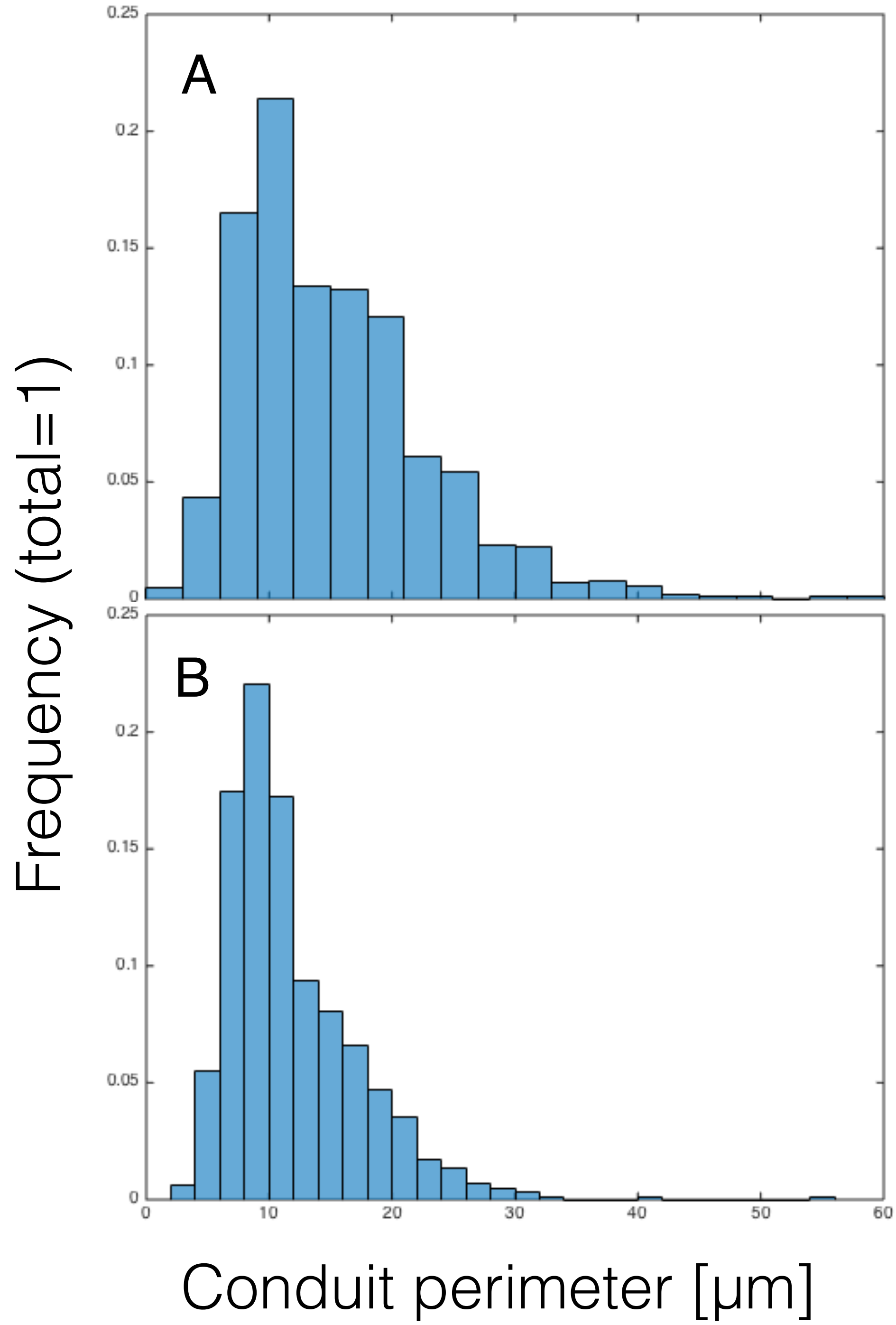


Figure S1. Normalized frequency distribution of perimeters for 2,278 xylem conduits from intrusive and bounding veins over all stages of collapse. A: Intrusive type, n=1241; B: Bounding type, n=1037.

