1	Optimal 2D-SIM reconstruction by two filtering steps with Richardson-
2	Lucy deconvolution
3	
4	Supplementary information
5	Victor Perez, Bo-Jui Chang and Ernst Hans Karl Stelzer*
6	Buchmann Institute for Molecular Life Sciences (BMLS)
7	Goethe Universität Frankfurt am Main
8	Max-von-Laue-Strasse 15, 60438 Frankfurt am Main, Germany
9	
10	
11	
12	

13 Supplementary Figures

- 14 Fig. S1 csiLSFM set-up.
- 15 Fig. S2 csiLSFM calibration curve.
- 16 Fig. S3 Apodization function.
- 17 Fig. S4 High vs. low gain reconstructions with Wiener×Apodization and our reconstruction
- 18 approach (RL reconstruction).
- 19 Fig. S5 Reconstruction dependence on the Apodization function parameters.
- Fig. S6 Extraction of reconstruction parameters, \vec{p}_i and ϕ_j , for the HUVEC data set.
- Fig. S7 HepG2 data set reconstructed with the fairSIM implementation and our
- 22 RL-reconstruction.
- Fig. S8 Deconvolution artifacts when applying a 3D initial deconvolution to SIM stacks.
- Fig. S9 3D continued deconvolution vs 2D continued deconvolution in plane-by-plane 2D-SIM reconstructions.
- 25 reconstructions.
- Fig. S10 Continued deconvolution enhances the reconstruction contrast.
- 27 Fig. S11 Initial Deconvolution enhances extraction of spatial frequencies.
- 28 Fig. S12 RL-reconstruction in external data set.
- 29 Table S1 Reconstruction parameters of Figure S7.
- 30 TableS2 Reconstruction parameters of Figure S12.
- 31
- 32
- 22
- 33
- 34
- 35
- 36
- 37
- 38
- 39
- 40
- 41
- 42



44

45 Figure S1 csiLSFM set-up. The beam splitter (BS) divides the laser beam into two coherent sources. Each beams is deflected individually into a polarization-preserving, single-mode fiber 46 (kineFLEX, Qioptiq). The fiber collimators (F220FC-543, Thorlabs) at the end of the fibers 47 deliver collimated beams and form two illumination paths. The chamber is filled with an 48 appropriate medium, e.g. phosphate buffered saline (PBS), for biological specimens. The angle α 49 defines the pattern period while ß defines its orientation. PP: piezo nano-positioner, CL: cylindrical 50 51 lens, L: achromatic lens, SM: scanning mirror, SL: $f - \theta$ lens, O1 and O2: illumination objectives, 52 O3: detection objective.

Two objectives are used for illumination (O1,O2) and one for detection (O3), the three foci co-53 localize (Fig. S1). The illumination objectives are spread by 130° between their long axes. All 54 objectives share the same specifications, i.e. water immersion, and a working distance of 2.1 mm 55 (63x/NA1.0, W Plan-Apochromat, 421480-9900-000, Carl Zeiss). The output of each illumination 56 objective is a light sheet shaped with a cylindrical lens (f=75 mm, F69-699, Edmund Optics) [1]. 57 Interference of the two light sheets generates a two-dimensional sinusoidal pattern in the focal 58 plane of the detection objective. The period T_o of the pattern is tuned by varying the half 59 interference angle θ , which is a function of the angular displacement α of the scanning mirror (S-60 334.2SL, Physik Instrumente). In each illumination arm the achromatic lens (f=200 mm, 61

G322327322, Qioptiq) and the f-theta scanning lens (f=60.5 mm, S4LFT0061, Sill optics) form a
4f-telecentric system that projects the light sheet into the back focal plane of the illumination
objective.

Rotation of the pattern is controlled by tilting the angle β in the scanning mirror, which correspondingly varies the height y' and y'' in each arm and anti-symmetrically tilts each light sheet on the focal plane of the detection objective. Three different orientations 0°, 49° and 133° of a pattern with $T_o = 301$ nm are shown. The pattern phase is controlled with the piezo nano-positioner (P-725.4CD, Physik Instrumente) translating the optical fiber collimator.

Features of the csiLSFM are the flexible controls of the pattern period and orientation. It allows resolution gains larger than the two-fold in a usual SIM due to the decoupling of the detection and illumination paths. The two-fold constraint is inherent in epi-fluorescence configurations when using linear emissions since the period T_o of the pattern is restricted by the angular aperture of the objective. Maximum resolution gain is achieved when the two light sheets are counter-propagating (θ =90°). In our experimental conditions (λ_{exc} =488nm, n=1.333) that angle corresponds to an illumination pattern with a period of 183 nm.

