A Solution procedure for the one-dimensional model

We recall the problem in its dimensional form:

$$\begin{cases} -c_f v + \partial_x \sigma = 0 & \text{in } (-L, L) \\ \tau_\alpha \dot{\sigma} - \tau_\alpha \sigma \partial_x v + \sigma = \tau_\alpha G \partial_x v + \sigma_\mathrm{a} & \text{in } (-L, L) \\ \sigma = 0 & \text{at } x = \pm L \end{cases}$$

Here we have grouped the bulk and shear viscosity terms, without loss of generality in 1D.

A.1 Viscous limit

Assume $\tau_\alpha \ll 1$. In an interval of $[-L, L]$ with a constant c_f, we get

$$L_f^2 \partial_{xx} v - v = 0$$

with $L_f = \sqrt{\frac{\tau_\alpha G}{c_f}}$, thus $v = -U_0 \sinh \frac{x}{L_f} + U_1 \cosh \frac{x}{L_f}$.

If c_f is constant on $[-L, L]$, $U_1 = 0$ by symmetry and $\sigma = \sigma_\mathrm{a} \left(1 - \frac{\cosh \frac{x}{L_f}}{\cosh \frac{L}{L_f}} \right)$, thus $V_0 = \frac{L_f \sigma_\mathrm{a}}{\tau_\alpha G \cosh \frac{L}{L_f}}$. These solutions are reported in figure 3*b* and figure 3*c*.

If $c_f = c_f^0$ for $|v| < v^*$ and $c_f = c_f^1$ for $|v| > v^*$, call $x^* > 0$ the point at which the increasing function v is such that $v(x^*) = -v^*$. Then if $v^* < \frac{L_f \sigma_\mathrm{a}}{\tau_\alpha G \cosh \frac{L}{L_f}}$, we have $x^* < L$, and on the interval $[x^*, L]$ we have:

$$v_{|_{[x^*,L]}} = -v^* \cosh \frac{\xi(x - x^*)}{L_f} + \left(v^* \sinh \frac{\xi(L - x^*)}{L_f} - \sigma_\mathrm{a} \frac{L_f}{\tau_\alpha G \xi} \right) \frac{\sinh \frac{\xi(x-x^*)}{L_f}}{\cosh \frac{\xi(L-x^*)}{L_f}}$$

and

$$v_{|_{[0,x^*]}} = -v^* \sinh \frac{\xi(x - x^*)}{L_f}$$

with $\xi = \sqrt{c_f^1 c_f^0}$.

We can prescribe v^* by imposing the continuity of stress at x^*, which is equivalent in 1D to impose the continuity of the derivative of v, we find :

$$v^* = \frac{L_f \sigma_\mathrm{a}}{\tau_\alpha G} \frac{\sinh \frac{x^*}{L_f}}{\xi \sinh \frac{x^*}{L_f} \sinh \frac{\xi(L-x^*)}{L_f} + \cosh \frac{x^*}{L_f} \cosh \frac{\xi(L-x^*)}{L_f}}$$

This result is shown in supplementary figure S1

If now we want to prescribe v^* rather than x^*, we can identify the desired value by numerical inversion, as v^* is strictly monotonic in x^*, see supplementary figure S2.

A.2 Full viscoelastic model

In permanent regime, $\dot{\sigma} = v\partial_x \sigma$, thus the system to solve is:

$$\partial_x v = \frac{\tau_\alpha c_f v^2 + \sigma - \sigma_\mathrm{a}}{\eta + \tau_\alpha \sigma}$$
$$\partial_x \sigma = c_f v$$

There is no known analytical solution to this system in general, we resort to numerical integration using Octave software in order to approximate its solution.

Results are shown in supplementary figure S3. The purely viscous approximation matches closely the solution of the visco-elastic problem for De $\lesssim 0.1$, and is still a fair approximation for De and up to 1.

B Two-dimensional problem

B.1 Variational formulation

In this section we detail how the problem is transformed into the variational form. Following the 1D results, we take $De = 0$. With the hypothesis that substrate deformation rate is small, $\tilde{\dot{\boldsymbol{\varepsilon}}} = \boldsymbol{D}(\boldsymbol{v})$, which is the symmetrical part of the tensor $\boldsymbol{\nabla}(\boldsymbol{v})$. So equations 4 become, dropping the $\tilde{\cdot}$ symbol:

$$-\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \zeta \boldsymbol{v} = 0, \tag{8}$$

with $\boldsymbol{\sigma} = \lambda \mathrm{tr}\,(\boldsymbol{D}(\boldsymbol{v}))\boldsymbol{I} + 2\boldsymbol{D}(\boldsymbol{v}) + \boldsymbol{\sigma}_\mathrm{a}$, where we have denoted $\lambda = \kappa / G$.