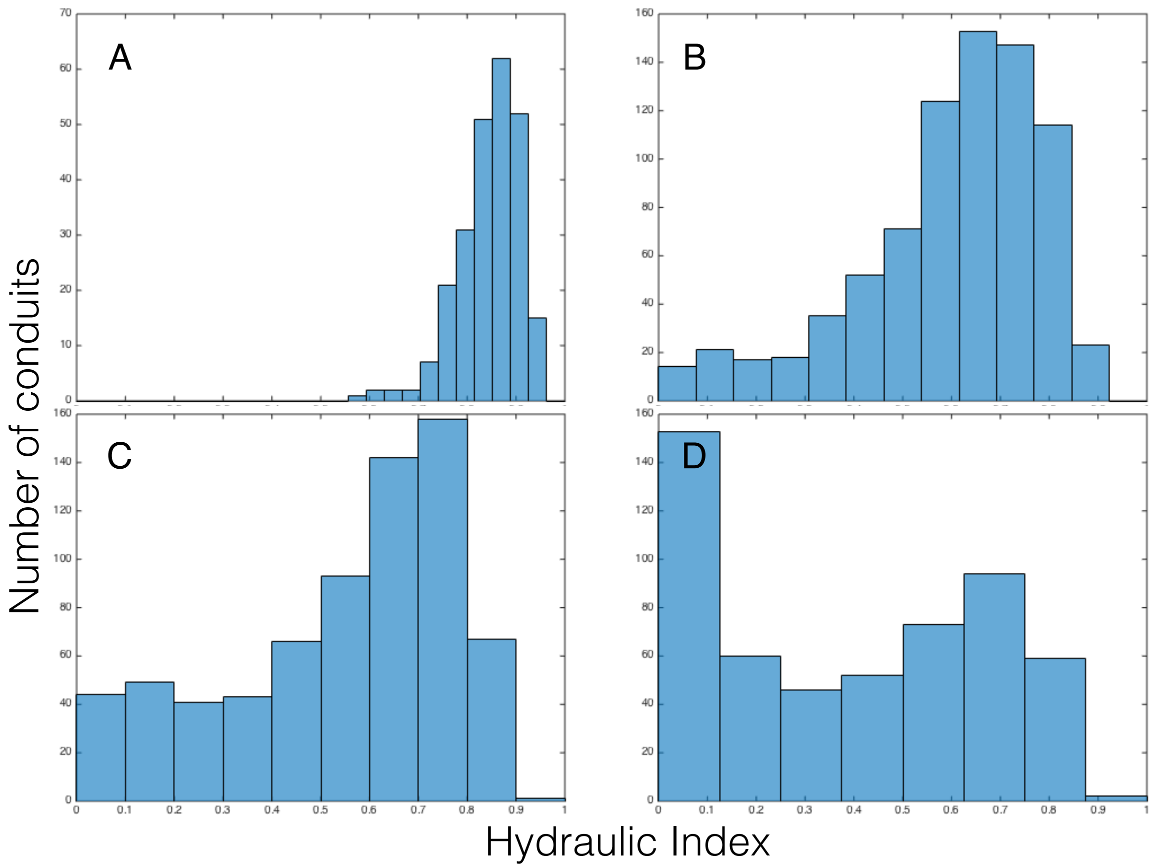


Figure S2. Frequency distribution for individual conduit hydraulic index values for both intrusive and boundary veins over progressive stages of water stress. A: -0.3 MPa to -1.99 MPa; B: -2 MPa to -2.49 MPa; C: -2.5 to -2.95 MPa; D: -2.96 to -3.9 MPa.

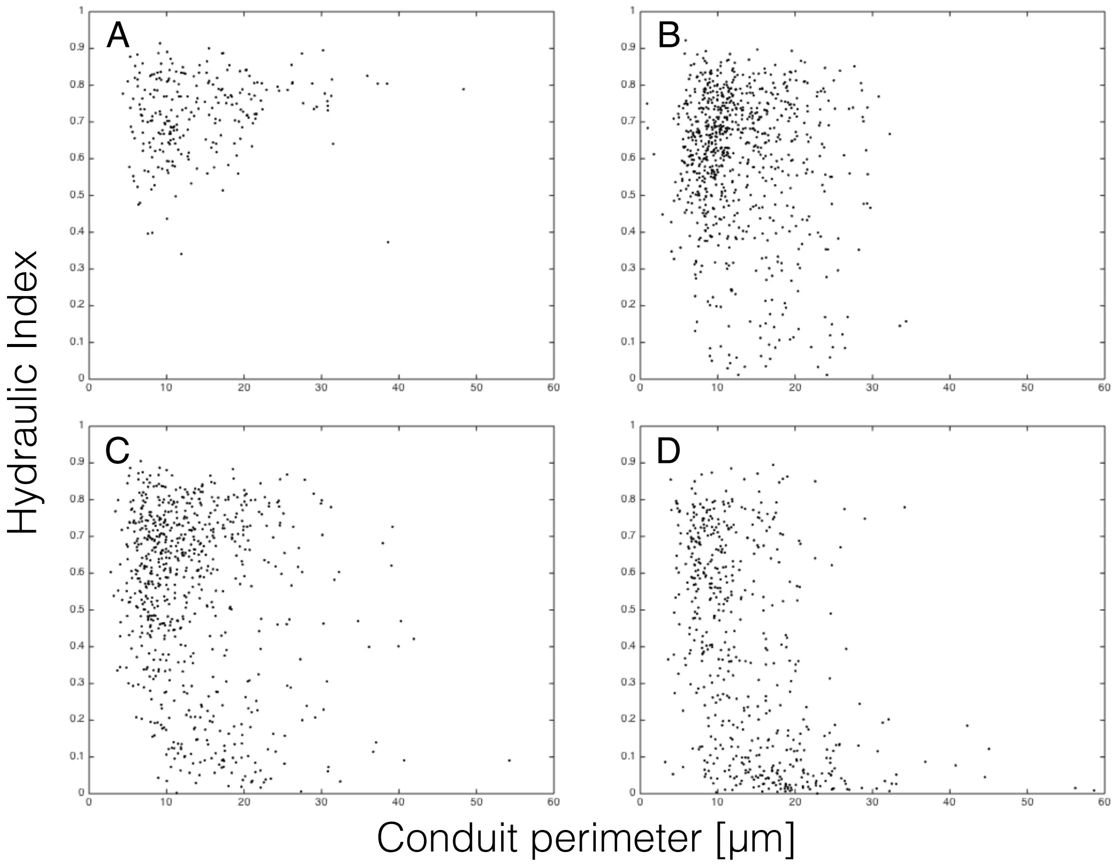


Figure S3. Conduit perimeters (μm) versus hydraulic index values for individual conduits from both intrusive and boundary veins over progressive stages of water stress. A: -0.3 MPa to -1.99 MPa; B: -2 MPa to -2.49 MPa; C:-2.5 to -2.95 MPa; D: -2.96 to -3.9 MPa.

