APPENDIX A: *Analytical calculation of SPEN diffusion effects upon employing linearly-swept 180° pulses*

This Appendix presents the results involved in calculating diffusion effects when employing 180° sweeps for performing the SPEN encoding, rather than the 90° chirp pulse used in the main text. Presenting such analysis is important, as 180° sweeps appear as the most promising way of decoupling the diffusion and the imaging gradients cross-coupling terms (*cf*. Fig. 4). By contrast to sequences using chirp 90° pulses to excite the spin packets, 180° sweeps in SPEN schemes are tuned to invert the spins in the transverse plane; excitation is thus created at the beginning of the experiment by a (usually slice-selective) homogeneous 90° pulse. As a result of this, the phase accumulated by spins during a gradient G_e of period T_e and a gradient G_a of duration T_a (*cf*. Fig. 2C) is

$$K_{local}^{180^\circ-SPEN}(t,z) = \frac{d(\phi_e(z)+\phi_a(t,z))}{dz} = \frac{d}{dz}\begin{cases} \gamma G_e z t & 0 \le t \le t_{180}(z) \\ \phi_{rf}\left[t_{180}(z)\right] + \left[\phi_{rf}\left[t_{180}(z)\right] - \gamma G_e z[t_{180}(z)]\right] + \gamma G_e z[t - t_{180}(z)], & t_{180}(z) \le t \le T_e \\ 2\phi_{rf}\left[t_{180}(z)\right] - \gamma G_e z\left[t_{180}(z) - t\right] + \gamma G_a z t, & T_e \le t \le T_a \end{cases} \quad \text{(A1)}$$

Expressions for the timing $t_{180}(z)$ when the adiabatic RF sweep reaches the resonance frequency of spin packet at a particular z coordinate, as well as for the phase ϕ_{RF} taken by the B$_1$ field at the time of this inversion, are similar to those given in the main text for the chirp 90° pulse. Taking the spatial derivatives involved in Eq. (A1) yields the relevant K_{local} wavenumbers,

$$K_{local}^{180^\circ-SPEN}(t,z) = \begin{cases} \gamma G_e t & 0 \le t \le t_{180}(z) \\ \gamma G_e[t - 2t_{180}(z)], & t_{180}(z) \le t \le T_e \\ \gamma G_e\left[T_e - 2t_{180}(z)\right] + \gamma G_a t, & T_e \le t \le T_a \end{cases} \quad \text{(A2)}$$

Substituting this expression into Eq. 8 provides the full argument of the exponential attenuation function,

$$A_{180^\circ-SPEN}(t,z) = \exp\left[-D\gamma^2\left(\begin{array}{c} \dfrac{t_{180}^3(z)G_e^2}{3} + \dfrac{G_e^2\left(t_{180}^3(z) + \left(-2t_{180}(z) + T_e\right)^3\right)}{3} + \\ \dfrac{\left(2t_{180}(z)G_e - G_e T_e\right)^3 + \left(-2t_{180}(z)G_e + G_a T_a + G_e T_e\right)^3}{3G_a} \end{array}\right)\right] \quad \text{(A3)}$$

These K_{local} and attenuation functions still have to account for the diffusion gradients G_d, and for the purging gradients G_{pr} that in 180°-encoded SPEN MRI are needed for shifting the symmetric phase parabola that the adiabatic sweep imparts, to one corner of the FOV (*cf*. Fig. 2C). Taking these additional factors into account, the total K_{local} wavenumber becomes

$$K_{local}(t,z) = \begin{cases} -\gamma G_{pr} t & 0 \le t \le T_{pr} \\ -\gamma G_{pr} T_{pr} + \gamma G_d (t - T_{pr}) & T_{pr} \le t \le T_{pr} + \delta \\ -\gamma G_{pr} T_{pr} + \gamma G_d \delta + \gamma G_e (t - (T_{pr} + \delta)) & T_{pr} + \delta \le t \le T_{pr} + \delta + t_{180}(z) \\ -\gamma G_{pr} T_{pr} + \gamma G_d \delta + \gamma G_e (t - 2t_{180}(z)) & T_{pr} + \delta + t_{180}(z) \le t \le T_{pr} + \delta + T_e \\ -\gamma G_{pr} T_{pr} + \gamma G_d \delta + \gamma G_e (T_e - 2t_{180}(z)) - \gamma G_d (t - (T_{pr} + \delta + T_e)) & T_{pr} + \delta + T_e \le t \le T_{pr} + \delta + \Delta \\ -\gamma G_{pr} T_{pr} + \gamma G_e (T_e - 2t_{180}(z)) + \gamma G_a t & T_{pr} + \delta + \Delta \le t \le T_{pr} + \delta + \Delta + T_a \end{cases}$$

(A4)

