Supporting Online Material

Wang, D., *et al*. 2009. How do variable substitution rates influence Ka and Ks calculations? *Genomics Proteomics Bioinformatics* 7: 116-127.
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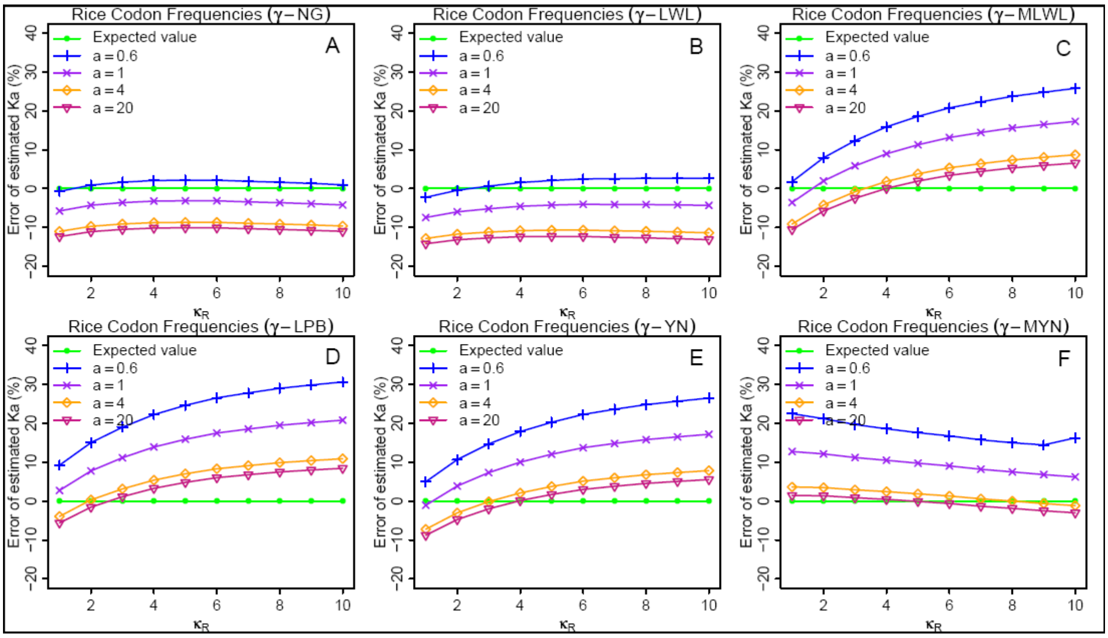

Figure S1 Percentage errors of estimated Ka in the condition of expected ω=0.3. We plotted percentage errors of Ka, averaging over 2,000 pairs of sequences by γ-NG, γ-LWL, γ-MLWL, γ-LPB, γ-YN, and γ-MYN, when κ_Y =3.75 and κ_R varies from 1 to 10, in the condition of expected ω=0.3. Sequences were simulated with the rice codon frequencies derived from rice protein-coding genes. The percentage error was calculated by the formula: 100% × [(estimated value) – (expected value)] / (expected value).

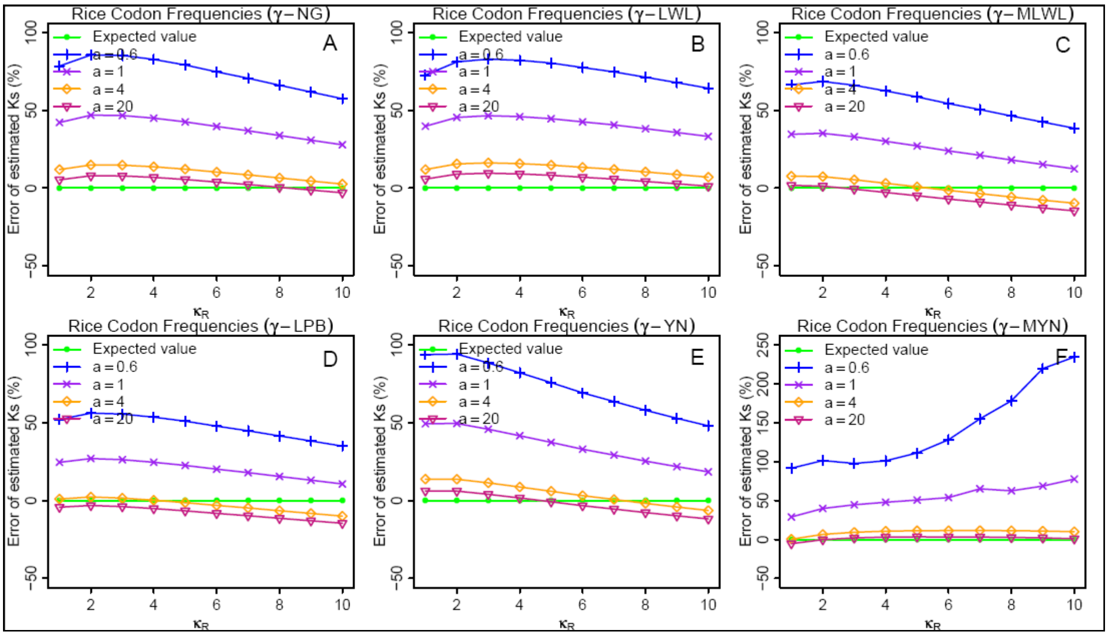

Figure S2 Percentage errors of estimated Ks in the condition of expected ω=0.3.