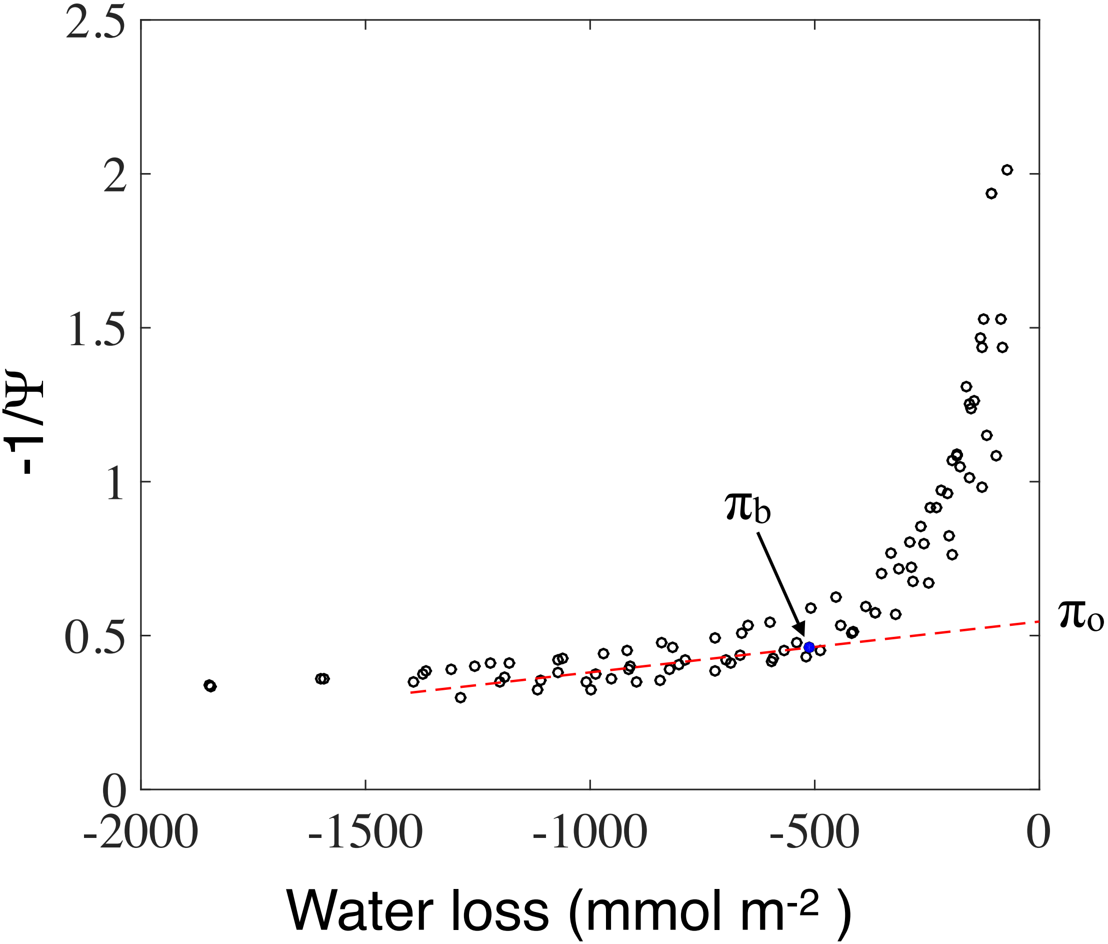


Figure S4. Pressure volume Höffler curve for red oak (n=6, July 2015). The buckling point (less generally known as the "turgor loss point") π_b was estimated as -2.17 MPa, and osmotic potential at full water content π_o= -1.83 MPa.

