Supporting Information

S2 Text

Equivalence of Eqs. (16) and (20). We show that the CERAMIC statistic of Eq. (16) can be rewritten as Eq. (20), in terms of the BLUP-imputed genotypes. It is sufficient to show that $\boldsymbol{Z}_W^T\hat{\boldsymbol{G}}_W = \boldsymbol{F}^T\boldsymbol{G}_R$. By a similar argument to that in S1 Text, we have that $\mathbf{1}_w^T\boldsymbol{Z}_W = 0$. By Eq. (18),
$\boldsymbol{Z}_W^T\hat{\boldsymbol{G}}_W = \boldsymbol{Z}_W^T[\mathbf{1}_w(\mathbf{1}_r^T\boldsymbol{\Phi}_R^{-1}\mathbf{1}_r)^{-1}\mathbf{1}_r^T\boldsymbol{\Phi}_R^{-1} + \boldsymbol{\Phi}_{WR}\boldsymbol{M}]\boldsymbol{G}_R$. Applying $\boldsymbol{Z}_W^T\mathbf{1}_w = 0$, we obtain $\boldsymbol{Z}_W^T\hat{\boldsymbol{G}}_W = \boldsymbol{Z}_W^T\boldsymbol{\Phi}_{WR}\boldsymbol{M}\boldsymbol{G}_R = \boldsymbol{F}^T\boldsymbol{G}_R$ by the definition of $\boldsymbol{F}$. From this, it is clear that the CERAMIC statistic of Eq. (16) can be rewritten as Eq. (20), in terms of the BLUP-imputed genotypes.