Then, we multiply the first equation by an arbitrarily test-function $\boldsymbol{w} \in W = H^1(\Omega_\mathrm{c})^2$ and we integrate over Ω_c:

$$\int_{\Omega_\mathrm{c}} -(\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) \cdot \boldsymbol{w}\,\mathrm{d}x = \int_{\Omega_\mathrm{c}} -\zeta \boldsymbol{v} \cdot \boldsymbol{w}\,\mathrm{d}x, \qquad \forall \boldsymbol{w} \in W$$

Using Green's formula, we can write:

$$\int_{\Omega_\mathrm{c}} (\boldsymbol{\nabla} \cdot \boldsymbol{\sigma}) \cdot \boldsymbol{w}\,\mathrm{d}x + \int_{\Omega_\mathrm{c}} \boldsymbol{\sigma} : \boldsymbol{D}(\boldsymbol{w})\,\mathrm{d}x = \int_{\partial\Omega_\mathrm{c}} \boldsymbol{\sigma} : (\boldsymbol{w} \otimes \boldsymbol{n})\,\mathrm{d}s$$

Remark that on $\partial\Omega_\mathrm{c}$, $\boldsymbol{\sigma} : (\boldsymbol{w} \otimes \boldsymbol{n}) = (\boldsymbol{\sigma}\boldsymbol{n}) \cdot \boldsymbol{w}$ which is equal to zero due to the boundary condition $\boldsymbol{\sigma}(\boldsymbol{v}) \cdot \boldsymbol{n} = 0$. So the integral over $\partial\Omega_\mathrm{c}$ is zero and the problem rewrites:

$$\int_{\Omega_\mathrm{c}} \boldsymbol{\sigma}(\boldsymbol{v}) : \boldsymbol{D}(\boldsymbol{w})\,\mathrm{d}x = \int_{\Omega_\mathrm{c}} \zeta \boldsymbol{v} \cdot \boldsymbol{w}\,\mathrm{d}x$$

From the constitutive equation, we have:

$$\begin{aligned}\boldsymbol{\sigma}(\boldsymbol{v}) : \boldsymbol{D}(\boldsymbol{w}) &= \lambda \mathrm{tr}\,(\boldsymbol{D}(\boldsymbol{v}))(I : \boldsymbol{D}(\boldsymbol{w})) + 2\boldsymbol{D}(\boldsymbol{v}) : \boldsymbol{D}(\boldsymbol{w}) + \boldsymbol{\sigma}_\mathrm{a} : \boldsymbol{D}(\boldsymbol{w}) \\ &= \lambda \mathrm{tr}\,(\boldsymbol{D}(\boldsymbol{v}))\boldsymbol{\nabla} \cdot \boldsymbol{w} + 2\boldsymbol{D}(\boldsymbol{v}) : \boldsymbol{D}(\boldsymbol{w}) + \boldsymbol{\sigma}_\mathrm{a} : \boldsymbol{D}(\boldsymbol{w})\end{aligned}$$

For now, we assume that the myosin pre-stress $\boldsymbol{\sigma}_\mathrm{a}$ is isotropic, thus $\boldsymbol{\sigma}_\mathrm{a} = \sigma_\mathrm{a}\boldsymbol{I}$. Then we finally have the variational formulation of the problem:

$$\int_{\Omega_\mathrm{c}} \lambda \mathrm{tr}\,(\boldsymbol{D}(\boldsymbol{v})) \cdot \boldsymbol{\nabla} \cdot \boldsymbol{w}\,\mathrm{d}x + \int_{\Omega_\mathrm{c}} 2\boldsymbol{D}(\boldsymbol{v}) : \boldsymbol{D}(\boldsymbol{w})\,\mathrm{d}x + \int_{\Omega_\mathrm{c}} \zeta \boldsymbol{v} \cdot \boldsymbol{w}\,\mathrm{d}x = -\sigma_\mathrm{a} \int_{\Omega_\mathrm{c}} \boldsymbol{\nabla} \cdot \boldsymbol{w}\,\mathrm{d}x \qquad \forall \boldsymbol{w} \in W. \tag{9}$$

Next we define the finite element space $W_h = \{\boldsymbol{w}_h \in W, w_h|_{K\in\mathcal{T}^i_{\mathrm{c},h}} \in P_2(K)\}$. The friction field ζ_h is calculated from a previous guess $\boldsymbol{v}_h^{k-1}$ as stated in the main text, and we solve equation (9) for $\boldsymbol{v}_h^k$ in W_h using test functions $\boldsymbol{w}_h \in W_h$ using the finite element library Rheolef.