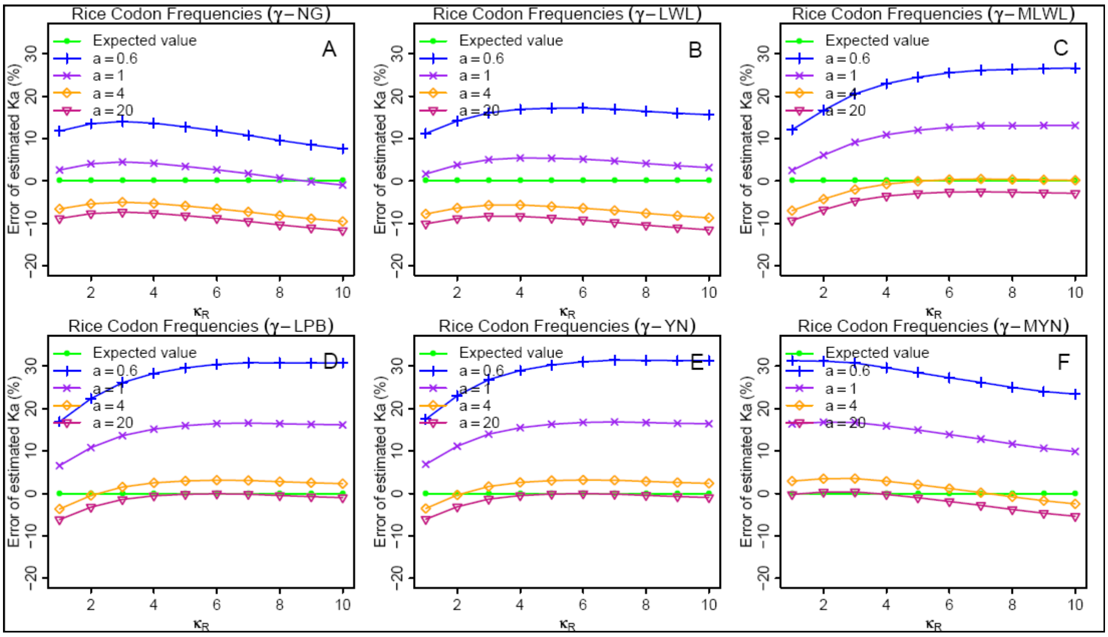

Figure S3 Percentage errors of estimated Ka in the condition of expected ω=1.

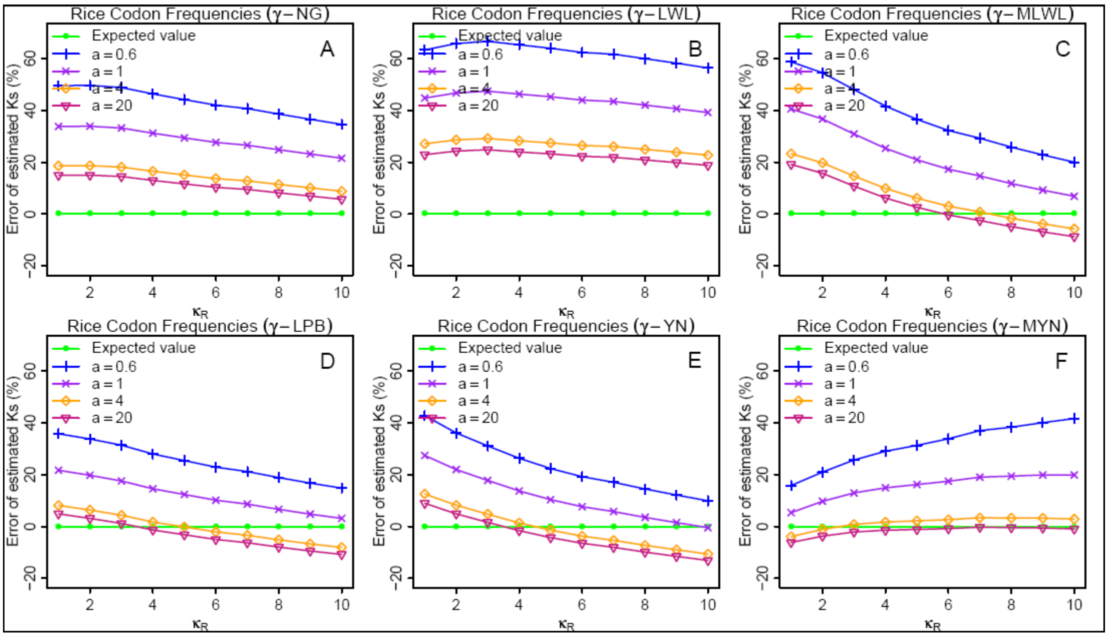

Figure S4 Percentage errors of estimated Ks in the condition of expected ω=1.

