Supplementary Text and Caption of Supplementary Figures

Text S1 Mathematical analysis of the slow variable
Here we considered the relaxation time around a fixed point. For simplicity we considered the case with $\gamma_i = 1$, to demonstrate that the relaxation time is longer even without the change in γ. The stability of the fixed point was given by eigenvalues of the Jacobian matrix W_{ij} where the diagonal component W_{ii} is given by -1 and the off-diagonal component W_{ij} is given by $J_{ij}\beta exp(-\beta X_i)/(1 + exp(-\beta X_i))^2$ where $X_i = \sum_j J_{ij}(x_j - \theta_j)$. If x_j's are close to 0 or 1, their deviation from θ_j is sufficiently larger than the detection threshold $1/\beta$, the off-diagonal elements are close to zero, and the eigenvalues are given by -1 (or -γ_i if it is not 1). When x_j's takes on intermediate values closer to θ_i, the off-diagonal elements assume larger values, and the eigenvalues are shifted from -1, either upwards or downwards. Hence, some exponents approach zero. As long as the real components of the eigenvalues are negative, the fixed point remains stable, but the stability is weaker, with the exponent closer to zero. This results in an increase in the timescale of the relaxation, given by the inverse of the real component of the eigenvalue. With this mechanism, the slowly changing variable is generated even without small γ_i.

Figure S1. Space-time diagrams of evolution, development and differences between the two.
(**A**)-(**H**) Eight additional space-time comparisons between evolution and development. Each consists of a space-time diagram of development, evolution, preset target pattern and difference between evolution and development. Space-time diagrams of evolution and development and their target pattern are plotted in the same way as in Figs.3 and 4. Calculated values of the difference Δ are shown below the diagram. For A-D, the evolution and development corresponded well, while a clear violation was observed for H.

Figure S2 Detailed analysis on feedback oscillation.
(**A**): network structure for data presented in Figure 8.
(**B**): Phase diagram of the expression dynamics. Two nullclines of gene expression cross at a single, unstable fixed point, and the cell state will oscillate on a limited cycle. Green circles represent the cell state at time step intervals

of 5, within a single cycle. The distance between two nullclines is shortest at the upper right and lower left corners so that cell state changes are slower at these corners.
(**C**): Time profile of the feedback oscillation for a specific cell. The abscissa represents developmental time and the ordinate is expression level. Gene A is plotted as a red line, while gene B plotted as a Blue line.

Figure S3 Temporal change of flow in the phase space over cells.
The oscillation fixation mechanism is revealed through comparison of flow temporal changes in the phase space over cells. The central cells of the first two stripes (cell indices 12 and 28) and valleys (cell indices 4 and 20).

(1)**t=0**:
Both gene A and gene B assume a null value, and gene B is inhibited by gene M within the first 8 cells. With these two initial conditions, flow in the phase space where cell index = 4 is different from other cells. At a stable fixed point therein, both the expressions of gene A and gene B are low (i.e., a low-low state). This fixed point is the root of the first valley.
(2)**t=80**:
As development begins, expression of non-inhibited cells begin to oscillate and move towards a state where both expressions of gene A and gene B are high (i.e., a high-high state), while cells of the first valley maintain the slow-low state. Thus, protein A diffuses from the first stripe to the first valley. Due to the incoming diffusion of protein expression of gene A, at cell index 4, the nullcline of gene A slides to the right, so that the expression of gene A assumes a higher value. Correspondingly, at cell index 8, the nullcline of gene A slides to the left, and crosses the nullcline of gene B to create a fixed point.
(3)**t=100**:
The expression of cell index 4 is constrained to the newly formed high-high fixed point. However, the expressions of cells at index 20 and 28 continued to oscillate.
(4)**t=180**:
Through the oscillation, cell index 20 approaches a low-low state for the second time, and at this time, protein B at the first stripe diffuses to the second valley. Thus, nullclines slide in the cells at indices 12 and 20 to the left and right, respectively. The nullcline of gene B then crosses, at a low-low state, in the cell at index 20.

(5)**t=200**:
The expression level at cell index 28 continued to oscillate while that at cell index 20 was constrained at the newly formed low-low state fixed point, similar to cell index 12 at t=80. Protein B subsequently diffused from the cell index 28 to the second valley, which resulted in the emergence of the second stripe. In this way stripes were shaped from the oscillation.

Figure S4 Network analysis of the extra example of the violation of evo-devo congruence 1.
In development, the third and fourth upper stripes stem from the some root, while in evolution the top three stems originate from the same root. This branching change occurs during generation 1549 where the second and third branches are clearly stabilized. In this case, unlike the former examples, topological changes in branching occur sequentially during three generations. A time-space diagram of the output and gene 7 are presented in Figure B. Gene 7 exhibits two stripes from generation 1549, which are driven by a feedforward mechanism. Due to the boundary effect of gene 7, the upper three stripes are generated in the output gene. Then, during generation 1550, part of the feedforward mechanism upstream of gene 7 is deactivated, which enhances the region expressed by the feedback oscillation mechanism. As a result, the upper four stripes that emerge share the same oscillation mechanism. At generation 1551, mutation occurs upstream of gene 7, so that the morphogen comes to inhibit the remaining feedforward mechanism. Before the mutation, gene 7 exhibits weak temporal expression in cell sites 30-85. After the mutation, this temporal expression is inhibited so that the expression region is restricted to cell sites 60-85. Due to this change, the third and the fourth upper stripes emerge faster than the first and second stripes, while the third stripe, generated in advance of the first two, provided a boundary to generate the second stripe. To summarize, the upper 4 stripes were generated by the feedback oscillation mechanism, but the change in the boundary condition due to mutation in the upstream feedforward mechanism introduced the branching combination.

Figure S5 Network analysis of an additional example of the violation of evo-devo congruence.
In evolution, the three central stripes are acquired nearly simultaneously, and two additional stripes are subsequently acquired independently. However, in development, at the final evolved generation, the 1st and 4th stripes were

generated from the same root at the same time, and subsequently the 2nd and 3rd stripes were generated from a common root. These two branchings follow the oscillation-fixation mechanism. Only the bottom stripe is generated independently. The developmental order of stripe formation was acquired, between generations 1104 and 1105. Genes that exhibited relevant change are displayed in Figure B. The expression of an upstream gene (green in the GRN figure below) and the downstream gene (red in the GRN figure) are plotted at the upper and lower columns, respectively. In this example, the feedforward mechanism worked only temporally, as shown in the transient expressed before time step = 100 (Figure C). This temporal expression region also corresponded to the region of feedback oscillation. Due to the mutation, the spatial domain of the transient expression was extended upward. Violation of evo-devo congruence was therefore induced by this expansion of the transient expression region.