The detection path (not shown) starts with O3 followed by an emission filter (FF02-525/50-25, Semrock) to block the laser for the observation of the fluorescence signal. A 1x tube lens (452960, Carl Zeiss) together with the objective is used to form the infinity corrected image. We use a CMOS camera with a pixel array of 1920x1440 and a pixel pitch of 3.63 μ m (C11440, ORCA-Flash 2.8, Hamamatsu). The three objectives are partially inserted into a customized sealed polyoxymethylene chamber with an open top for the sample entry. The sample is embedded in phytagel, agarose, or a coverslip, and is mounted on a rod-like holder parallel to the *y* axis. The 84 holder is attached to a 4-axis (xyz translation and a rotation around y) motorized stage (custom-

85 designed, SmarAct).

86



Figure S2 csiLSFM calibration curve. Images of beads are taken at different angular 89 displacements α , and the norm of the spatial frequency \vec{p}_i is calculated with equation 8 (black 90 dots). Inverting the values of this curve times 2π yields the period T_0 (red dots). Comparing 91 experimental data (red dots) with theoretical values (red solid line) allows an estimation of the half 92 interference angle θ . The center of the black squares represents the spatial frequency values 93 94 obtained from the range of angles α where the pattern is visible in the detection objective. A good match is found between these optically detected values and the cross-correlation estimates. Finally, 95 from this calibration curve an estimated spatial frequency value expected for other samples can be 96 obtained. The estimated spatial frequency corresponding to angular displacements α is used as the 97 reference radius for the mask shown in Figure 5c. 98





Figure S3 Apodization function applied to the spectra of the reconstructed image when using 103 a Wiener filter to assemble the domains. This function is formed by the product of z_1 and z_2 . z_1 104 cuts off the frequencies larger than the parameter ω_a , whose value is chosen to be around the 105 expected largest frequency. Furthermore, the parameter a determines the concavity (a < 2) or 106 convexity (a>2) of the function surface. A concave surface enhances the high frequencies while a 107 convex one hinders them. Proper tuning of the parameter *a* enhances the image quality by 108 109 suppressing artifacts such as side lobes (Figure S4). z_2 is a function that suppresses out-of-focus signal by reducing the central value of the spectra. The parameter b defines the strength of the 110 suppression and *c* determines the size of the suppressed area. 111

- 112
- 113
- 114





Figure S4 Reconstructions at high and low resolution gains. 116 Comparison between reconstructions using the Wiener-filter×Apodization approach in references [2], [3] and our 117 reconstruction method using the initial and continued deconvolution with the Richardson-Lucy 118 algorithm (labeled as RL reconstruction). The comparison concerns the reconstruction effects on 119 a point source, which is a bead cropped from a larger field of view with 40 nm fluorescent beads 120 embedded in phytagel. (a) Wide-field and reconstructions of a sample illuminated with a pattern 121 122 period of 183 nm to achieve a 2.4 resolution gain. Below each image is the corresponding (b) spectrum of the whole field of view. (c) Sample under a 307 nm pattern period, corresponding to 123

a 1.9 resolution gain and (d) the spectrum of the whole field of view. Two issues highlight the 124 importance of this figure: The reconstructions using only the Wiener filter are not optimal, the 125 bead with 2.4 resolution gain features a petal-like artifact as consequence of its very patchy power 126 127 spectrum. For the 1.9 resolution gain no such artifacts occur because the overlap between the central and extended domains is larger and generates a less patchy spectrum, nevertheless, due to 128 the out-of-focus background in the wide-field image, the background in the reconstruction contains 129 features that are not there originally. After applying the apodization function to the spectra of the 130 Wiener filtered reconstructions and empirically adjusting the parameters a, b and c, it is possible 131 132 to diminish the aforementioned artifacts and improve the reconstructions as demonstrated in the Wiener×Apodization images. Finally, our approach (RL reconstruction) offers a straightforward 133 artifact-minimized reconstruction, which is comparable to the one carried out by the 134 Wiener×Apodization method. The black circle on the spectrums delimits the cut-off frequency of 135 our detection objective. Log₂ was applied on the spectrums for better visualization. Scale bar: 136 100 nm. 137