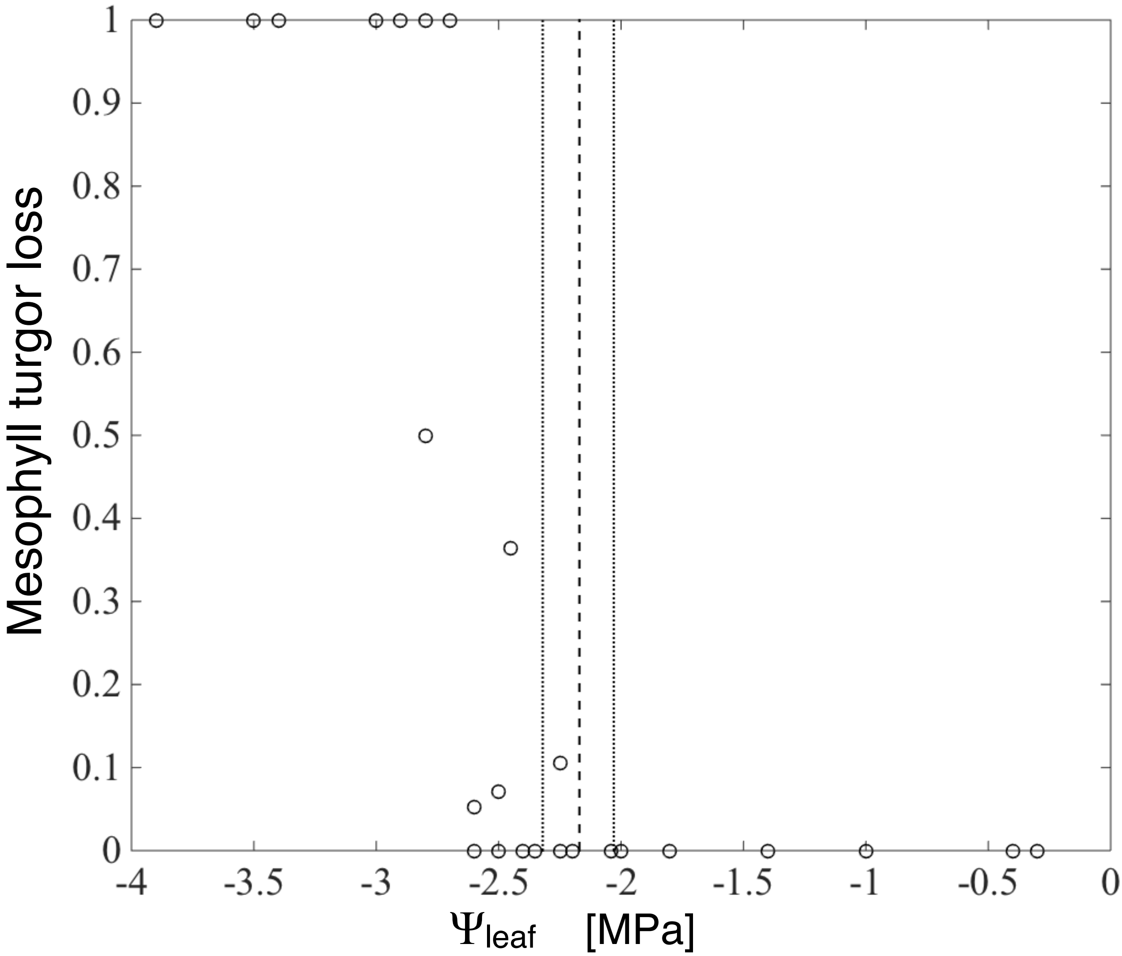


Figure S5. Proportion of intrusive vein images showing evidence of buckling in the surrounding mesophyll plotted as a function of sample (leaf) water potential. The average buckling point defined by pressure-volume curves for five leaves was -2.17 MPa (dashed line), with the range given by -2.03 to -2.32 MPa (dotted lines). The imaging-based data transition from few signs of wrinkling or folding of mesophyll cell surfaces to pervasive signs of deformation between -2.6 and -2.7 MPa.

Supplemental Text S1: Model with cavitation only.
Code may be run in Mathematica, Wolfram Research.

```
Remove["Global`*"]
```

Define initial steady state and parameters

```
gs = 0.3(* stomatal conductance mol m-2 s-1 , .305 for oak*)
0.3

χair = 0.0166338
0.0166338

χl = 0.0166338 + 0.0222693740992867 (*match model*)
0.0389032

Δχi = χl - χair
0.0222694
```

$$ETi = \Delta\chi i\,(1/gs + 1/1.5968)^{-1}$$

```
0.00562417

KVT = ETi / (-1.29 - (-1.69)) (*molar conductance*)
0.0140604

RVT = 1 / KVT
71.1216

PRMV = .001;

RV = (1 - PRMV) RVT;
RMV = PRMV RVT;

KMVi = 1 / RMV
14.0604

KVi = 1 / RV
0.0140745
```

$$\left(\frac{1}{KVi} + \frac{1}{KMVi}\right)^{-1}$$

```
0.0140604

ψLi = -1.69;

ψPi = -1.29;
```

$$\psi Vi = \psi Pi - \frac{ETi}{KVi}$$

```
-1.6896
```

KS = ETi / (-.1 - ψPi)

0.00472619

$$\text{KVMAX} = \text{KVi} \Big/ \left(\frac{\left(1 + 95.0927\, e^{-0.00185975\, (-\psi\text{Vi})^{5.53753}}\right)}{100} \right) \text{(*for petiole*)}$$

0.0151471

$$\text{KMVMAX} = \text{KMVi} \Big/ \left(\frac{\left(0.15929 + 0.59860\, e^{-0.002537444201795871\grave{}\, (-\psi\text{Li})^{6.121770706997838\grave{}}}\right)}{0.15929 + 0.59860} \right) \text{(*.15929 for fine veins*)}$$

14.773

$$\text{KV}[\text{x\_}] := \text{KVMAX} \frac{\left(1 + 95.0927\, e^{-0.00185975\, (-x)^{5.53753}}\right)}{100} \text{(*for petiole*)}$$

KV[ψVi] - KVi

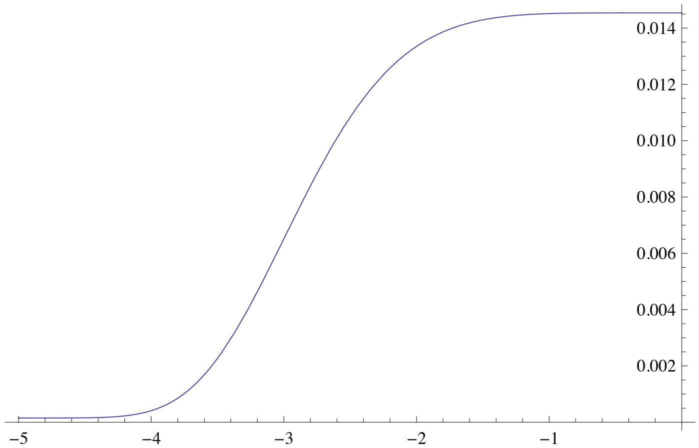
0.