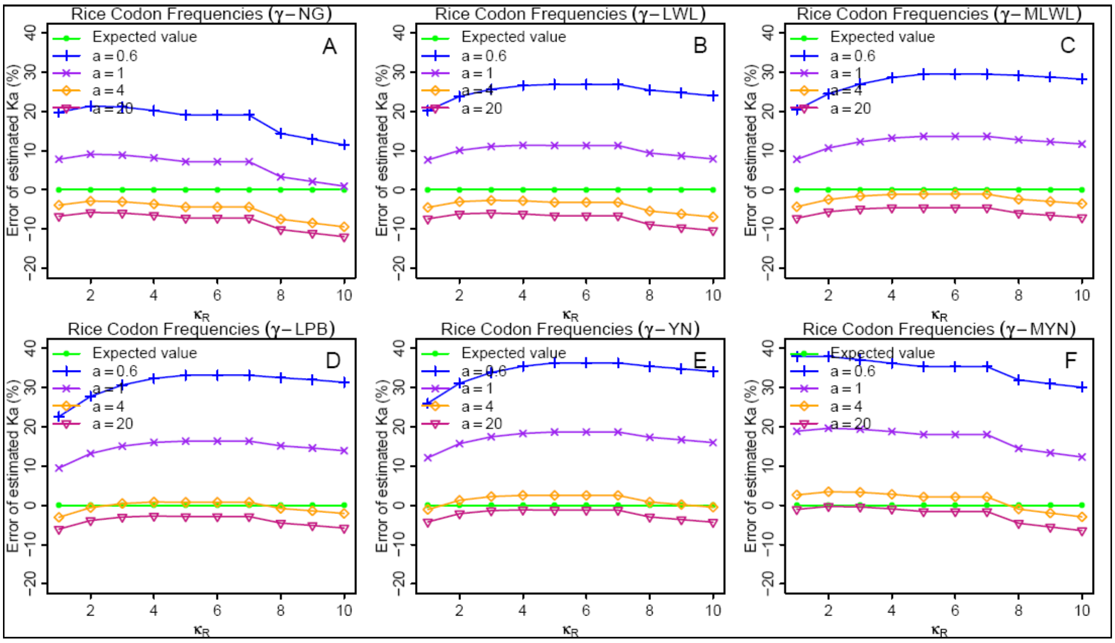

Figure S5 Percentage errors of estimated Ka in the condition of expected ω=3.

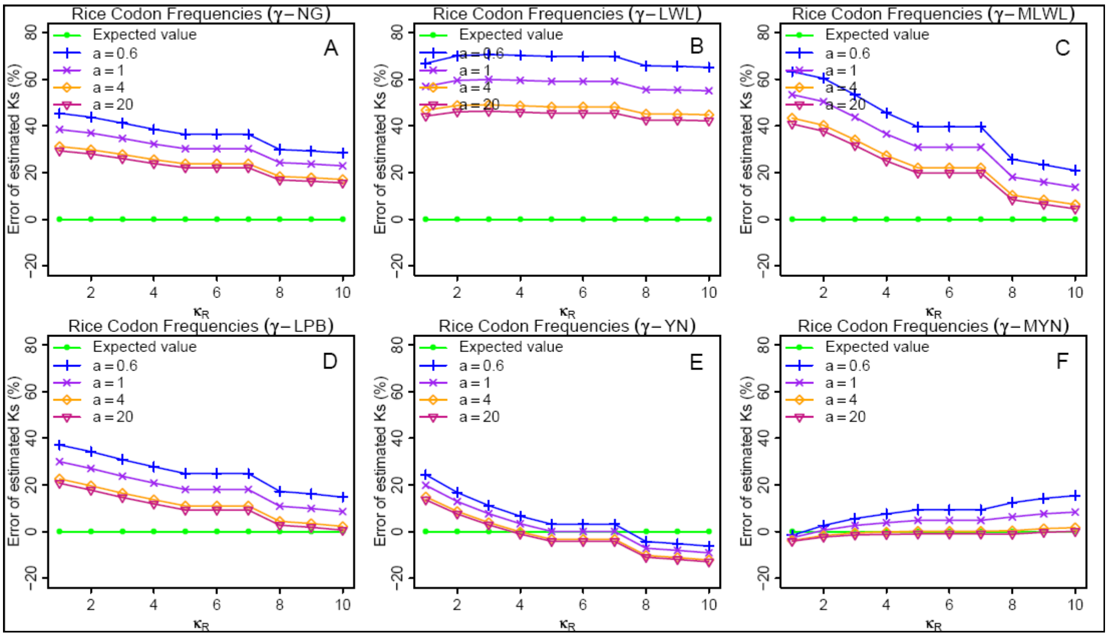

Figure S6 Percentage errors of estimated Ks in the condition of expected ω=3.