138





Figure S5 Comparison of Wiener×Apodization and RL reconstructions on images of 142 mitochondria in HepG2 cells. Left side panel: (i) Wide-Field, (ii) RL reconstruction and 143 deconvolved wide-Field with (iii) RL (10 iterations) and (iv) Wiener algorithms. Right side panel: 144 (v-xii) Several Wiener×Apodization reconstructions are displayed as a function of the parameters 145 a, b and c of the apodization function (Fig. S3). Their tuning is necessary to obtain a good 146 reconstruction. For instance, suppression of the central value of the spectrum with b=0.9 (vii-xii) 147 provides a better contrast than the images with no suppression, b=0 (v-vi). Also the suppressed 148 area of the spectrum, proportional to the c parameter, influences the reconstruction. The images 149 with c=1 (ix-x) have their contrast further improved in comparison to the ones with c=0.5 (vii-viii), 150 choosing a slightly larger value, c=2, induces artifacts such as the ones pointed by the red arrows 151 in (xi). These artifacts are boosted when trying to make the images sharper by emphasizing the 152 high frequency content by setting a=0.1 (xii). Choosing a reconstruction becomes a user-dependent 153 matter when using the parameter tuning of the apodization function, this issue is avoided with our 154 RL reconstruction method (ii). Scale bar: 500nm. 155

156



Figure S6 Reconstruction parameters extraction for the HUVE cell data set. (a) Sharp peaks are produced when correlating each of the extended domains with the central one. The position of those peaks determine the spatial frequencies $\pm \vec{p}_j$ of the illumination pattern at each *j* orientation (j=1,2,3). (b) The initial phases ϕ_j of the illumination pattern are determined from the curves $R_j(\Phi)$. The argument of the maximum in the curves (red dot) defines ϕ_j at each orientation. (c) Deconvolved wide-Field with 10 iterations of the Richardson-Lucy algorithm. (d) Super-resolved image obtained through our reconstruction method using the initial and continued deconvolution

- steps. Magnifications of the selected areas in the red and blue boxes are shown in the rightmost
- 167 column. Scale bar: 1 μ m.



Figure S7 HepG2 data set reconstructed with an open source implementation and with our 171 method. (a), (b) and (c) images reconstructed with the ImageJ/Fiji plug-in fairSIM [4]. For each 172 of these reconstructions a different set of tunable parameters was used (Table S1). Optimization of 173 174 the reconstruction was performed by trying out several parameter sets offered by the plug-in to tune the Wiener filter and the apodization function. About 9 different sets were used. To our 175 176 judgement the best reconstructions the plug-in could produce are the ones in (b) and (c). Notice that in (b) there are less periodic artifacts though less resolution gain than in (c). In (c) the 177 resolution is higher but artifact occurrence is obvious. On the other hand (d) the image generated 178 179 with our RL reconstruction produces, without the need of any parameter tuning, an image with high 180 contrast/resolution and no visible artifacts. Scale bar: 1 µm.



Figure S8 2D vs 3D initial deconvolution for 3D-SIM stacks. Applying a 3D initial 182 183 deconvolution to data illuminated with a structured pattern produces artifacts that, in order to be 184 avoided, might require the use of 3D spatially variant PSF. This situation is demonstrated with a 3D stack of 100 nm fluorescent beads imaged in an OMX commercial set-up. Orthogonal view of: 185 (a) Wide field. (b) 3D-SIM reconstruction applying our 2D initial deconvolution approach. (c) 3D-186 SIM reconstruction applying a 3D initial deconvolution. The 3D initial deconvolution was applied 187 to the 5 stacks produced at each phase step, i.e. all the planes comprising a given stack have the 188 same illumination phase. Reconstructions were carried using the 3D-SIM equations in [2]. Beads 189 190 in (b) are properly reconstructed with no artifacts present in the image. In contrast, the reconstruction in (c) displays an intensity variation across the whole field of view, reaching a 191 dramatic reduction in the center of the beads cluster (white arrows). Such situation points out to 192 the need of a 3D spatially variant PSF to compensate for those variations. To avoid such 193 complication in the initial deconvolution and keep our pipeline robust we stick to a 2D 194 195 deconvolution, which as observed in (b) does not lead to any deconvolution related artifacts. Scale 196 bar: 1 μm.