And the ensuing exponential attenuation function

$$A(t,z)=\exp\left[-D\gamma^2\left(\begin{array}{l}\dfrac{G_{pr}^2T_{pr}^3}{3}+\dfrac{\delta\left(G_d^2\delta^2-3G_d\delta G_{pr}T_{pr}+3G_{pr}^2T_{pr}^2\right)}{3}+\\ \dfrac{\left(G_d\delta+t_{180}(z)G_e-G_{pr}T_{pr}\right)^3+\left(-G_d\delta+G_{pr}T_{pr}\right)^3}{3G_e}+\\ \dfrac{\left(t_{180}(z)G_e-\delta(G_d+G_e)+(-G_e+G_{pr})T_{pr}\right)^3}{3G_e}+\\ \dfrac{\left(G_d\delta-2t_{180}(z)G_e-G_{pr}T_{pr}+G_e(\Delta+T_{pr})\right)^3}{3G_e}\\ \dfrac{\left(-2t_{180}(z)G_e+G_eT_e+G_d\left(-\Delta+2\delta+T_e\right)-G_{pr}T_{pr}\right)^3}{3G_d}+\\ \dfrac{\left(2t_{180}(z)G_e+G_d\left(\Delta-\delta-T_e\right)-G_eT_e+G_{pr}T_{pr}\right)^3}{3G_d}\\ \dfrac{\left(-2t_{180}(z)G_e+G_aT_a+G_eT_e-G_{pr}T_{pr}\right)^3}{3G_a}+\\ \dfrac{\left(2t_{180}(z)G_e-G_eT_e+G_{pr}T_{pr}\right)^3}{3G_a}\end{array}\right)\right].$$

(A5)

APPENDIX B: *Experimental validation of exact vs. PGSE-based b-value effects in dSPEN sequence*

Figure B.1 assesses experimentally the efficiency of all the DW pulse sequences variants explored, with a series of single-scan 2D MRI comparisons conducted for water samples. Diffusion-sensitizing gradients were applied separately along the readout, the low-bandwidth (phase-encode or SPEN) and the slice-selection directions, such that the final ADC map is a geometric mean of the three directions. In all cases, a high consistency is evidenced by the diffusion coefficient values arising over the entire phantom sample for all five sequences –regardless of the diffusion-gradient's encoding axis– once the correct *b*-values have been computed and are taken into account for computing the corresponding maps.

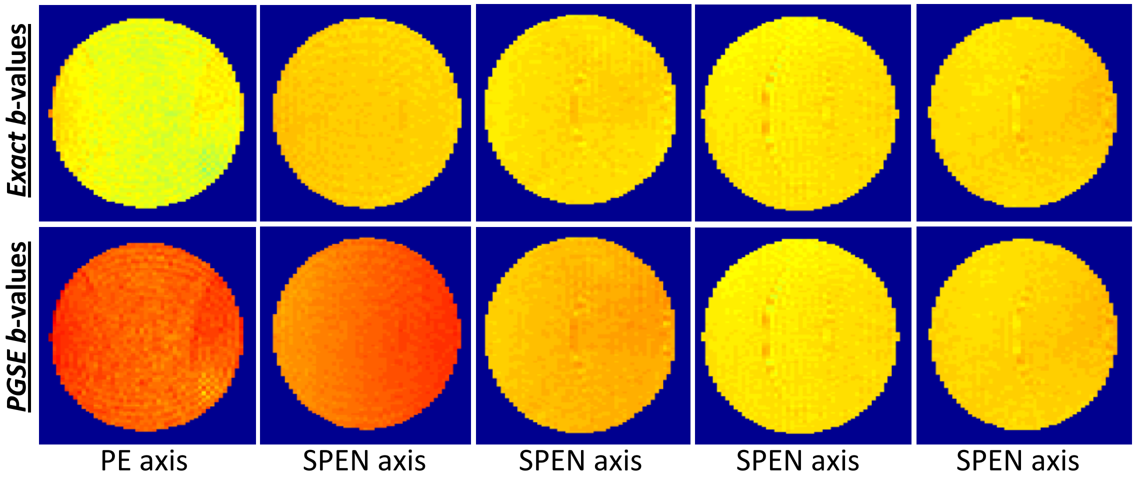

Figure B1. ADC maps derived from b^{Exact}- (top) and from b^{PGSE}-values (bottom) for a neat H_2O sample scanned by the five pulse sequences described in Fig. 2. PGSE parameters were δ= 3ms, Δ = 14ms (except for Fig. 1D where Δ = 11.4ms), $0.8 \leq G_d \leq 3.1$ G/cm gradients applied along all three measurements directions (readout, PE/SPEN and slice-selection). The maps represent an average of these three directions. The mean diffusion values arising over the entire phantom from b^{PGSE}- / b^{Exact}-values are: 2.16±0.03 / 1.9±0.03 (A); 2.15±0.03 / 1.99±0.03 (B); 2.03±0.03 / 1.97±0.03 (C); 1.96.03±0.03 / 1.96±0.03 (D); 1.99±0.03 / 1.98±0.03 (E) (all values $\times 10^{-3}$ mm^2/s). Minor features disturbing the flatness of the images reflect artifacts arising from the various ultrafast imaging modalities. Common scanning parameters: square FOV = 30x30 mm^2, nominal resolution = 0.4×0.4 mm^2, 2 mm slice. Other gradient and timing values: (A) T_{90}=T_{180}=2ms, T_a=21ms, total scan duration = 51.5ms; (B) T_{90}=T_a=21ms, G_e=1.2 G/cm, G_a=4.5 G/cm, total scan duration = 60ms; (C) T_{90}=2ms, T_{180}=T_a/2=10.5ms, G_e=0.8 G/cm, G_a=3 G/cm, total scan duration = 51ms; (D) T_{90}=2ms, T_{180}=T_a/2=15.4ms, G_e=0.4 G/cm, G_a=4.4 G/cm, total scan duration = 65ms; (E) T_{90}=2ms, T_{180}=T_a/2=10.5ms, G_e=0.8 G/cm, G_a=3 G/cm, total scan duration = 62ms.