B.2 Fitting of the myosin pre-stress parameter σ_a

From the solution of the 1D problem in the viscous case, we see that the myosin pre-stress σ_a appears as a multiplying factor of the solution. This is convenient in terms of data fitting, since this allows to calculate the traction field $\boldsymbol{T}_1$ for a unit choice of $\sigma_\mathrm{a} = \sigma_\mathrm{a}^1$, and then to minimise explicitly the difference between $\boldsymbol{T}_1$ and $\boldsymbol{T}_\mathrm{exp}$. Here is the precise derivation of this result.

Let $\boldsymbol{v}_1$ be solution of the problem:

$$\zeta_{v^*}(|\boldsymbol{v}|)\boldsymbol{v} - \lambda\boldsymbol{\nabla}\cdot\boldsymbol{v} - \boldsymbol{\nabla}\cdot 2\boldsymbol{D}(\boldsymbol{v}) = 0 \quad \text{on } \partial\Omega_\mathrm{c} \tag{10a}$$

with $v^* = v_1^*$ and the boundary condition:

$$\boldsymbol{D}(\boldsymbol{v}_1)\boldsymbol{n} = -\sigma_\mathrm{a}^1\boldsymbol{n} \quad \text{on } \partial\Omega_\mathrm{c} \tag{10b}$$

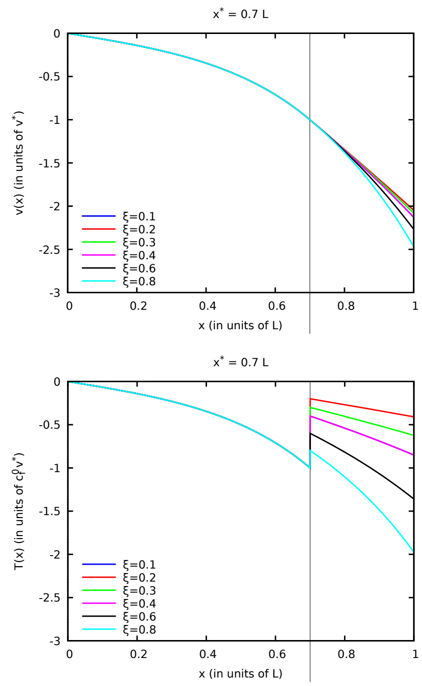
Then by plugging $\boldsymbol{v}_1 = \frac{\sigma_\mathrm{a}^1}{\sigma_\mathrm{a}^2}\boldsymbol{v}_2$ in equation (10a), we get:

$$\zeta_{v_1^*}\left(\frac{\sigma_\mathrm{a}^1}{\sigma_\mathrm{a}^2}|\boldsymbol{v}_2|\right)\boldsymbol{v}_2 - \boldsymbol{\nabla}\cdot 2\eta\boldsymbol{D}(\boldsymbol{v}_2) = 0 \quad \text{on } \partial\Omega_\mathrm{c}$$

and thus $\boldsymbol{v}_2$ is solution of equation (10a) with $v^* = v_2^* = \frac{\sigma_\mathrm{a}^2}{\sigma_\mathrm{a}^1}\boldsymbol{v}_1^*$ and the boundary condition:

$$\boldsymbol{D}(\boldsymbol{v}_2)\boldsymbol{n} = -\sigma_\mathrm{a}^2\boldsymbol{n} \quad \text{on } \partial\Omega_\mathrm{c}$$

if and only if the choice of the friction law is scale invariant, i.e. $\zeta_{\lambda v^*}(\lambda|\boldsymbol{v}|) = \zeta_{v^*}(|\boldsymbol{v}|)$.



(*a*)

(*b*)

Figure S1: (*a*) Actin velocity and (*b*) traction forces for various values of $\xi = \sqrt{c_f^1/c_f^0}$, see supplementary text A

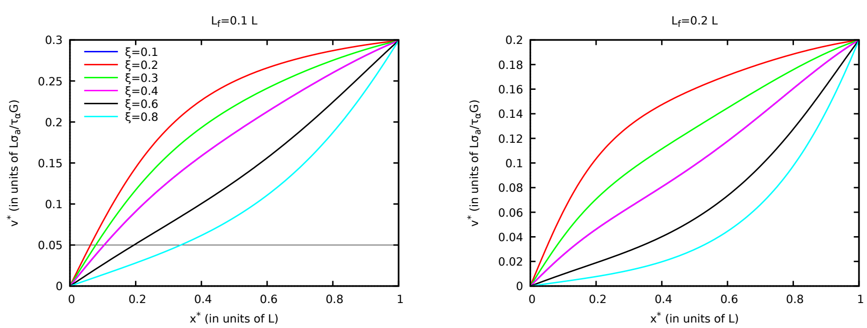
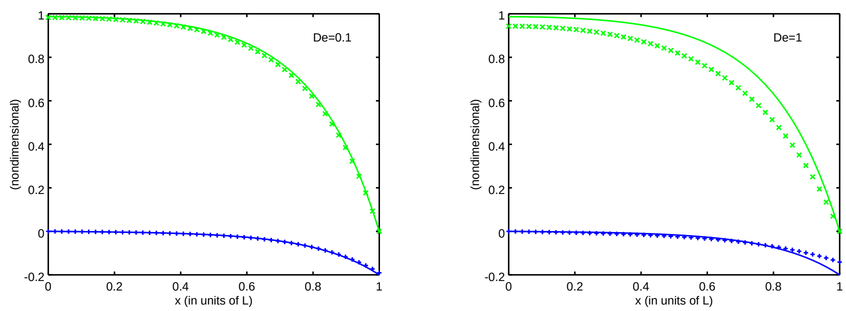