- 197
- 198
- 199
- 200
- 201



Figure S9 Maximum Projections in xy, xz and yz planes of a Yeast 3D stack. The images display the resolution enhancement when applying a plane-by-plane 2D deconvolution or a 3D deconvolution to the wide field image. Although the 2D deconvolution does not efficiently eliminate the out-of-focus background it emphasizes the in-focus signals over it. The 3D deconvolution allows easier identification of structures along the z-axis. In the same manner, a 3D continued deconvolution can be applied to the plane-by-plane 2D-SIM reconstruction to improve the axial resolution in comparison to the 2D continued deconvolution. Images were acquired with our set-up. Scale bar: 500 nm.



Figure S10 The continued deconvolution enhances contrast by re-distributing the spectra of 217 the reconstruction. The circular average of the power spectrum of the wide field (black line), RL 218 reconstruction (I_{SIM} , red line) and I_{SIM} with continued deconvolution (blue line) is presented for 219 three different image data sets: (a) HepG2 cell, (b) HUVE cell and (c) TIRF-SIM microtubules. 220 221 The complete power spectrum of the reconstruction is shown in the top left corner insets. Upper box: I_{SIM} . Lower box: deconvolved I_{SIM} . Vertical lines indicate the cut-off frequency of the wide 222 field (ω_o) and the cut-off frequency ($\omega_o + p$) of the SIM reconstructions, with p denoting the norm 223 of the pattern spatial frequency. Constants δ and l in the x-axis label denote the pixel-size and the 224 225 image length in pixels. Plots (a) and (b) show that the blue curve lies above the red curve in the 226 range of middle and high spatial frequencies, only to converge to the same noise baseline passing the frequency $\omega_o + p$. This situation indicates a contrast improvement of the deconvolved I_{SIM} image 227 rather than an increase in frequencies beyond $\omega_o + p$. In plot (c) the blue curve is found above the 228 red one beyond $\omega_o + p$. Such case might indicate a slight resolution gain due to the non-negativity 229 constraint of the RL algorithm [5,6]. 230

- 231
- 232
- 233
- 234
- 235





Figure S11 The initial deconvolution enhances the extraction of high frequencies. The 237 extended domains \tilde{S}_i^{\pm} that carry the high spatial frequencies are formed by the differences between 238 the $\tilde{\boldsymbol{O}}_{i,m}$ images acquired with the different *m* phases of the pattern (Eq. 7). The initial 239 deconvolution increases the modulation, i.e. it increases these differences over a given background. 240 These represents a two-fold advantage, because it enhances the extraction of high frequencies and 241 also allows the recovery of features in the image where the modulation in the raw images was not 242 enough to represent a resolution gain. To illustrate this, the absolute value of the intensity 243 differences $O_{1,m} - O_{1,n}$ has been calculated for the raw images in (a), (c), and (e) and also for the 244 deconvolved raw images in (b), (d), and (f). Respectively their power spectrum is shown in (g), 245 (i), (k) and (h), (j), (l). Consistently one can observe that the background level (black arrows) 246 remains the same for each pair (a)-(b), (c)-(d) and (e)-(f), but using the deconvolved raw images to 247 calculate the differences allows to salvage signals from background levels in comparison to the 248 differences calculated only with the raw images. This effect is very obvious in (c)-(d), since in (c) 249 17 image features are barely recognizable, whereas in (d) many features have been successfully recovered and lie above background levels. Such recovered features represent more information that can be incorporated into the extended domains \tilde{S}_{j}^{\pm} as seen in the spectra (i)-(j). Hence the initial deconvolution allows the recovery of image features that might be buried in the background due to a low pattern modulation, this is specially useful when using resolution gains ≥ 2 , since this involves illumination patterns with a spatial frequency in the limits of the cut-off frequency ω_{0} , situation that inherently leads to low modulations. Scale bar: 2 µm.

