Supporting Information

Supporting Information is provided with this submission.

Supporting Figure S1

For constant applied RF excitation field B_1 with no R_1 relaxation or off-resonance, R_2 determines the excitation dynamic for any pulse duration p or nominal excitation angle $\alpha = \gamma B_1 p$. The effect varies over the range of relative R_2 (thick lines). To achieve any particular $\alpha < 2\pi$, the rotation is truncated at the appropriate p (blue $\rightarrow$ yellow gradient). For magnetization across the range of R_2, the final orientations (dashed lines tracing the range over R_2) can differ greatly from the nominal excitation angle (dotted lines).

Supporting Figure S2

The trace of angles from 0° to 90° can be used to compute efficiency of a hard pulse excitation for any T_2 and nominal excitation angle α. Excitation efficiency is $M_{xy}/\sin(\alpha)$ for M_{xy} immediately following the excitation.

Supporting Figure S3

The trace of angles from 0° to 90° can be used to chart efficiency of a hard pulse saturation for any T_2 and nominal saturation angle α. Saturation efficiency is $M_{xy}/(1-\cos(\alpha))$ for M_{xy} immediately following the saturation.

Supporting Figure S4

The trace of angles from 90° to 180° can be used to compute effectiveness of a hard pulse inversion for any T_2 and nominal inversion angle α. Inversion effectiveness is $M_z/\cos(\alpha)$ for M_z immediately following the inversion.

Supporting Information for *Improved Cortical Bone Specificity in UTE MR Imaging*

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This work was supported by NIH P01 CA159992 and P41 EB015891 and by GE Medical Systems.

Approximate Word Count: 772

Submitted September 22, 2015, to *Magnetic Resonance in Medicine* as a Supporting Information for *Improved Cortical Bone Specificity in UTE MR Imaging.*

Concurrent excitation and relaxation dynamics

During RF excitation by constant applied B_1, if there is no R_1 relaxation or off-resonance, the ratio of R_2 to R_2^{crit} or 'relative R_2' determines the excitation dynamic. Tracing the nominal 2π rotation induced by the field over a duration $(\gamma B_1)^{-1} = 2T_2^{\mathrm{crit}}$ fully illustrates the magnetization orientations (Sup. Fig. S1) effected by any pulse duration p or nominal excitation angle $\alpha = \gamma B_1 p$. Therefore this trace describes the effect of any hard pulse; the 90° rotation discussed in *Improved Cortical Bone Specificity in UTE MR Imaging* is a special case of such. Without incorporation of other considerations, such as off-resonance or spatial selectivity, but given a maximum-B_1 constraint, a hard pulse is the maximally-efficient excitation, *i.e.* it is the best-possible approximation to an ideal excitation.