Figure S2: Determination of the position x^* such that $v(x^*) = v^*$: plots of the threshold velocity v^* corresponding to solutions obtained with given parameters L_f and ξ and choice of x^*, see supplementary text A.



(*a*) (*b*)

Figure S3: Comparison of results with the full visco-elastic model equation (5) with the solution in the viscous limit De $= 0$, for two values of the Deborah number. Green '$\times$' symbols, stress σ, blue '+' symbols, velocity v, see supplementary text A.

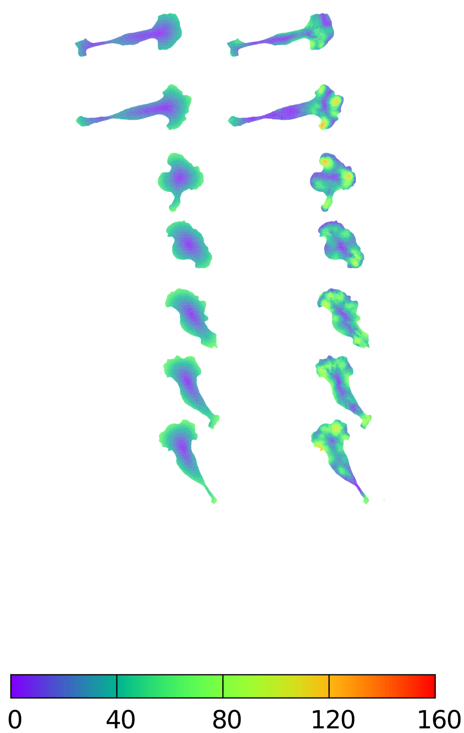


Figure S4: Comparison of intensity of traction field predicted by the model (left) and calculated from experimental observations (right) during a migration experiment of T24 cell number 2, instants shown are $t = 0$, 8, 16, 24, 32, 40 and 48 min. Magnitudes are given in Pascals.

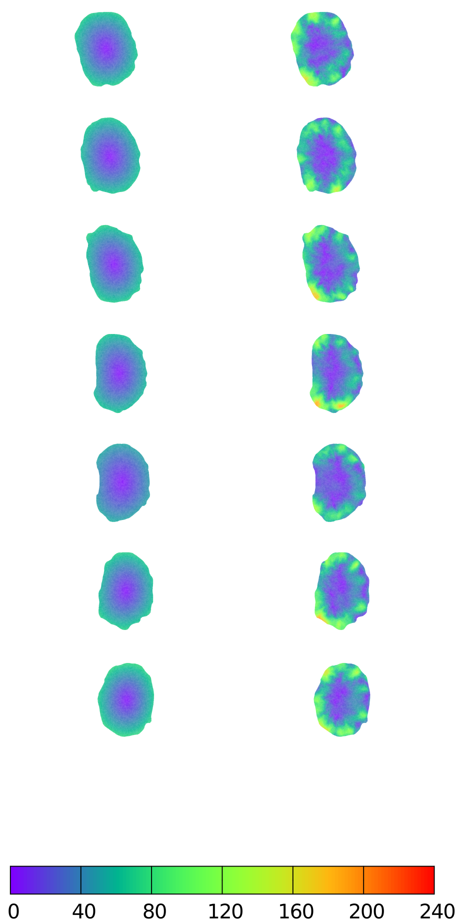


Figure S5: Comparison of intensity of traction field predicted by the model (left) and calculated from experimental observations (right) during a migration experiment of an RT112 cell, instants shown are $t = 0$, 12, 24, 36 48 60 and 72 min ($t = 0$ is also show in figure 4*b*) The RT112 morphology is much simpler and does not change in the course of migration. The model predicts a centripetal traction field which globally agrees with observations throughout migration. The same shortcomings of the model as for the lamella region in T24 cells, figure 4*a*, are noted. Here no parameter is adjusted, ζ_0 and σ_a are taken to the value chosen for T24 cell experiment at $t = 0$, figure 4*a*. Magnitudes are given in Pascals.