257

258



261

Figure S12 Testing RL reconstruction with an external data set. A TIRF-SIM data set of 262 tubulin emitting at 525 nm was reconstructed with the fairSIM implementation and our two step 263 deconvolution approach. The first row shows the (a) wide field and expansions of the its (b) upper 264 and (c) lower halves. Similarly the second and third row display the (d) RL and (g) fairSIM 265 266 reconstructions, with their corresponding power spectra on the top left corner inset. The region in 267 (b) has a cleaner background in contrast with (c) which presents a high background signal. These two regions are used to compare slight differences between the two reconstruction approaches for 268 this particular image set. Firstly, microtubules in region (h) appear more continuous and with 269

270	higher intensity than in (e), to the point that for some microtubules (white arrows) these two
271	features do not match with the intensities displayed in the wide field (b). This difference can be
272	explained if one observes that the spectra of (d) seems more evened out in comparison to the spectra
273	of (g) which has more emphasis on the low central frequencies and a ring-like area of dimmed
274	middle range frequencies. A second difference is noticed in the high background areas (c), e.g.
275	due to the initial deconvolution the reconstruction in (f) appears to have a cleaner background than
276	(i). In spite of these differences, both reconstructions are practically the same in terms of overall
277	structures and features present in the images. The data set as well as the input parameters for the
278	fairSIM reconstruction where downloaded from the fairSIM website [4]. These parameters along
279	with the parameters extracted by our reconstruction approach can be found in Table S2. Scale bar:
280	1 μm.
281	
282	
283	
284	
285	
286	
207	
287	
288	
289	
290	

Reconstruction	Period ₁ (nm) / angle (°)	Period ₂ (nm) / angle(°)	Period ₃ (nm) / angle(°)	strength	FWHM	Wiener Parameter	Apodization cut-off
(a)fairSIM*	305.57 / 0.04	293.94 / -26.99	292.28 / 26.74	0.99	0.15	0.05	2
(b)fairSIM*	305.57 / 0.04	293.94 / -26.99	292.28 / 26.74	0.99	1.2	0.2	1.5
(c) fairSIM*	305.57 / 0.04	293.94 / -26.99	292.28 / 26.74	0.99	0.15	0.2	2
(d) RL Reconstruction*†	307.20 / 0	295.71 / -26.82	293.086 / 26.5651	-	_	-	_

*PSF of NA 1 and emission wavelength of 515 nm. † 10 iterations for the initial deconvolution. 5 iterations for the continued deconvolution.

Table S1 Reconstruction parameters of the HepG2 data set (Fig. S7). The step size of the pattern phases in all reconstructions is 120°. The initial phase is not presented since it is a relative value whose value depends on the chosen range of angles.

Reconstruction	Period ₁ (nm) / angle (°)	Period ₂ (nm) / angle(°)	Period ₂ (nm) / angle(°)	strength	FWHM	Wiener Parameter	Apodization cut-off
fairSIM*	179.70 / -89.26	178.61 / -28.90	178.26 / 30.50	_	_	0.1	2
RL Reconstruction*†	180.18 / -89.04	178.83 / -28.84	178.10 / 30.53	_	_	-	_

*PSF of NA 1.49 and emission wavelength of 525 nm. † 10 iterations for the initial deconvolution. 5 iterations for the continued deconvolution.

Table S2 Reconstruction parameters of the TIRF-SIM data set (Fig. S12). The step size of the pattern phases in all reconstructions is 120°. The initial phase is not presented since it is a relative value whose value depends on the chosen range of angles. As suggested by reference [4] a correction of 405 was applied to the raw images to compensate for camera noise.

- 310
- 311
- 312
- 313
- 1. Greger, K., Swoger, J. & Stelzer, E. H. K. Basic building units and properties of a fluorescence single plane illumination microscope. *Rev. Sci. Instrum.* **78**, 23705 (2007).
- 316 2. Gustafsson, M. G. L. *et al.* Three-dimensional resolution doubling in wide-field 317 fluorescence microscopy by structured illumination. *Biophys. J.* **94**, 4957–70 (2008).
- 318 3. Wicker, K., Mandula, O., Best, G., Fiolka, R. & Heintzmann, R. Phase optimisation for 319 structured illumination microscopy. *Opt. Express* **21**, 2032–2049 (2013).
- Müller, M., Mönkemöller, V., Hennig, S., Hübner, W. & Huser, T. Open-source image reconstruction of super-resolution structured illumination microscopy data in ImageJ. *Nat. Commun.* 7, 10980 (2016).
- Sementilli, P. J., Hunt, B. R. & Nadar, M. S. Analysis of the limit to superresolution in incoherent imaging. J. Opt. Soc. Am. A 10, 2265 (1993).
- 325 6. VERVEER, P. J., GEMKOW, M. J. & JOVIN, T. M. A comparison of image restoration
 326 approaches applied to three-dimensional confocal and wide-field fluorescence microscopy.
 327 *J. Microsc.* 193, 50–61 (1999).
- 328
- 329
- 330
- 331