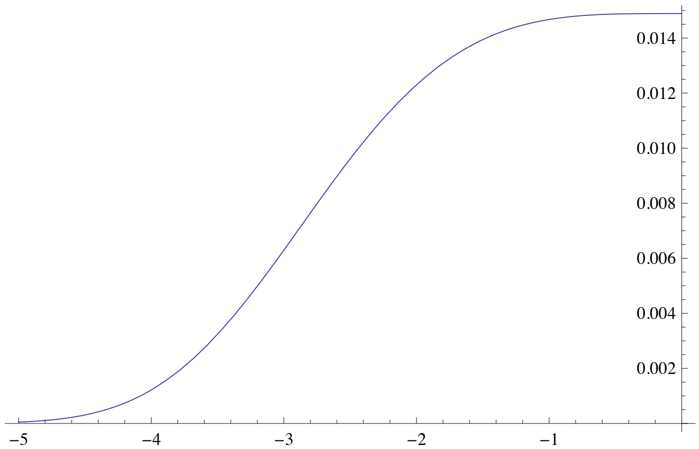
$$\text{KMV}[\text{x\_}] := \text{KMVMAX} \frac{\left(0.15929 + 0.59860\, e^{-0.002537444201795871\grave{}\, (-x)^{6.121770706997838\grave{}}}\right)}{0.15929 + 0.59860} \text{(*.15929 for fine veins*)}$$

$$\text{Kleaf}[\text{x\_}] := \left(\frac{1}{\text{KV}[x]} + \frac{1}{\text{KMV}[x]} \right)^{-1}$$

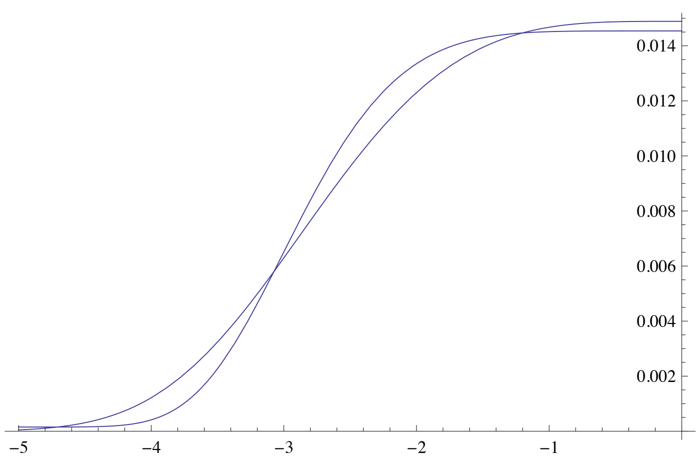
$$\text{KL}[\text{x\_}] := .01488936683642644\grave{}\, e^{-0.014551849381043652\grave{}\, (-x)^{3.713774342889727\grave{}}}$$

```
p1 = Plot[Kleaf[x], {x, -5, 0}]
p2 = Plot[KL[x], {x, -5, 0}]
```





```
Show[p1, p2, PlotRange → All]
```



```
CL[x_] := If[x > -4, -.104 x^4 - .7588 x^3 - 1.6778 x^2 - 1.46 x - .0653, 13.9056 / x^2]
 (*mol m-2 MPa-1*)
```

Define perturbation

```
Δχ = .044;
```

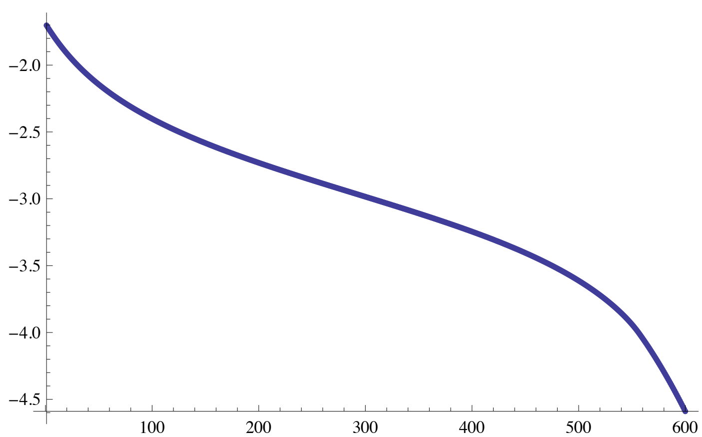
```
Δt = 1;
```

Define time length

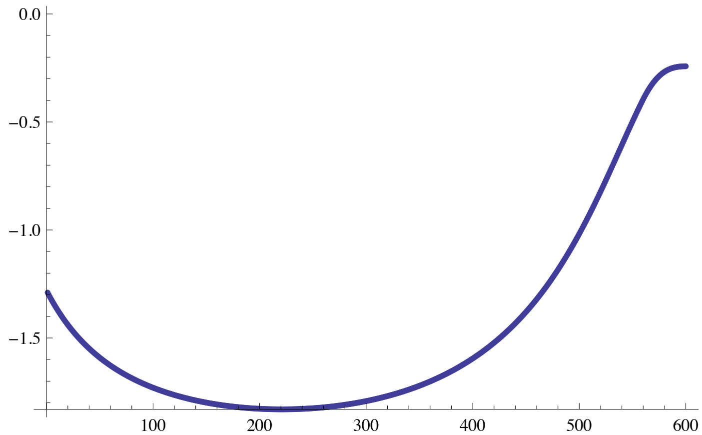
```
T = 600;

ψL = Table[0, {T}];

ψP = Table[0, {T}];

ψV = Table[0, {T}];

CAP = Table[0, {Length[ψL]}];

KVC = Table[0, {Length[ψL]}];

KVX = Table[KVi, {Length[ψL]}];
KMVX = Table[KMVi, {Length[ψL]}];

KEFF = Table[0, {Length[ψL]}];

ETX = Table[0, {Length[ψL]}];

ETC = Table[0, {Length[ψL]}];

ET = Δχ (1 / gs + 1 / 1.5968)^-1
0.0111123
```

Iterate local solution over all time steps: In this model driving vein K due to cav with wp of middle of vein - half way between stem and minor vein.

