 True :

Statement 19. A square matrix with coefficients in a principal ideal domain is invertible if and only if its determinant is invertible.

Statement 20. For even n, any sub-algebra of $M_n(\mathbb{C})$ of dimension ≤ 4 admits a non-trivial centralizer.

Statement 21. The square matrices with coefficients in a field that are equivalent to a nilpotent matrix are the non-invertible matrices.

Statement 22. Up to conjugacy, there only exists 5 crystallographic groups of the plane.

Statement 23. There exists a 13-dimensional algebra of 4×4-complex matrices.

Statement 24. $\mathbb{Q}$ can be canonically embedded into any field of characteristic zero.

2.2 False :

Statement 25. There exists a group of order 169 whose center is reduced to one element.

Statement 26. Any matrix with coefficients in a principal ideal is equivalent to a companion matrix.

Statement 27. A group of which all proper subgroups are abelian is abelian.

Statement 28. In the algebra $M_n(\mathbb{C})$, if two sub-algebras commute, the sum of their dimensions is not greater than n^2.

Statement 29. Any square matrix is equivalent to a permutation matrix.

Statement 30. There exists an infinite order group that admits a finite number of subgroups.

2.3 Meaningless :

Statement 31. Any square invertible ring admits a hexadecimal expansion.

Statement 32. Any matrix with cardinality greater than 3 is factorial.

Statement 33. The field of fractions of an immatricial algebra is embedded in the space of projections.

Statement 34. Any algebra of dimension not greater than 4 is a linear combination of three projections.

Statement 35. There only exists 5 nilpotent canonically additive groups.

Statement 36. The field $\mathbb{R}[i]$ admits a free noetherian centralizer over $\mathbb{Q}$.

3 Topology

3.1 True :

Statement 37. A finite left-invariant measure over a compact group is bi-invariant.

Statement 38. The boundary of the Cantor set equals itself.

Statement 39. There exists non-discrete spaces whose connected components are reduced to one point.

Statement 40. The union of a family of pairwise non-disjoint connected subsets of $\mathbb{C}$ is connected.

Statement 41. Any locally finite bounded set of $\mathbb{R}$ is finite.

Statement 42. The quotient of a topological group by its identity component is totally disconnected.

3.2 False :

Statement 43. Any continuous bijection between two Hausdorff spaces is a homeomorphism.

Statement 44. There exists a continuous function from the unit sphere onto itself without any fixed point.

Statement 45. Any convex compact set of a euclidean space is the intersection of a family of closed balls.

Statement 46. In any topological space, every subspace homeomorphic to an open set is also an open set.

Statement 47. Every complete graph can be embedded into the unit sphere of $\mathbb{R}^3$.

Statement 48. Any inifinite set of real numbers admits at least one accumulation point.

3.3 Meaningless :

Statement 49. Every non-decreasing morphism of the Cantor set is conjugated to a homeomorphism of the unit ball.

Statement 50. Every finite measure on a Hopf algebra is locally modelled on the Haar measure.

Statement 51. The boundary of a homeomorphism has empty interior.

Statement 52. A subset of $\mathbb{C}$ is always left-invariant and right-continuous.

Statement 53. The graph of the completion of a compact group is dense in a partially connected open set.

Statement 54. Evey non-countable measure is the intersection of a family of compact groups.

4 Geometry

4.1 True :

Statement 55. Any vector field on an even-dimensional sphere vanishes.

Statement 56. The eccentricity of a rectangular hyperbola equals $\sqrt{2}$.

Statement 57. In an ellipse, the ratio of the distance from the center to the directrix equals half the major axe over the eccentricity.

Statement 58. The set of points that are equidistant from two given disjoint lines of $\mathbb{R}^3$ is an hyperbolic paraboloid.

Statement 59. A vector bundle whose base is contractible (for instance, a ball) is trivializable.

Statement 60. The euclidean orthogonal group has exactly two connected components.

4.2 False :

Statement 61. The stereographic projection of the sphere minus one point in the Euclidean space is bounded.

Statement 62. A holomorphic function on a Riemann surface is constant.

Statement 63. Any compact surface is diffeomorphic to an algebraic surface.

Statement 64. At any point P of a directrix of a hyperbola, two tangent lines intersect.

Statement 65. The orthogonal projection of the focus of a parabola on one of its tangent is on the directrix.

Statement 66. Any C^1 vector field on a torus admits a singularity.

4.3 Meaningless :

Statement 67. Any Riemannian metric is conjugated to the Haar measure.

Statement 68. The stereographic projection admits $\sqrt{2}$ as Euler characteristic.

Statement 69. The set of points equidistant from two Riemann surfaces is compatible with a paraboloid.

Statement 70. Any holomorphic compact fiber bundle is a particular sphere.

Statement 71. Any variety locally contractible is included in a two-sheeted hyperboloid.

Statement 72. Any locally ellipsoidal submersion is the exponential of a Riemann surface.

5 Non-math

5.1 True :

Statement 73. In all Ancient Mediterranean cultures, bulls were considered deities.

Statement 74. In Ancient Greece, a citizen who could not pay his debts was made a slave.

Statement 75. The VAT is a French invention and is a direct consumption tax.

Statement 76. The flag of the Esperanto community is predominantly green.

Statement 77. Apart from the Vatican, Gibraltar is the world's smallest country.

Statement 78. The concept of robots and avatars was already present in Greek mythology.

5.2 False :

Statement 79. The Paris metro was built before the Istanbul one.

Statement 80. All borders in Europe, except for Yugoslavia, were set at the end of World War II.

Statement 81. The poet Aragon never joined the Communist party.

Statement 82. The end of the Council of Trent coincides with the fall of the Western Roman Empire.

Statement 83. All members of the Club des Cordeliers were guillotined during the "Terror".

Statement 84. In every society, the market is considered an essential and founding institution.

5.3 Meaningless :

Statement 85. The potato flag was guillotined at the end of the Council of Trent.

Statement 86. The institutionalized market drinks Western Roman avatars.

Statement 87. Every indebted green beans have a scientific background.

Statement 88. The Greek mythology is the smallest alcohol derived from the VAT.

Statement 89. Most of the robotic bulls never met Yugoslavia.

Statement 90. A poet is a predominantly green tax over the metro.