Table S1 The optimal indexes of Ka, Ks and ω based on a combination of six values of "*a*" and seven methods

A. Optimal indexes of Ka estimates when ω=0.3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	11547.54	19270.02	45690.11	68170.24	65472.78	59409.25	1615916
0.6	**26.762**	**46.5106**	3583.898	5906.709	5509.021	4103.508	3214.378
1	151.3296	241.4875	1452.381	2500.191	2288.53	1536.083	926.1002
4	865.4107	1259.782	384.0482	577.932	518.8555	302.3096	46.34939
20	1141.536	1643.657	291.8221	340.7176	314.1287	206.6636	**26.0993**
∞	1215.59	1745.952	**279.856**	**297.192**	**278.252**	**195.967**	33.0314

B. Optimal indexes of Ks estimates when ω=0.3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	3082012	2981870	2258079	1735173	1787809	3610240	34605799
0.6	55902.95	57327.73	31747.3	23120.76	24272.01	55233.91	226132.5
1	15783.32	17221.99	6899.777	4379.574	4774.601	13507.57	30977.05
4	1168.573	1673.119	**353.971**	**255.337**	**257.101**	686.887	962.5671
20	263.1357	468.3207	660.1346	791.9355	734.2831	**432.199**	82.79042
∞	**168.859**	**298.789**	825.3646	997.0964	926.6511	501.2602	**57.3466**

C. Optimal indexes of ω estimates when ω=0.3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	0.520786	0.49793	0.400133	0.324493	0.333247	0.429116	0.308941
0.6	0.140898	0.14589	0.050369	0.022864	0.026797	0.074154	0.128693
1	0.076312	0.086779	0.018305	**0.0075**	**0.00907**	0.028887	0.047647
4	0.023863	0.035786	**0.01493**	0.019197	0.017591	**0.01104**	0.003026
20	0.014569	0.025608	0.020829	0.027804	0.025273	0.013745	**0.00104**
∞	**0.0126**	**0.02331**	0.022782	0.030323	0.027555	0.014966	0.001218

D. Optimal indexes of Ka estimates when ω=1

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	58907.95	122534.3	152935.5	190618.5	186805	209668.6	1767742
0.6	1346.055	2518.576	5405.957	7853.904	7672.49	8201.335	7759.976
1	**84.5399**	**190.445**	1231.588	2171.984	2107.548	2257.878	2009.991
4	490.2283	488.9044	**71.9116**	69.31302	69.25104	71.45276	**56.4888**
20	846.2706	931.8537	211.2094	**52.528**	**61.723**	**50.4726**	77.65364
∞	948.914	1061.436	269.6196	76.96815	88.08028	74.54203	110.9996

E. Optimal indexes of Ks estimates when ω=1

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	254219.3	476868.4	292138.5	178142.5	179026.7	141584.2	6071823
0.6	18824.58	38940.68	15259.76	6646.666	6760.638	6390.708	10466.78
1	8305.24	19374.12	5804.733	1780.699	1863.829	1943.952	2602.04
4	2183.109	7035.53	1340.185	**280.365**	**342.608**	**538.866**	71.47309
20	1305.79	4997.238	927.8209	380.2644	438.6278	636.4287	**62.364**
∞	**1127.42**	**4554.74**	**865.451**	431.4219	488.9049	686.2089	93.00846

F. Optimal indexes of ω estimates when ω=1

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	0.680797	0.84767	0.345015	0.323869	0.341138	1.127662	1.372097
0.6	0.3752	0.694656	0.175829	0.117227	0.129481	0.234247	0.077914
1	0.326145	0.656782	0.149473	0.094841	0.106057	0.172474	0.043912
4	0.276189	0.612876	0.123537	0.074518	0.084615	0.123632	0.022785
20	0.263804	0.601034	0.117321	0.069847	0.079656	0.113416	0.019249
∞	**0.26077**	**0.59807**	**0.1158**	**0.06872**	**0.07846**	**0.11102**	**0.01847**

G. Optimal indexes of Ka estimates when ω=3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	117427.5	268509.4	283279.6	326755.9	321937.9	389226.3	2517063
0.6	3290.478	6330.325	7626.924	9599.631	9529.177	11594.53	12160.55
1	459.1958	1017.903	1504.38	2180.039	2171.417	2906.486	2975.361
4	**318.135**	**188.506**	**58.6364**	**19.4999**	**20.0034**	**34.0555**	**64.5459**
20	673.8138	570.9903	316.4082	174.011	171.1972	71.22115	103.1384
∞	783.6474	696.6727	413.0874	249.2517	245.3681	120.4233	152.7502

H. Optimal indexes of Ks estimates when ω=3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	55840.05	191942.6	102904.4	50192.05	47548.5	7604.48	428626.3
0.6	13680.85	46729.61	18906.76	6913.527	6467.506	1159.572	955.1489
1	9492.459	33548.46	12275.49	3896.237	3639.276	829.01	266.5879
4	5916.31	22341.78	7075.586	1756.811	1659.059	**719.589**	**22.1801**
20	5140.708	19895.41	6023.675	1372.149	1309.021	733.6384	29.34963
∞	**4957.59**	**19315.7**	**5779.83**	**1286.85**	**1231.92**	739.8674	35.28303

