Definition of the different contributions in the approximation of the SVM classifiers.

This appendix summarizes how the contributions $f^{(p)}$ and $f^{(p,q)}$ to the latent variable can be calculated for the linear, polynomial and RBF kernel.

1 Linear kernel

For the linear kernel, contributions involving more than two inputs do not exist, so $f^{(p,q)}$ should not be defined. Contributions involving only one input are defined in the main paper as:

$$f^{(p)} = \sum_{i=1}^{N} \alpha_i y_i x_i^{(p)} x^{(p)} .$$

2 Polynomial kernel

The expansion of the polynomial kernel with $a = 1$ equals:

$$\begin{aligned}
K_{\text{poly}}(x_i, x) &= (x_i^T x + c)^\delta = \left(\sum_{p=1}^{d} x_i^{(p)} x^{(p)} + c \right)^\delta \\
&= c^\delta + \sum_{p=1}^{d} {x_i^{(p)}}^\delta {x^{(p)}}^\delta + \\
&\quad \sum_{p=1}^{d} \sum_{\substack{k_p + k_c = \delta \\ k_p, k_c \neq \delta}} \binom{\delta}{k_p, k_c} {x_i^{(p)}}^{k_p} {x^{(p)}}^{k_p} c^{k_c} + \\
&\quad \sum_{p=1}^{d} \sum_{q \neq p} \sum_{\substack{k_p + k_q = \delta \\ k_p, k_q \neq \delta}} \binom{\delta}{k_p, k_q} {x_i^{(p)}}^{k_p} {x^{(p)}}^{k_p} {x_i^{(q)}}^{k_q} {x^{(q)}}^{k_q} + \\
&\quad \sum_{p=1}^{d} \sum_{q \neq p} \sum_{\substack{k_p + k_q + k_c = \delta \\ k_p, k_q, k_c \neq \delta \\ k_p + k_q \neq \delta \\ k_p + k_c \neq \delta \\ k_q + k_c \neq \delta}} \binom{\delta}{k_p, k_q, k_c} {x_i^{(p)}}^{k_p} {x^{(p)}}^{k_p} {x_i^{(q)}}^{k_q} {x^{(q)}}^{k_q} c^{k_c} + \Delta \\
&= \sum_{p=1}^{d} g^{(p)} \left(x_i^{(p)}, x^{(p)} \right) + \sum_{p=1}^{d} \sum_{q \neq p} g^{(p,q)} \left(x_i^{(p,q)}, x^{(p,q)} \right) + b' + \Delta .
\end{aligned}$$

The kernel is hereby decomposed in terms $g^{(p)}$ that solely depend on the p^{th} input variable:

$$g^{(p)}\left(x_i^{(p)},x^{(p)}\right) = \left(x_i^{(p)}x^{(p)}\right)^{\delta} + \sum_{\substack{k_p+k_c=\delta\\ k_p,k_c\neq\delta}} \binom{\delta}{k_p,k_c} {x_i^{(p)}}^{k_p}{x^{(p)}}^{k_p}c^{k_c}$$

$$= K_{\text{poly}}(x_i^{(p)},x^{(p)}) - c^{\delta}\,,$$

and terms that depend on two input variables $x^{(p)}$ and $x^{(q)}$:

$$g^{(p,q)}\left(x_i^{(p,q)},x^{(p,q)}\right) = \sum_{\substack{k_p+k_q=\delta\\ k_p,k_q\neq\delta}} \binom{\delta}{k_p,k_q}\left(x_i^{(p)}x^{(p)}\right)^{k_p}\left(x_i^{(q)}x^{(q)}\right)^{k_q} +$$

$$\sum_{\substack{k_p+k_q+k_c=\delta\\ k_p,k_q,k_c\neq\delta\\ k_p+k_q\neq\delta\\ k_p+k_c\neq\delta\\ k_q+k_c\neq\delta}} \binom{\delta}{k_p,k_q,k_c} {x_i^{(p)}}^{k_p}{x^{(p)}}^{k_p}{x_i^{(q)}}^{k_q}{x^{(q)}}^{k_q}c^{k_c}$$

$$= K_{\text{poly}}(x_i^{(p,q)},x^{(p,q)}) - g^{(p)}\left(x_i^{(p)},x^{(p)}\right) - g^{(q)}\left(x_i^{(q)},x^{(q)}\right) - c^{\delta}\,.$$