Excitations, saturations and inversions

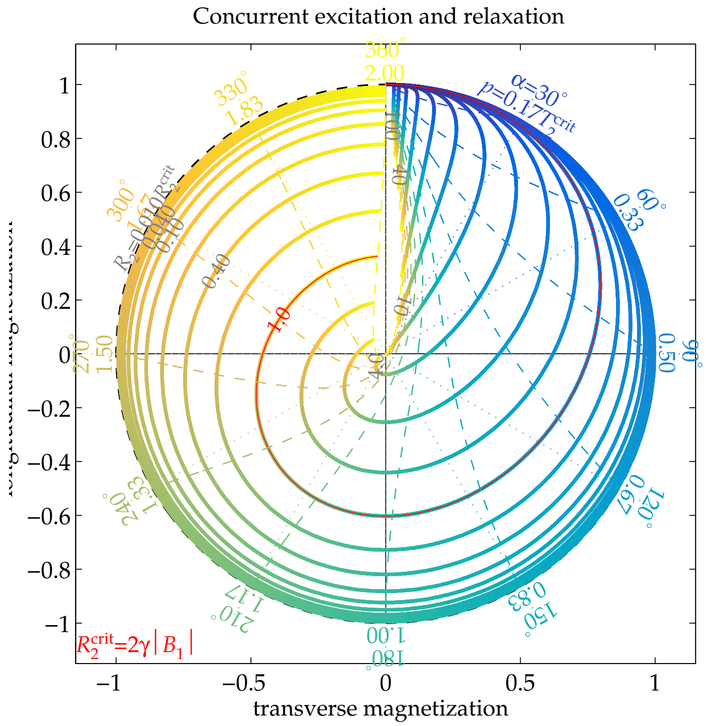
The trace of excitation effects can be used to chart the efficacy of excitation, saturation and inversion hard pulses, which vary with T_2 and nominal excitation angle α. An ideal excitation tips magnetization to angle α in the range of 0 to $\pi/2$, which creates transverse magnetization of $\sin(\alpha)$. The excitation efficiency of a real excitation is then $M_{xy}/\sin(\alpha)$ for M_{xy} immediately following the excitation (Sup. Fig. S2). For example, a pair of excitations using 14 μT and 1.4 μT pulses for 22.5° tip angles might be used to target magnetization from collagen-bound water in compact bone, which is attributed a T_2 of 0.4 ms. The pulses impose critical T_2 rates of 0.84 ms and 8.4 ms; the targeted magnetization then has relative T_2 of 0.48 and 0.05, and it is excited with efficiency of 87% and 35% under the excitations. If magnetization with T_2 of 10 ms is also present, it will be excited with efficiencies of 99% and 95%. Therefore a difference image will give the targeted T_2 an intensity weighting more than $10\times$ that of the longer T_2.

Analogously, a saturation pulse is intended to diminish the M_z component of magnetization by tipping the magnetization to angle α in the range of 0 to $\pi/2$, which ideally leaves a longitudinal component of $1 - \cos(\alpha)$. Therefore the saturation efficiency of a real saturation pulse is $(1 - M_z)/(1 - \cos(\alpha))$ for M_z immediately following the saturation (Sup. Fig. S3). This can be used, for example, to specify parameters for a simple rectangular long-T_2 suppression pulse, as well as to delineate its fundamental performance limits. To suppress longer-T_2 magnetization while retaining that with shorter-T_2, high and low efficiencies would be desired at the longer and shorter T_2 values respectively. For 10 ms and 0.4 ms T_2 magnetizations, the B_1 amplitude and tip angle can be chosen by centering the 1.4 decade difference in relative T_2 over the steepest change in efficiency. On this principle, B_1 should be set to place the longer T_2 just below the critical threshold and a tip angle near 90° should be used. With a 1.07 μT, 85° saturation pulse applied, the remaining normalised M_z for the short-T_2 component will be 0.632 greater than that of the long-T_2 component. This can additionally be used to facilitate accurate T_1 mapping for short-T_2 components by saturation-recovery-type methods, as estimation of T_1 is premised upon knowledge of the M_z remaining after the interrogation pulse.