I. Optimal indexes of ω estimates when ω=3

a	γ-NG	γ-LWL	γ-MLWL	γ-LPB	γ-MLPB	γ-YN	γ-MYN
0.2	7.395717	**3.81038**	25.56014	57.74035	61.63727	250.1332	742.1982
0.6	**0.80107**	4.44375	**1.24481**	1.42028	1.838754	13.50247	10.38286
1	1.96461	6.751934	1.858692	**0.40606**	**0.58259**	5.513446	3.636866
4	3.668106	9.447704	3.212627	0.692306	0.6752	1.42975	0.535923
20	4.171876	10.16467	3.649382	0.94639	0.888573	0.939573	0.22437
∞	4.300364	10.34319	3.761977	1.019101	0.951723	**0.84778**	**0.17253**

Table S2 Substitution rate matrices for Markov models of nucleotide substitution used in this study (*1*)

Model	From	To: T	C	A	G
JC69 (Jukes and Cantor 1969) (*2*)	T	–	α	α	α
	C	α	–	α	α
	A	α	α	–	α
	G	α	α	α	–
K80 (Kimura 1980) (*3*)	T	–	α	β	β
	C	α	–	β	β
	A	β	β	–	α
	G	β	β	α	–
F84 (Felsenstein 1984) (*4*)	T	–	$(1+\kappa/g_Y)\ \beta g_C$	βg_A	βg_G
	C	$(1+\kappa/g_Y)\ \beta g_T$	–	βg_A	βg_G
	A	βg_T	βg_C	–	$(1+\kappa/g_R)\ \beta g_G$
	G	βg_T	βg_C	$(1+\kappa/g_R)\ \beta g_A$	–
HKY85 (Hasegawa *et al*. 1984, 1985) (*5*)	T	–	αg_C	βg_A	βg_G
	C	αg_T	–	βg_A	βg_G
	A	βg_T	βg_C	–	αg_G
	G	βg_T	βg_C	αg_A	–
TN93 (Tamura and Nei 1993) (*6*)	T	–	$\alpha_2 g_C$	βg_A	βg_G
	C	$\alpha_2 g_T$	–	βg_A	βg_G
	A	βg_T	βg_C	–	$\alpha_1 g_G$
	G	βg_T	βg_C	$\alpha_1 g_A$	–

Note: The diagonals of the matrix are determined by the requirement that each row sums to 0. The equilibrium distribution is $\pi = (1/4, 1/4, 1/4, 1/4)$ under JC69 and K80, and $\pi = (g_T, g_C, g_A, g_G)$ under F84, HKY85, and TN93. α, transitional rate; β, transversional rate; α_1, transitional rate between purines; α_2, transitional rate between pyrimidines; g_N, frequencies of nucleotide N, where $N \in \{T, C, A, G\}$; $g_R = g_A + g_G$; $g_Y = g_T + g_C$.

A survey of gamma distribution

We assume that the rate of nucleotide substitution λ approximately follows the gamma distribution (*1*):

$$f(\lambda) = \frac{b^a}{\tau(a)} e^{-b\lambda} \lambda^{a-1} \qquad \text{(S1)}$$

where $a = \overline{\lambda}^2 / V(\lambda)$ and $b = a / \overline{\lambda}$, $\overline{\lambda}$ and $V(\lambda)$ is the mean and variance of λ, respectively, and $\tau(a)$ is the gamma function. Here note that a is the square of the inverse of the coefficient of variation. To avoid using too many parameters, we set $b = a$ so that the mean of the distribution is 1, with variance $1/a$. The shape parameter a is then inversely related to the extent of rate variation at sites.

Deductions of related parameters based on gamma distribution

(1) JC69 model

We directly describe the proportion of nucleotide sites showing differences (*P*) between two sequences that diverged *t* evolutionary time units ago:

$$P = \frac{3}{4} - \frac{3}{4}\exp(-4\alpha t) \qquad \text{(S2)}$$

Then, we can get the transformation:

$$\alpha t = -\frac{1}{4}\log_e\left(1 - \frac{4}{3}P\right) \qquad \text{(S3)}$$

Hence, the formula for estimating *d* is as follows:

$$d = 3\alpha t = -\frac{3}{4}\log_e\left(1 - \frac{4}{3}P\right) \qquad \text{(S4)}$$

Therefore, if λ or α follows the gamma distribution, the mean of *P* is given by (*7*):

$$\overline{P} = \int_0^\infty P f(\lambda) d\lambda = \frac{3}{4} - \frac{3}{4}\left[\frac{a}{a + 4\overline{\alpha t}}\right]^a \qquad \text{(S5)}$$

Then, we can get the transformation:

$$\overline{\alpha t} = \frac{a}{4}\left[\left(1 - \frac{4}{3}\overline{P}\right)^{-1/a} - 1\right] \qquad \text{(S6)}$$

Hence, the formula for estimating $\overline{d}$ is as follows:

$$\overline{d} = 3\overline{\alpha t} = \frac{3a}{4}\left[\left(1-\frac{4}{3}\overline{P}\right)^{-1/a} - 1\right] \qquad \text{(S7)}$$

