Supplemental Material for Nuismer et al.

An imperfect vaccine

In order to investigate the potential impact of imperfect vaccination, we consider a variation of our model in which immunity to the infectious disease conferred by vaccination is not permanent. Instead, individuals that have been vaccinated lose their immunity to the infectious disease at some constant rate, f, such that the system is now described by:

$$\frac{dS}{dt} = b(1-\sigma) - \beta_V SV - \beta_W SW - dS + fR_V \quad \text{(S1a)}$$

$$\frac{dV}{dt} = b\sigma + \beta_V SV - \delta_V V - dV \quad \text{(S1b)}$$

$$\frac{dW}{dt} = \beta_W SW - \delta_W W - (d+v)W \quad \text{(S1c)}$$

$$\frac{dR_V}{dt} = \delta_V V - dR_V - fR_V \quad \text{(S1d)}$$

$$\frac{dR_W}{dt} = \delta_W W - dR_W \quad \text{(S1e)}$$

where R_V is the abundance of individuals who have been vaccinated and are currently resistant to the infectious disease and R_W is the abundance of individuals who have been previously infected and are permanently resistant to the infectious disease.

Much as we saw previously for the model with a perfect vaccine, mathematical analyses of equations (S1) reveals three possible equilibria, only two of which exist. The first equilibrium is given by:

$$\hat{S} = \frac{d+v+\delta_W}{\beta_W} \quad \text{(S2a)}$$

$$\hat{W} = \frac{(d+f)k\beta_V(dk-b\beta_W)+\beta_W(-d(d+f)k(d+\delta_V)+b\beta_W(-d(d+f)(-1+\sigma)+(d+f-d\sigma)\delta_V))}{\beta_W(-k\beta_V+\beta_W(d+\delta_V))((d+f)(d+v)+d\delta_W)} \quad \text{(S2b)}$$

$$\hat{V} = \frac{b\sigma\beta_W}{\beta_W(d+\delta_V)-\beta_V(d+v+\delta_W)} \quad \text{(S2c)}$$

$$\hat{R}_V = \frac{dk^2\beta_V\delta_W - k\beta_W(b\beta_V + d(d+\delta_V))\delta_W + b\beta_W^2((d+v)\sigma\delta_V + (d-d\sigma+\delta_V)\delta_W)}{\beta_W(-k\beta_V + \beta_W(d+\delta_V))((d+f)(d+v)+d\delta_W)} \quad \text{(S2d)}$$

where $k = d + v + \delta_W$. The second equilibrium is given by:

$$\hat{S} = \frac{d^3 + bf\beta_V + d^2(f+\delta_V) + d(b\beta_V + f\delta_V) - \sqrt{(d+f)(4bd^2\sigma\beta_V(d+f+\delta_V) + (d+f)(b\beta_V - d(d+\delta_V))^2)}}{2d(d+f)\beta_V} \quad \text{(S3a)}$$

$$\widehat{W} = 0 \quad \text{(S3b)}$$

$$\hat{V} = \frac{-d^3 + bf\beta_V - d^2(f+\delta_V) + d(b\beta_V - f\delta_V) + \sqrt{(d+f)(4bd^2\sigma\beta_V(d+f+\delta_V) + (d+f)(b\beta_V - d(d+\delta_V))^2)}}{2d\beta_V(d+f+\delta_V)} \quad \text{(S3c)}$$

$$\hat{R}_V = \frac{\delta_V(-(d+f)(d^2 - b\beta_V) - d(d+f)\delta_V + \sqrt{(d+f)(4bd^2\sigma\beta_V(d+f+\delta_V) + (d+f)(b\beta_V - d(d+\delta_V))^2))}}{2d(d+f)\beta_V(d+f+\delta_V)} \quad \text{(S3d)}$$

Local stability analyses of (S2-S3) reveal that the equilibrium where the infectious disease is extinct (S3) is stable if:

$$\sigma > \frac{d+f}{d+f+\delta_V}\frac{(d(d+v+\delta_W) - b\beta_W)(\beta_V(d+v+\delta_W) - \beta_W(d+\delta_V))}{bd\beta_W^2} \quad \text{(S4)}$$

When this stability condition holds, the alternative equilibrium where the infectious disease is present (S2) is locally unstable. If local stability condition (S4) does not hold, the situation reverses and only the equilibrium with a positive density of individuals infected by the disease is locally stable. Thus, condition (S4) is sufficient to completely predict the qualitative dynamics of this system.