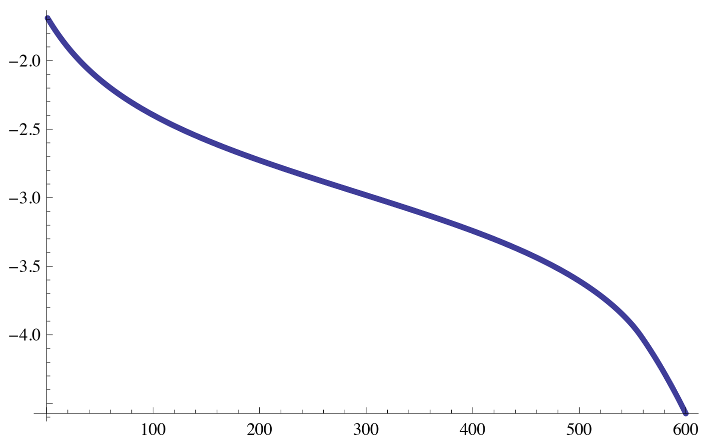
```
Do[CAP[[i]] = CL[ψLi]; (* log values of cap and KV lagged 1 step *)
 KVX[[i]] = KV[ψVi];
 (* calculate vein conductance from vulner curve cavitation*)
 (*KMVX[[i]]=KMV[ψLi];(* get minor vein K *)*)
 KVC[[i]] = Min[KVX]; (* set cav K inc hysteresis effects *)
 ETX[[i]] = (-.1 - ψLi) (1/KS + 1/KVC[[i]] + 1/KMVX[[i]])^-1;
 (* flow from soil lagged 1 step,
 cond is collapse and cavitation effects in series *)
 KEFF[[i]] = (1/KVC[[i]] + 1/KMVX[[i]])^-1;
 (*record eff cond of veins due to cav and collapse*)
 (* Update water potentials for next step *)
 ETC[[i]] = ET - ETX[[i]];
 (* calculate discharge to meet E not met by lagged ETX *)
 ψL[[i]] = -(Δt (ETC[[i]])/CL[ψLi] - ψLi); (* new wpL set by discharge to satisfy ET*)
 ψP[[i]] = -.1 - (ETX[[i]] / KS); (* pet wp req to produce ETX from soil *)
 ψV[[i]] = -.1 - (ETX[[i]] / KS) - (ETX[[i]] / Min[KVX]); (* find new psi V*)
 ψPi = ψP[[i]];
 ψVi = ψV[[i]];
 ψLi = ψL[[i]], (* update water potential for calc cap,
 K, EX, ψP next time step *)
 {i, Length[ψL]}]

ListPlot[ψL, PlotRange → All]
```