(2) K80 (K2P) model

We directly describe the proportion of nucleotide sites showing transitional differences (*P*) and of those showing transversional differences (*Q*) between two sequences that diverged *t* evolutionary time units ago:

$$P = \frac{1}{4} + \frac{1}{4}\exp(-4\beta t) - \frac{1}{2}\exp[-2(\alpha+\beta)t] \qquad \text{(S8)}$$

$$Q = \frac{1}{2} - \frac{1}{2}\exp(-4\beta t) \qquad \text{(S9)}$$

Then, we can get the transformations:

$$\alpha t = -\frac{1}{2}\log_e(1-2P-Q) + \frac{1}{4}\log_e(1-2Q) \qquad \text{(S10)}$$

$$\beta t = -\frac{1}{4}\log_e(1-2Q) \qquad \text{(S11)}$$

Hence, the formulae for estimating *d* and *κ* are as follows:

$$d = (\alpha + 2\beta)t = -\frac{1}{2}\log_e(1-2P-Q) - \frac{1}{4}\log_e(1-2Q) \qquad \text{(S12)}$$

$$\kappa = \frac{\alpha}{\beta} = \frac{2\log_e(1-2P-Q)}{\log_e(1-2Q)} - 1 \qquad \text{(S13)}$$

Therefore, if λ or *α*, *β* follows the gamma distribution, the means of *P* and *Q* are given by (*8*):

$$\overline{P} = \int_0^\infty Pf(\lambda)d\lambda = \frac{1}{4} + \frac{1}{4}\left[\frac{a}{a+4\overline{\beta t}}\right]^a - \frac{1}{2}\left[\frac{a}{a+2\overline{(\alpha+\beta)t}}\right]^a \qquad \text{(S14)}$$

$$\overline{Q} = \int_0^\infty Qf(\lambda)d\lambda = \frac{1}{2} - \frac{1}{2}\left[\frac{a}{a+4\overline{\beta t}}\right]^a \qquad \text{(S15)}$$

Then, we can get the transformations:

$$\overline{\alpha t} = \frac{a}{2}\left[\left(1-2\overline{P}-\overline{Q}\right)^{-1/a} - 1\right] - \frac{a}{4}\left[\left(1-2\overline{Q}\right)^{-1/a} - 1\right] \qquad \text{(S16)}$$

$$\overline{\beta t} = \frac{a}{4}\left[\left(1-2\overline{Q}\right)^{-1/a} - 1\right] \qquad \text{(S17)}$$

Hence, the formulae for estimating $\overline{d}$ and $\overline{\kappa}$ are as follows:

$$\overline{d} = \overline{(\alpha + 2\beta)t} = \frac{a}{2}\left[\left(1 - 2\overline{P} - \overline{Q}\right)^{-1/a} + \frac{1}{2}\left(1 - 2\overline{Q}\right)^{-1/a} - \frac{3}{2}\right] \quad \text{(S18)}$$

$$\overline{\kappa} = \frac{\overline{\alpha}}{\overline{\beta}} = \frac{2\left[\left(1 - 2\overline{P} - \overline{Q}\right)^{-1/a} - 1\right]}{\left(1 - 2\overline{Q}\right)^{-1/a} - 1} - 1 \quad \text{(S19)}$$

(3) F84 model

We directly describe the proportion of nucleotide sites showing transitional differences (P) and of those showing transversional differences (Q) between two sequences that diverged t evolutionary time units ago:

$$P = 2\left(g_T g_C + g_A g_G\right) + 2\left(\frac{g_T g_C g_R}{g_Y} + \frac{g_A g_G g_Y}{g_R}\right)\exp(-\beta t) - 2\left(\frac{g_T g_C}{g_Y} + \frac{g_A g_G}{g_R}\right)\exp\left[-(\kappa + 1)\beta t\right]$$

(S20)

$$Q = 2 g_Y g_R \left[1 - \exp(-\beta t)\right] \quad \text{(S21)}$$

Let

$$h = (\kappa + 1)\beta t = -\log_e\left(1 - \frac{1}{2\left[(g_T g_C / g_Y) + (g_A g_G / g_R)\right]}P - \frac{\left[(g_T g_C g_R / g_Y) + (g_A g_G g_Y / g_R)\right]}{2\left(g_T g_C g_R + g_A g_G g_Y\right)}Q\right)$$

(S22)

$$i = \beta t = -\log_e\left(1 - \frac{1}{2 g_Y g_R}Q\right) \quad \text{(S23)}$$

Then we get

$$\begin{aligned} d &= 2\left(g_T g_C + g_A g_G + g_Y g_R\right)\beta t + 2\left(g_T g_C / g_Y + g_A g_G / g_R\right)\kappa\beta t \\ &= 2\left(\frac{g_T g_C}{g_Y} + \frac{g_A g_G}{g_R}\right)h - 2\left(\frac{g_T g_C g_R}{g_Y} + \frac{g_A g_G g_Y}{g_R} - g_Y g_R\right)i \end{aligned} \quad \text{(S24)}$$

$$\kappa = \frac{(\kappa + 1)\beta t - \beta t}{\beta t} = \frac{(\kappa + 1)\beta t}{\beta t} - 1 = \frac{h}{i} - 1 \quad \text{(S25)}$$