Defining the basic reproductive numbers of the vaccine and disease as before allows condition (S4) to be re-written in a more insightful form defining the critical level of direct vaccination required for disease eradication:

$$\sigma_{crit} = \frac{(d+f)(d+\delta_V)}{d(d+f+\delta_V)}\left(1 - \frac{R_{0,V}}{R_{0,W}}\right)\left(1 - \frac{1}{R_{0,W}}\right) \quad \text{(S5)}$$

In the absence of vaccine transmission, (S5) reduces to:

$$\sigma_{crit}^{*} = \frac{(d+f)(d+\delta_V)}{d(d+f+\delta_V)}\left(1-\frac{1}{R_{0,W}}\right) \quad \text{(S6)}$$

As before, we calculate the percentage reduction in vaccination effort provided by a transmissible vaccine relative to a traditional, non-transmissible, vaccine:

$$f_{Eradicate} = 1-\frac{\sigma_{crit}}{\sigma_{crit}^{*}} = 1-\frac{\frac{(d+f)(d+\delta_V)}{d(d+f+\delta_V)}\left(1-\frac{R_{0,V}}{R_{0,W}}\right)\left(1-\frac{1}{R_{0,W}}\right)}{\frac{(d+f)(d+\delta_V)}{d(d+f+\delta_V)}\left(1-\frac{1}{R_{0,W}}\right)} = \frac{R_{0,V}}{R_{0,W}} \quad \text{(S7)}$$

This is equation (2) of the text and demonstrates that the relative performance of a transmissible vaccine is unaffected by the durability of vaccine induced immunity.

Integrating vaccine reversion

In order to investigate the impact of vaccine reversion we modified the model of the main text to include reversion from the vaccinated class (V) to the disease class (W) at rate r:

$$\frac{dS}{dt} = b(1-\sigma) - \beta_V SV - \beta_W SW - dS \quad \text{(S8a)}$$

$$\frac{dV}{dt} = b\sigma + \beta_V SV - \delta_V V - dV - rV \quad \text{(S8b)}$$

$$\frac{dW}{dt} = \beta_W SW - \delta_W W - (d+v)W + rV \quad \text{(S8c)}$$

$$\frac{dR}{dt} = \delta_V V + \delta_W W - dR \quad \text{(S8d)}$$

Stochastic simulations of this model were run as described in the main text, and disease eradication was considered to have occurred if no infected individuals were present within the population 150 years after initiation of the vaccination program. Of course, because reversion occurs within these simulations, disease eradication is never permanent and individuals infected with disease will appear

from time to time. Consequently, Figure S1 represents the proportion of simulations in which the disease was transiently eradicated at that particular point in time.

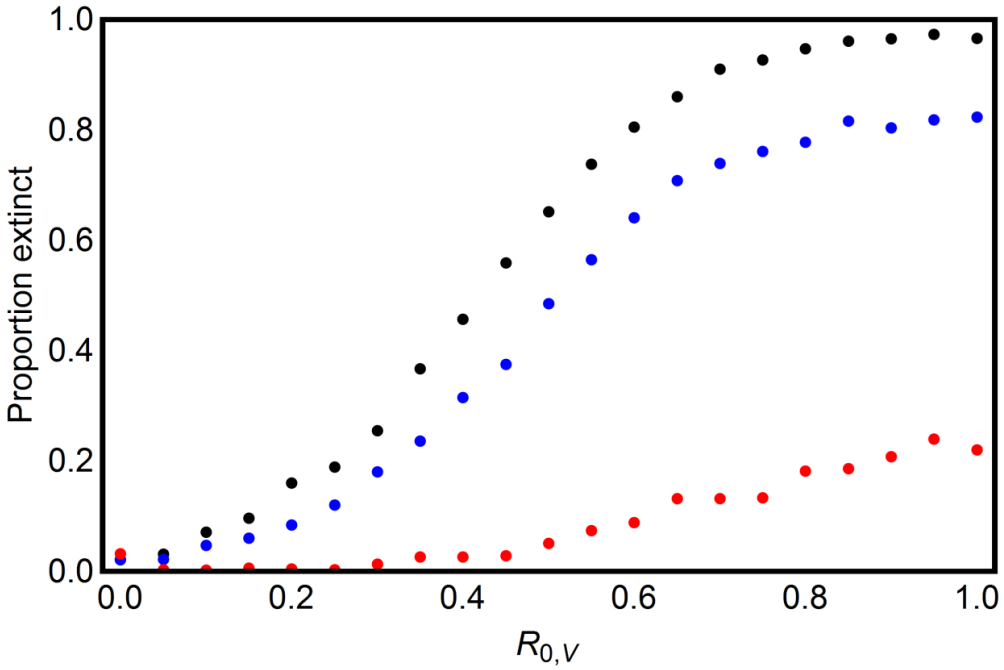

Figure S1. The proportion of stochastic simulations in which no individuals were infected with the disease 150 years after vaccine introduction as a function of vaccine R_0. Parameters are identical to Figure 3 in the main text, with the exception of β_V which varied from 0 to 0.0003 in .000015 increments. Reversion rates were: r = 0.00001 (black dots), r = 0.0001 (blue dots), and r = 0.001 (red dots). Note that when reversion becomes substantial (red dots) low rates of vaccine transmission actually promote the persistence of the infectious disease relative to a non-transmissible vaccine.