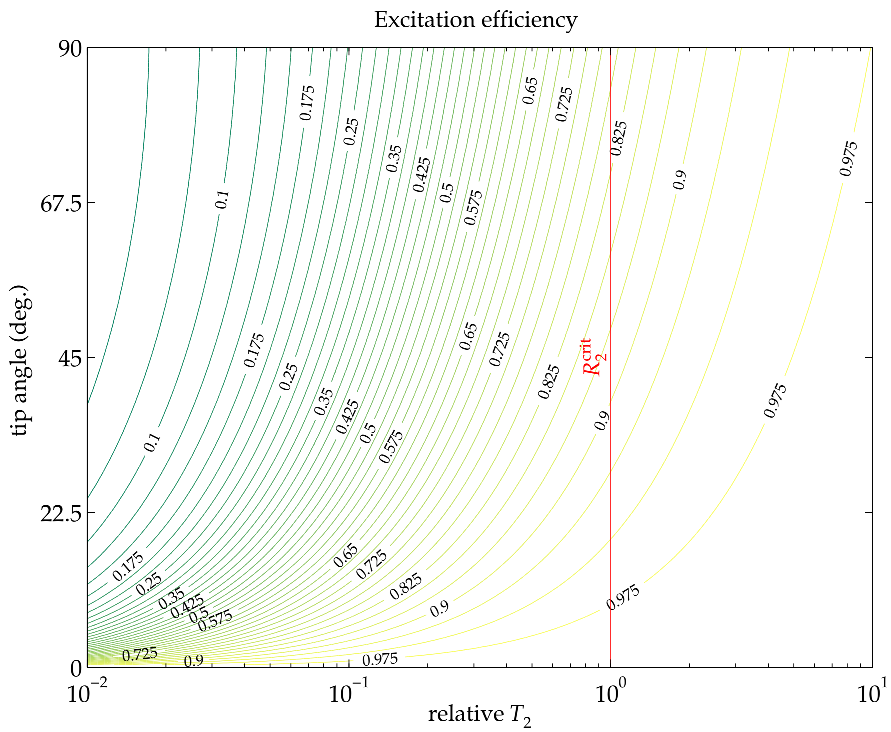
Finally, an inversion ideally flips magnetization to an angle α in the range of $\pi/2$ to π so that its polarity is negated. The effectiveness of an inversion is $M_z/\cos(\alpha)$ for M_z immediately following the inversion (Sup. Fig. S4), which is 1 when maximally effective or is negative when ineffective. Analysis of contrast in pulse sequences that apply inversion pulses with short-T_2 magnetization present depends upon characterization of the influence that an inversion pulse has upon the short-T_2 components. In such analysis, the $\cos(\alpha)$ term that typically characterises inversion effects must be scaled by the efficacy to accurately represent M_z. For instance, a sequence that applies an inversion pulse and then performs one or more excitations and encodes will have some T_2-dependence in its T_1 contrast. Setting sequence parameters such as inversion time or flip angle to exploit the dependence necessitates incorporation of the inversion effectiveness. Following a 180° inversion pulse using B_1 of 20 μT, magnetization with T_2 of 0.4 ms, a relative T_2 of 0.7, is actually inverted with effectiveness of 0.46, which is much greater than 0, which might otherwise be assumed for a short T_2.

As a consequence, if choosing parameters under an assumption of no inversion for short-T_2 components, an inversion time may be chosen that unintentionally nulls the component of interest!

List of Figures



Supporting Figure S1: For constant applied RF excitation field B_1 with no R_1 relaxation or off-resonance, R_2 determines the excitation dynamic for any pulse duration p or nominal excitation angle $\alpha = \gamma B_1 p$. The effect varies over the range of relative R_2 (thick lines). To achieve any particular $\alpha < 2\pi$, the rotation is truncated at the appropriate p (blue $\rightarrow$ yellow gradient). For magnetization across the range of R_2, the final orientations (dashed lines tracing the range over R_2) can differ greatly from the nominal excitation angle (dotted lines).



