

Supplementary Materials:  
**Improving Estimation of Fiber Orientations in  
Diffusion MRI Using Inter-Subject Information  
Sharing**

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### Proof for Reorientation Equation

Applying  $\mathbf{A}^{-1}(\mathbf{x}_j)$  to  $\hat{\mathbf{u}}$  on both sides of Eq. 5, we have

$$\begin{aligned}
& \psi \left( \frac{\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|}, q' \right) \\
& \approx \int E(\mathbf{q}, x_j) \delta \left( \hat{\mathbf{q}}^T \frac{\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|} \right) \delta(\|\mathbf{q}\| - q') d\mathbf{q} \\
& = \int E(\mathbf{q}, x_j) \delta \left( (\mathbf{A}^{-T}(\mathbf{x}_j)\hat{\mathbf{q}})^T \frac{\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|} \right) \times \delta(\|\mathbf{q}\| - q') d\mathbf{q}.
\end{aligned} \tag{14}$$

Performing change of variable  $\frac{\mathbf{A}^{-T}(\mathbf{x}_j)\hat{\mathbf{q}}}{\|\mathbf{A}^{-T}(\mathbf{x}_j)\hat{\mathbf{q}}\|} \rightarrow \hat{\mathbf{q}}$ , (14) becomes

$$\begin{aligned}
& \psi \left( \frac{\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|}, q' \right) \\
& = \int E \left( \frac{\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}}{\|\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}\|} \|\mathbf{q}\|, x_j \right) \delta \left( \frac{\hat{\mathbf{q}}^T}{\|\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}\|} \frac{\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|} \right) \\
& \quad \times |\mathbf{A}^T(\mathbf{x}_j)| |\mathbf{J}_{\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}} | \delta(\|\mathbf{q}\| - q') d\mathbf{q}.
\end{aligned} \tag{15}$$

Using the generalized scaling property of the Dirac-delta function [1], we have

$$\delta\left(\frac{\hat{\mathbf{q}}^T}{\|\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}\|}\frac{\hat{\mathbf{u}}}{\|\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}\|}\right) = |\mathbf{J}_{\mathbf{A}^T(\mathbf{x}_j)\hat{\mathbf{q}}}|^{-1}|\mathbf{J}_{\mathbf{A}^{-1}(\mathbf{x}_j)\hat{\mathbf{u}}}|^{-1}\delta(\hat{\mathbf{q}}^{-T}\hat{\mathbf{u}}). \quad (16)$$

Substituting (16) in (15), we have Eq. 8.

## References

- [1] IM Gelfand and GE Shilov. *Generalized functions. Vol. 3, Theory of differential equations*. Academic Press, 1967.