Therefore, if λ or β follows the gamma distribution, the means of P and Q are given by (*1*):

$$\begin{aligned} \overline{P} &= \int_0^\infty P f(\lambda) d\lambda \\ &= 2\left(g_T g_C + g_A g_G\right) + 2\left(\frac{g_T g_C g_R}{g_Y} + \frac{g_A g_G g_Y}{g_R}\right)\left(\frac{a}{a + \overline{\beta t}}\right)^a - 2\left(\frac{g_T g_C}{g_Y} + \frac{g_A g_G}{g_R}\right)\left[\frac{a}{a + \overline{(\kappa + 1)\beta t}}\right]^a \end{aligned}$$

(S26)

$$\overline{Q} = \int_0^\infty Q f(\lambda) d\lambda = 2 g_Y g_R \left[1 - \left(\frac{a}{a + \overline{\overline{\beta t}}} \right)^a \right] \quad \text{(S27)}$$

Let

$$\overline{h} = \overline{(\kappa+1)\beta t} = a \left\{ \left[1 - \frac{1}{2(g_T g_C / g_Y + g_A g_G / g_R)} \overline{P} - \frac{g_T g_C g_R / g_Y + g_A g_G g_Y / g_R}{2(g_T g_C g_R + g_A g_G g_Y)} \overline{Q} \right]^{-1/a} - 1 \right\} \quad \text{(S28)}$$

$$\overline{i} = \overline{\beta t} = a \left[\left(1 - \frac{1}{2 g_Y g_R} \overline{Q} \right)^{-1/a} - 1 \right] \quad \text{(S29)}$$

Then we get

$$\begin{aligned} \overline{d} &= 2(g_T g_C + g_A g_G + g_Y g_R)\overline{\beta t} + 2(g_T g_C / g_Y + g_A g_G / g_R)\overline{\kappa \beta t} \\ &= 2\left(\frac{g_T g_C}{g_Y} + \frac{g_A g_G}{g_R} \right)\overline{h} - 2\left(\frac{g_T g_C g_R}{g_Y} + \frac{g_A g_G g_Y}{g_R} - g_Y g_R \right)\overline{i} \end{aligned} \quad \text{(S30)}$$

$$\overline{\kappa} = \frac{\overline{(\kappa+1)\beta t} - \overline{\beta t}}{\overline{\beta t}} = \frac{\overline{(\kappa+1)\beta t}}{\overline{\beta t}} - 1 = \frac{\overline{h}}{\overline{i}} - 1 \quad \text{(S31)}$$

(4) HKY85 model

We can get estimated κ_{HKY85} and $\overline{\kappa}_{\mathrm{HKY85}}$ using the following formulae (*9, 10*):

$$\kappa_{HKY85} = 1 + \frac{g_T g_C / g_Y + g_A g_G / g_R}{g_T g_C + g_A g_G} \kappa_{F84} \quad \text{(S32)}$$

$$\overline{\kappa}_{HKY85} = 1 + \frac{g_T g_C / g_Y + g_A g_G / g_R}{g_T g_C + g_A g_G} \overline{\kappa}_{F84} \quad \text{(S33)}$$

(5) TN93 model

We directly describe the proportion of nucleotide sites showing transitional differences between purines (P_1) and between pyrimidines (P_2) and of those showing transversional differences (Q) between two sequences that diverged t evolutionary time units ago:

$$P_1 = \frac{2 g_A g_G}{g_R} \{ g_R + g_Y \exp(-\beta t) - \exp[-(g_R \alpha_1 + g_Y \beta) t] \} \quad \text{(S34)}$$

$$P_2 = \frac{2 g_T g_C}{g_Y} \{ g_Y + g_R \exp(-\beta t) - \exp[-(g_Y \alpha_2 + g_R \beta) t] \} \quad \text{(S35)}$$

$$Q = 2g_R g_Y \left[1 - \exp(-\beta t)\right] \quad \text{(S36)}$$

Then, we can get the transformations:

$$\alpha_1 t = \frac{1}{g_R}\left[g_Y \log_e\left(1 - \frac{1}{2g_R g_Y}Q\right) - \log_e\left(1 - \frac{g_R}{2g_A g_G}P_1 - \frac{1}{2g_R}Q\right)\right] \quad \text{(S37)}$$

$$\alpha_2 t = \frac{1}{g_Y}\left[g_R \log_e\left(1 - \frac{1}{2g_R g_Y}Q\right) - \log_e\left(1 - \frac{g_Y}{2g_T g_C}P_2 - \frac{1}{2g_Y}Q\right)\right] \quad \text{(S38)}$$

$$\beta t = -\log_e\left(1 - \frac{1}{2g_R g_Y}Q\right) \quad \text{(S39)}$$

Let

$$h = \log_e\left(1 - \frac{g_R}{2g_A g_G}P_1 - \frac{1}{2g_R}Q\right) \quad \text{(S40)}$$

