
Approximate Backbone Tree Algorithm

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1: Input: Complete undirected graph:  $G \equiv (V(G), E(G))$ 
2: Input: Distance attribute:  $d : V(G) \times V(G) \rightarrow \mathbb{R}^+$ 
3: Input: Distance threshold:  $\delta$ 
4: Input: Outlier tolerance:  $\rho$ 
5:  $T := \text{minimum\_spanning\_tree}(G)$ 
6:  $B := \text{Graph}(\{T.\text{root}\}, \emptyset)$   $\triangleright$  Init backbone tree with the root of MST
7:  $S := \emptyset$   $\triangleright$  Set of vertices close enough to the backbone
8: while  $|V(B)| + |S| < (1 - \rho)|V(G)|$  do
9:    $V_s := T.\text{neighbours}(V(B))$   $\triangleright$  Next branch need to start near current
   backbone
10:   $R := T.\text{induced\_subgraph}(V(T) \setminus V(B))$   $\triangleright$  Create (disconnected) subgraph
   by removing backbone vertices
11:   $P := \max_{v_1 \in V_s \cap V(R), v_2 \in V(R)} R.\text{shortest\_path}(v_1, v_2)$   $\triangleright$  Longest shortest path
   between starting set and non-backbone vertices
12:   $B.\text{add\_path}(P)$   $\triangleright$  Add branch to backbone
13:   $S := \{v \in V(G) \setminus V(B) \mid \exists v_b \in V(B), d(v_b, v) < \delta\}$ 
14: end while
15: for  $u := B.\text{backbone}_D\text{FS}()$  do  $\triangleright$  Depth-first-search through all backbone
   vertices
16:   for  $v := u.\text{backbone}_c\text{children}()$  do  $\triangleright$  Remove close contiguous backbone
   nodes
17:     if  $d(u, v) < \delta$  then
18:       if  $|\{w \in V(G) \setminus V(B) \mid \exists v_b \in V(B) \setminus v, d(v_b, w) < \delta\}| < \rho|V(G)|$  then
19:          $\text{merge\_backbone}_v\text{ertices}(u, v)$   $\triangleright$  remove  $v$  from backbone
20:       else
21:         if  $|\{w \in V(G) \setminus V(B) \mid \exists v_b \in V(B) \setminus u, d(v_b, w) < \delta\}| < \rho|V(G)|$ 
then
22:            $\text{merge\_backbone}_v\text{ertices}(v, u)$   $\triangleright$  remove  $u$  from backbone
23:         end if
24:       end if
25:     end if
26:   end for
27: end for
28: Output:  $B$ 
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