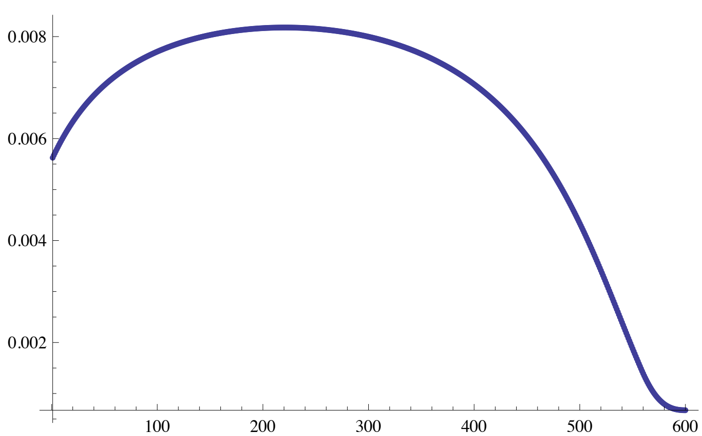
```
ListPlot[ψP]
```



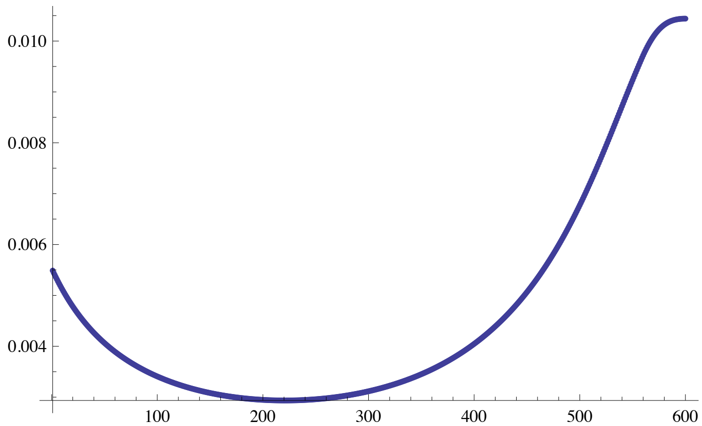

```
ListPlot[ψV]
```



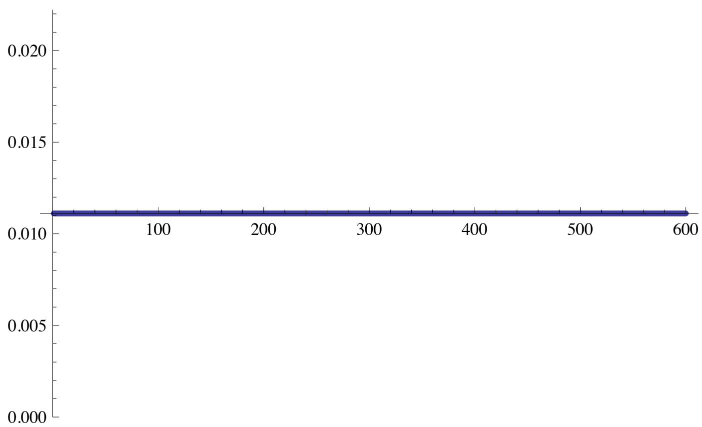

```
ListPlot[ETX, PlotRange → All]
```



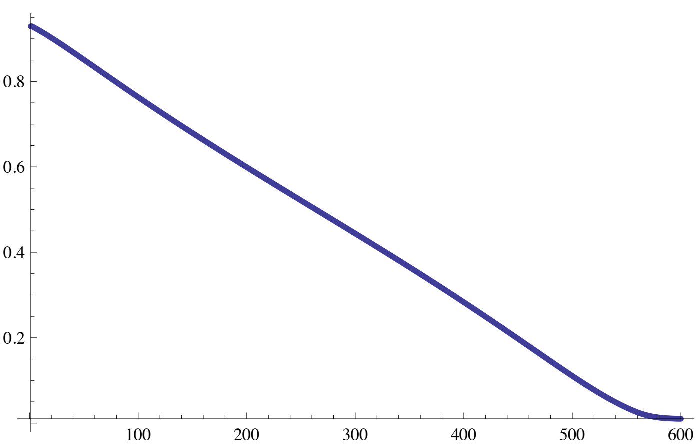
```
ListPlot[ETC, PlotRange → All]
```



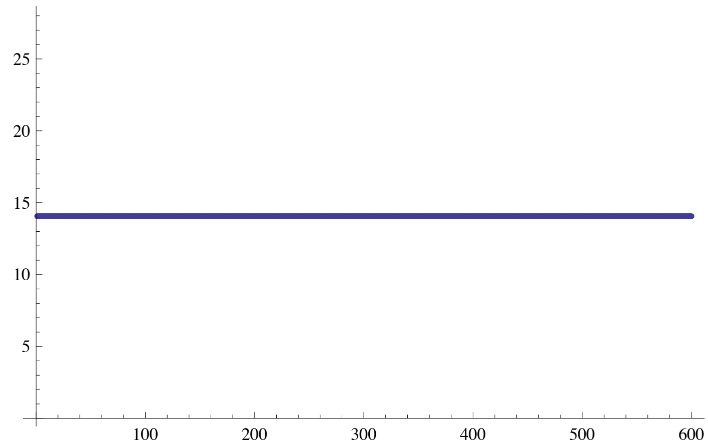

```
ListPlot[ETX + ETC]
```



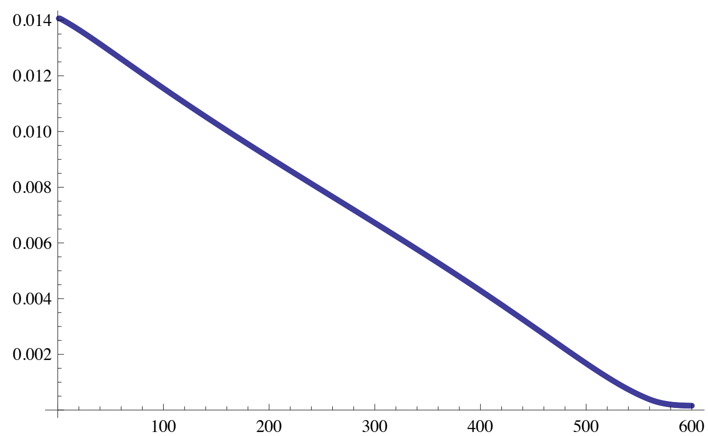

```
ListPlot[KVC / KVMAX, PlotRange → All]
```



ListPlot[KMVX]



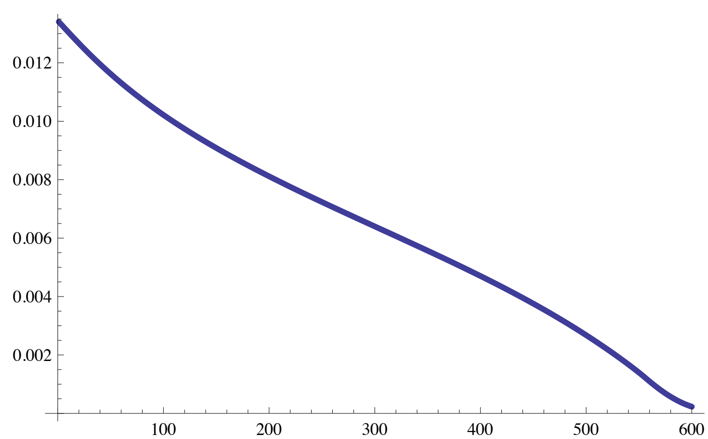
p3 = ListPlot[KEFF]