$$i = \log_e\left(1 - \frac{g_Y}{2g_T g_C}P_2 - \frac{1}{2g_Y}Q\right) \quad \text{(S41)}$$

$$j = \log_e\left(1 - \frac{1}{2g_R g_Y}Q\right) \quad \text{(S42)}$$

We can get

$$\begin{aligned} d &= 2g_A g_G \alpha_1 t + 2g_T g_C \alpha_2 t + 2g_R g_Y \beta t \\ &= -\frac{2g_A g_G}{g_R}h - \frac{2g_T g_C}{g_Y}i + \left(\frac{2g_A g_G g_Y}{g_R} + \frac{2g_T g_C g_R}{g_Y} - 2g_R g_Y\right)j \end{aligned} \quad \text{(S43)}$$

$$\kappa_R = \frac{\alpha_1}{\beta} = \frac{h - g_Y \times j}{g_R \times j} \quad \text{(S44)}$$

$$\kappa_Y = \frac{\alpha_2}{\beta} = \frac{i - g_R \times j}{g_Y \times j} \quad \text{(S45)}$$

Therefore, if λ or α_1, α_2 and β follows the gamma distribution, the means of P_1, P_2, and Q are given by (*6*):

$$\overline{P_1} = \int_0^\infty P_1 f(\lambda) d\lambda = \frac{2g_A g_G}{g_R}\left\{g_R - \left[\frac{a}{a + (g_R \overline{\alpha_1} + g_Y \overline{\beta})t}\right]^a + g_Y\left(\frac{a}{a + \overline{\beta}t}\right)^a\right\} \quad \text{(S46)}$$

$$\overline{P_2} = \int_0^\infty P_2 f(\lambda) d\lambda = \frac{2g_T g_C}{g_Y}\left\{g_Y - \left[\frac{a}{a + (g_Y \overline{\alpha_2} + g_R \overline{\beta})t}\right]^a + g_R\left(\frac{a}{a + \overline{\beta}t}\right)^a\right\} \quad \text{(S47)}$$

$$\overline{Q} = \int_0^\infty Q f(\lambda) d\lambda = 2 g_R g_Y \left[1 - \left(\frac{a}{a + \overline{\beta t}} \right)^a \right] \qquad \text{(S48)}$$

Then, we can get the transformations:

$$\overline{\alpha_1 t} = \frac{a}{g_R} \left[(1 - \frac{1}{2g_R} \overline{Q} - \frac{g_R}{2 g_A g_G} \overline{P_1})^{-1/a} - g_Y (1 - \frac{1}{2 g_R g_Y} \overline{Q})^{-1/a} - g_R \right] \qquad \text{(S49)}$$

$$\overline{\alpha_2 t} = \frac{a}{g_Y} \left[(1 - \frac{1}{2g_Y} \overline{Q} - \frac{g_Y}{2 g_T g_C} \overline{P_2})^{-1/a} - g_R (1 - \frac{1}{2 g_R g_Y} \overline{Q})^{-1/a} - g_Y \right] \qquad \text{(S50)}$$

$$\overline{\beta t} = a \left[(1 - \frac{1}{2 g_R g_Y} \overline{Q})^{-1/a} - 1 \right] \qquad \text{(S51)}$$

Let

$$\overline{h} = \left(1 - \frac{g_R}{2 g_A g_G} \overline{P_1} - \frac{1}{2 g_R} \overline{Q} \right)^{-1/a} \qquad \text{(S52)}$$

$$\overline{i} = \left(1 - \frac{g_Y}{2 g_T g_C} \overline{P_2} - \frac{1}{2 g_Y} \overline{Q} \right)^{-1/a} \qquad \text{(S53)}$$

$$\overline{j} = \left(1 - \frac{1}{2 g_R g_Y} \overline{Q} \right)^{-1/a} \qquad \text{(S54)}$$

We can get

$$\begin{aligned} \overline{d} &= 2 g_A g_G \overline{\alpha_1 t} + 2 g_T g_C \overline{\alpha_2 t} + 2 g_R g_Y \overline{\beta t} \\ &= 2a \left[\frac{g_A g_G}{g_R} \overline{h} + \frac{g_T g_C}{g_Y} \overline{i} + \left(g_R g_Y - \frac{g_A g_G g_Y}{g_R} - \frac{g_T g_C g_R}{g_Y} \right) \overline{j} - g_A g_G - g_T g_C - g_R g_Y \right] \end{aligned}$$

(S55)

$$\overline{\kappa_R} = \overline{\alpha_1} / \overline{\beta} = \frac{\overline{h} - g_Y \times \overline{j} - g_R}{g_R \times \overline{j} - g_R} \qquad \text{(S56)}$$

$$\overline{\kappa_Y} = \overline{\alpha_2} / \overline{\beta} = \frac{\overline{i} - g_R \times \overline{j} - g_Y}{g_Y \times \overline{j} - g_Y} \qquad \text{(S57)}$$

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