The latent variable of the SVM model using this kernel can then be decomposed in terms $f^{(p)}$ depending on the p^{th} input variable:

$$f^{(p)}\left(x_i^{(p)},x^{(p)}\right) = \sum_{i=1}^{N}\alpha_i y_i g^{(p)}\left(x_i^{(p)},x^{(p)}\right)$$

and terms depending on two input input variables:

$$f^{(p,q)}\left(x_i^{(p,q)},x^{(p,q)}\right) = \sum_{i=1}^{N}\alpha_i y_i g^{(p,q)}\left(x_i^{(p,q)},x^{(p,q)}\right).$$

3 RBF kernel

The RBF kernel can be expanded as:

$$K_{\text{RBF}}(x_i,x) = \sum_{n=0}^{\infty}\frac{(-1)^n\gamma^n}{n!}\left[\sum_{p=1}^{d}(x_i^{(p)}-x^{(p)})^{2n} + \sum_{p=1}^{d}\sum_{q\neq p}\sum_{\substack{k_p+k_q=n\\ k_p,k_q\neq n}}\binom{n}{k_p,k_q}(x_i^{(p)}-x^{(p)})^{2k_p}(x_i^{(q)}-x^{(q)})^{2k_q}\right] + \Delta\,,$$

$$= \sum_{p=1}^{d} g^{(p)}\left(x_i^{(p)},x^{(p)}\right) + \sum_{p=1}^{d}\sum_{q\neq p} g^{(p,q)}\left(x_i^{(p,q)},x^{(p,q)}\right) + \Delta\,.$$

The kernel is hereby decomposed in terms $g^{(p)}$ that solely depend on the p^{th} input variable:

$$g^{(p)}\left(x_i^{(p)},x^{(p)}\right) = \textstyle\sum_{n=0}^{\infty}\frac{(-1)^n\gamma^n}{n!}\sum_{p=1}^{d}(x_i^{(p)}-x^{(p)})^{2n}$$

$$= K_{\text{RBF}}(x_i^{(p)},x^{(p)})\,,$$

and terms that depend on two input variables $x^{(p)}$ and $x^{(q)}$:

$$g^{(p,q)}\left(x_i^{(p,q)},x^{(p,q)}\right) = \textstyle\sum_{n=0}^{\infty}\frac{(-1)^n\gamma^n}{n!}\displaystyle\sum_{\substack{k_p+k_q=n\\ k_p,k_q\neq n}}\binom{n}{k_p,k_q}(x_i^{(p)}-x^{(p)})^{2k_p}(x_i^{(q)}-x^{(q)})^{2k_q}$$

$$= K_{\text{RBF}}(x_i^{(p,q)},x^{(p,q)}) - K_{\text{RBF}}(x_i^{(p)},x^{(p)}) - K_{\text{RBF}}(x_i^{(q)},x^{(q)})\,.$$

The latent variable of the SVM model using this kernel can then be decomposed in terms $f^{(p)}$ depending on the p^{th} input variable:

$$f^{(p)}\left(x_i^{(p)}, x^{(p)}\right) = \sum_{i=1}^{N} \alpha_i y_i g^{(p)}\left(x_i^{(p)}, x^{(p)}\right)$$

and term depending on two input input variables:

$$f^{(p,q)}\left(x_i^{(p,q)}, x^{(p,q)}\right) = \sum_{i=1}^{N} \alpha_i y_i g^{(p,q)}\left(x_i^{(p,q)}, x^{(p,q)}\right).$$