Appendix 1: Equations and parameters used in simulations

The equations used for all simulations were drawn from Buxton et al., 1998; Glover, 1999; Friston et al., 2000; Mildner et al., 2001; Buxton et al., 2004; and Havlicek et al., 2015; and are reproduced here for completeness.

1 Canonical linear model

The predicted frequency response from canonical models was computed by convolving a sinusoidal oscillation at the frequency of interest with one of three different hemodynamic response functions (HRFs).

The SPM HRF:

$$HRF_{SPM}(x) = \frac{1}{\Gamma(a)b^a}x^{a-1}e^{-\frac{x}{b}} - k\frac{1}{\Gamma(c)d^c}x^{c-1}e^{-\frac{x}{d}} \tag{1}$$

With $a = 6, b = 1, c = 16, d = 1, k = 1/6$.

The single gamma HRF:

$$HRF_{SG}(x) = \frac{1}{\Gamma(a)b^a}x^{a-1}e^{-\frac{x}{b}} \tag{2}$$

With $a = 6, b = 1$.

The Glover 1999 HRF:

$$HRF_{GL}(x) = c_1 x^{n_1} e^{\frac{-x}{t_1}} - a_2 c_2 x^{n_2} e^{\frac{-x}{t_2}} \tag{3}$$

$$c_i = \max(x^{n_i} e^{\frac{-x}{t_i}}) \tag{4}$$

With $n_1 = 6, t_1 = 0.9, a_2 = 0.35, n_2 = 12, t_2 = 0.9$.

2 Balloon model

The balloon model (Buxton et al., 1998; Obata et al., 2004) was implemented as follows, with neurovascular coupling equations (Friston et al., 2000) included when simulations aimed to predict the BOLD response to neural activity, and not included when flow input was modeled directly (e.g. Fig. S3d-f). $E(t)$ is oxygen extraction fraction; $r(t)$ is $CMRO_2$; $f_{in}(t)$ is inflow; $f_{out}(v,t)$ is outflow; $q(t)$ is deoxyhemoglobin concentration; $v(t)$ is blood volume; $u(t)$ is neural activity; $s(t)$ is the inducing signal for flow.

$$E(t) = 1 - (1 - E_0)^{1/f_{in}(t)} \tag{5}$$

$$f_{out}(v,t) = \frac{1}{(\tau_v + \tau_{MTT})}(\tau_{MTT} v(t)^{1/\alpha} + \tau_v f_{in}(t)) \tag{6}$$

$$r(t) = f_{in}(t)\frac{E(t)}{E_0} \tag{7}$$

$$\frac{dq}{dt} = \frac{1}{\tau_{MTT}}(r(t) - f_{out}(v,t)\frac{q(t)}{v(t)}) \tag{8}$$

$$\frac{dv}{dt} = \frac{1}{\tau_{MTT}}(f_{in}(t) - f_{out}(v,t)) \tag{9}$$

$$\frac{df_{in}}{dt} = s(t) \tag{10}$$

$$\frac{ds}{dt} = \epsilon u(t) - s(t)\tau_s - \frac{f_{in}(t) - 1}{\tau_f} \tag{11}$$

Coupling parameters: $\epsilon = 0.5, \tau_s = 0.8, \tau_f = 0.4$.
Hemodynamic fixed parameters: $E_0 = 0.4, \alpha = 0.4$.
Hemodynamic variable parameters: τ_v was varied between 0 and 30, τ_{MTT} was varied between 1 and 4.

BOLD signal:

$$y = V_0(k_1(1 - q) + k_2(1 - \frac{q}{v}) + k_3(1 - v)) \tag{12}$$

BOLD signal parameters: $V_0 = 0.03, k_1 = \frac{6.7}{0.4}E_0, k_2 = \frac{2.73}{0.4}E_0, k_3 = 0.57$.

The BOLD signal parameters were drawn from Mildner et al., 2001, via Chen and Glover, 2015, to simulate expected results at 3T. Alternate parameter settings from Stephan et al., 2007, and Buxton et al., 2004, were also tested and produced qualitatively similar results.