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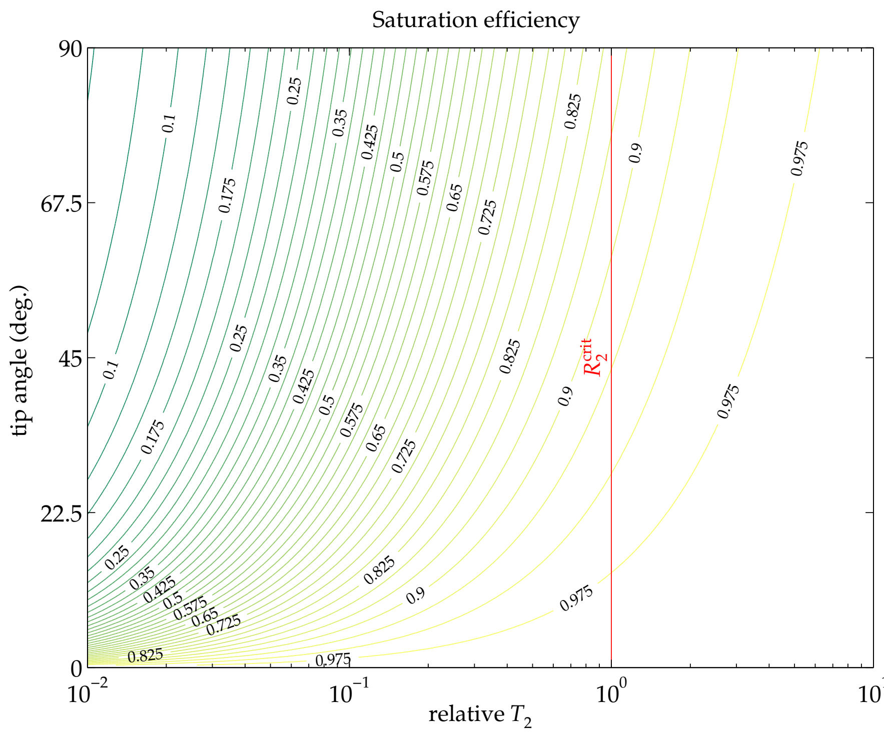
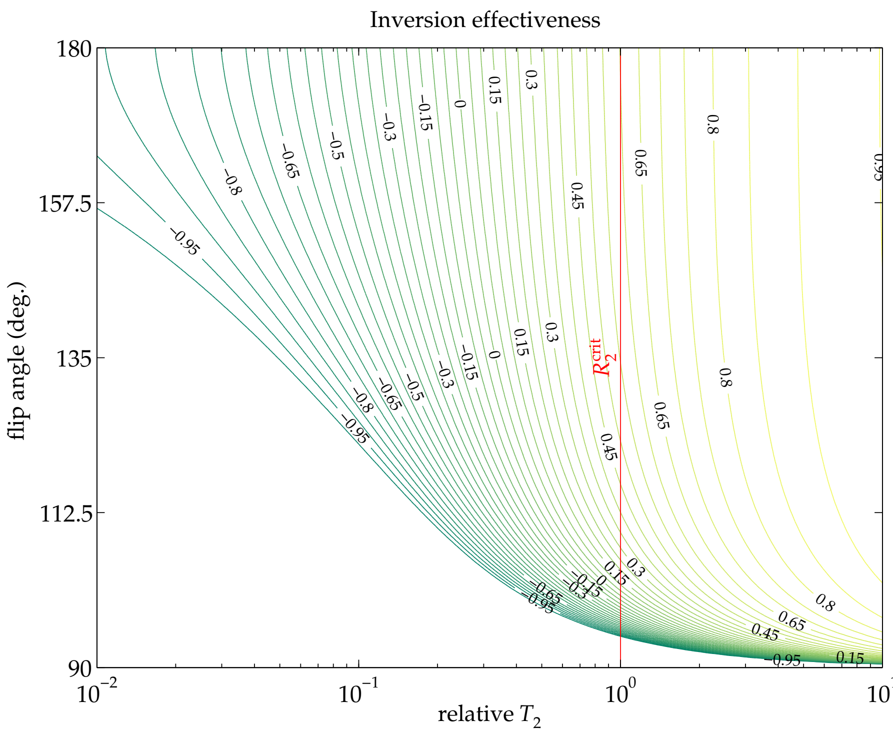


Supporting Figure S3: The trace of angles from 0° to 90° can be used to chart efficiency of a hard pulse saturation for any T_2 and nominal saturation angle α. Saturation efficiency is $M_{xy}/(1 - \cos(\alpha))$ for M_{xy} immediately following the saturation.



Supporting Figure S4: The trace of angles from 90° to 180° can be used to compute effectiveness of a hard pulse inversion for any T_2 and nominal inversion angle α. Inversion effectiveness is $M_z/\cos(\alpha)$ for M_z immediately following the inversion.