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Supplemental Information

Shape Selection of Surface-Bound Helical Filaments: Biopolymers on Curved Membranes

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Shape selection of surface-bound helical filaments: biopolymers on curved membranes

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Supplementary Material

Here we examine dependence of the filament twist and tilt on the potential in the weak adhesion limit. The Hamiltonian in the general case is given by

$$\mathcal{H} = \int_0^L ds \left[\frac{C}{2}(\theta')^2 + \frac{C}{2}\frac{\sin^4(\theta)}{r^2} + \frac{K}{2}\left(\psi' - \frac{\sin(2\theta)}{2r} - \omega_0\right)^2 + \frac{V}{2}\sin^2(\psi)\right] \tag{1}$$

This yield the corresponding equations of motion

$$C\theta'' = 2C\frac{\sin^3(\theta)cos(\theta)}{r^2} - K\frac{\cos(2\theta)}{2r}(\psi' - \frac{\sin(2\theta)}{2r} - \omega_0) \tag{2}$$

$$K(\psi' - \frac{\sin(2\theta)}{2r})' = \frac{V}{2}\sin(2\psi) \tag{3}$$

We now consider the limit of weak binding potential i.e. the $V \to 0$ limit. Here, the filament is close to its unperturbed state with $\psi \to \omega_0 s$ and $\theta \to 0$. This allows us to rewrite the equations of motion, (2) and (3), to linear order in θ, ψ as

$$C\theta'' = -\frac{K}{2r}\big(\psi' - \frac{\theta}{r} - \omega_0\big) \tag{4}$$

$$K\Big(\psi' - \frac{\theta}{r}\Big)' = \frac{V}{2}\sin(2\psi) \tag{5}$$

We look for the solution of these equations for θ, ψ in the form $\theta = \theta_0 + \delta\theta$ and $\psi' = \omega + \delta\omega$, where θ_0, ω are constants and $\delta\theta, \delta\omega$ are oscillating parts, such that $\langle\delta\theta\rangle = \langle\delta\omega\rangle = 0$. The twist equation of motion, eq.5 implies

$$K\Big(\delta\omega - \frac{\delta\theta}{r}\Big)' = \frac{V}{2}\sin(2\omega s) \tag{6}$$

which yields upon integration

$$\delta\omega - \frac{\delta\theta}{r} = -\frac{V}{4\omega K}\cos(2\omega s) \tag{7}$$

This relates the difference between the oscillating part of the twist rate and Frenet torsion from the oscillating part of the tilt to the strength of the potential. From the tile equation of motion, eq.4, keeping terms of the lowest order in $\theta_0, \omega, \delta\theta, \delta\omega$, we get a constant part

$$0 = -\frac{K}{2r}\Big(\omega - \frac{\theta_0}{r} - \omega_0\Big) \tag{8}$$

and an oscillating part

$$C\delta\theta'' = -\frac{K}{2r}\Big(\delta\omega - \frac{\delta\theta}{r}\Big) \tag{9}$$

Using eq.7, this reduces to

$$C\delta\theta'' = \frac{V}{8\omega r}\cos(2\omega s) \tag{10}$$

yielding

$$\delta\theta = -\frac{V}{32\omega^3 rC}\cos(2\omega s) \tag{11}$$

We can now use the relation between the oscillating parts of the twist and tilt, eq.7, to get

$$\delta\omega = -\frac{V}{4\omega}\cos(2\omega s)\Big(K^{-1} + \frac{C^{-1}}{8\omega^2 r^2}\Big) \tag{12}$$

This allows us to write the oscillatory part of the twist angle $\delta\psi$ as

$$\delta\psi = -\frac{V}{8\omega^2}\sin(2\omega s)\Big(K^{-1} + \frac{C^{-1}}{8\omega^2 r^2}\Big) \tag{13}$$

Thus we see that both the tilt angle and twist vary sinusoidally along the filament with the variations having magnitude of order V. We now consider the total energy of the filament in this weak adhesion limit.

$$\mathcal{E} = \int_0^L ds\Big[\frac{C}{2}(\delta\theta')^2 + \frac{C}{2}\frac{\theta^4}{r^2} + \frac{K}{2}\Big(\omega + \delta\omega - \frac{\theta_0 + \delta\theta}{r} - \omega_0\Big)^2 + \frac{V}{4}[1 - \cos(2\psi)]\Big] \tag{14}$$

Taking $\psi \sim \omega s + \delta\psi$ and using equations 11,8,7,12,13, and keeping terms to order V, we get

$$\begin{aligned}\mathcal{E} = \int_0^L ds\Big[\frac{C}{2}\Big(\frac{V}{16\omega^2 rC}\Big)^2\sin^2(2\omega s) + \frac{C}{2}\frac{\theta_0^4}{r^2} + \frac{K}{2}\Big(-\frac{V}{4\omega K}\cos(2\omega s)\Big)^2 \\ + \frac{V}{4}\sin(2\omega s)\frac{V}{4\omega^2}\sin(2\omega s)\Big(K^{-1} + \frac{C^{-1}}{8\omega^2 r^2}\Big)\Big]\end{aligned} \tag{15}$$

and averaging over one period

$$\langle\mathcal{E}\rangle/L = \Big[\frac{C}{4}\big(\frac{V}{16\omega^2 rC}\big)^2 + \frac{C}{2}\frac{\theta_0^4}{r^2} + \frac{V^2}{64\omega^2 K} + \frac{V^2}{32\omega^2}\Big(K^{-1} + \frac{C^{-1}}{8\omega^2 r^2}\Big)\Big] \tag{16}$$

Using 8 and setting $\theta_0 = (\omega - \omega_0)r$, this reduces to

$$\langle\mathcal{E}\rangle/L = \frac{C}{2}(\omega - \omega_0)^4 r^2 - \frac{V^2}{16\omega^2}\Big[\frac{K^{-1}}{4} + \frac{3C^{-1}}{64\omega^2 r^2}\Big] \tag{17}$$

Minimizing this with respect to ω allows us to compute the non-trivial constant contribution to the twist rate in the weak adhesion limit (for finite curvature $(\omega_0 r)^{-1}$).

$$\omega \simeq \omega_0 - \Big[\frac{V^2}{32C\omega_0^3 r^2}\Big(\frac{K^{-1}}{2} + \frac{3C^{-1}}{16\omega^2 r^2}\Big)\Big]^{1/3};\ \text{for finite } r \tag{18}$$

Comparing this to the perturbative solution for flat interfaces (taking the limit of $1/r \to 0$ in eqs. 13 and 15) we arrive and the effective mean energy,

$$\langle \mathcal{E} \rangle / L = \frac{K}{2}(\omega - \omega_0)^2 + \frac{V^2}{32K\omega^2}; \text{ for r} \to \infty. \tag{19}$$

Minimizing with respect to ω for weak binding, we find

$$\omega \simeq \omega_0 - \frac{V^2}{16K^2\omega_0^3}; \text{ for } r \to \infty. \tag{20}$$

Relative to the $V^{2/3}$ scaling of strain on curved surfaces, the V^2 dependence indicates a weaker coupling to surface potential on flat surfaces.