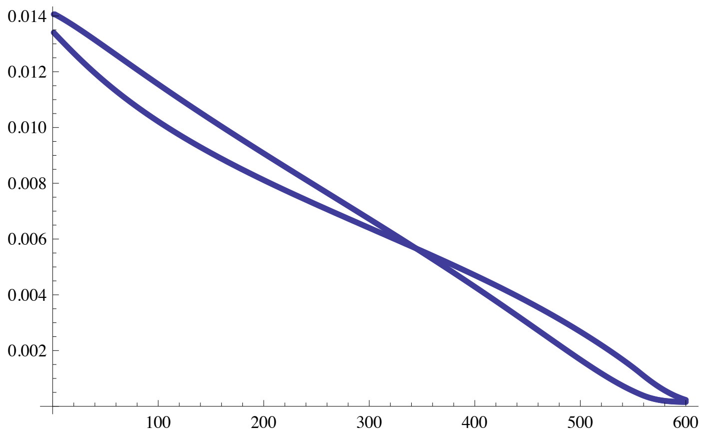
KEFF[[1]]

0.0140604

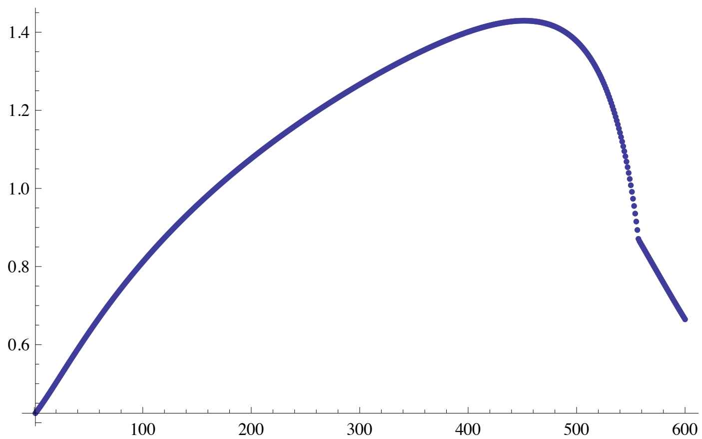
p4 = ListPlot[KL[ψL]]



```
Show[p3, p4]
```




```
ListPlot[CAP, PlotRange → All]
```




```
SetDirectory[ToFileName[NotebookDirectory[]]]
/Users/Tony/Dropbox/John Taxus/Oak ms Literature/mathematica code/Collapse models

outfile = "cavitate_"
cavitate_

Export[outfile <> "PL.xls", ψL]
cavitate_PL.xls

Export[outfile <> "PV.xls", ψV]
cavitate_PV.xls

Export[outfile <> "PP.xls", ψP]
cavitate_PP.xls
```

```
Export[outfile <> "KV.xls", KVC]
cavitate_KV.xls

Export[outfile <> "KMV.xls", KMVX]
cavitate_KMV.xls

Export[outfile <> "KEFF.xls", KEFF]
cavitate_KEFF.xls

Export[outfile <> "EX.xls", ETX]
cavitate_EX.xls

Export[outfile <> "EC.xls", ETC]
cavitate_EC.xls
```

Supplemental Text S2: Model with cavitation and collapse.
Code may be run in Mathematica, Wolfram research.

```
Remove["Global`*"]
```

Remove::rmnsm : There are no symbols matching "Global`*". ≫

Define initial steady state and parameters

```
gs = 0.3(* stomatal conductance mol m-2 s-1 , .305 for oak*)
0.3

χair = 0.0166338
0.0166338

χl = 0.0166338 + 0.0222693740992867 (*match model*)
0.0389032

Δχi = χl - χair
0.0222694

ETi = Δχi (1 / gs + 1 / 1.5968)^-1
0.00562417

KVT = ETi / (-1.29 - (-1.69)) (*molar conductance*)
0.0140604

RVT = 1 / KVT
71.1216

PRMV = .5;

RV = (1 - PRMV) RVT;
RMV = PRMV RVT;

KMVi = 1 / RMV;

KVi = 1 / RV;
```

$$\left(\frac{1}{KVi} + \frac{1}{KMVi}\right)^{-1}$$

```
0.0140604

ψLi = -1.69;

ψPi = -1.29;
```

$$\psi Vi = \psi Pi - \frac{ETi}{KVi}$$

```
-1.49

KS = ETi / (-.1 - ψPi)
0.00472619
```

```
KVMAX = KVi / ((1 + 95.0927 ⅇ^(-0.00185975 (-ψVi)^5.53753)) / 100) (*for petiole*)
```

0.0297585

```
KMVMAX = KMVi /
  ((0.15929 + 0.59860 ⅇ^(-0.002537444201795871` (-ψLi)^6.121770706997838`)) / (0.15929 + 0.59860)) (*.15929 for fine veins*)
```

0.0295461

```
KV[x_] := KVMAX (1 + 95.0927 ⅇ^(-0.00185975 (-x)^5.53753)) / 100 (*for petiole*)

KV[ψVi] - KVi
```

-3.46945×10^{-18}

```
0.15929 / (0.15929 + 0.59860)
```

0.210176

```
KMV[x_] :=
  KMVMAX (0.15929 + 0.59860 ⅇ^(-0.002537444201795871` (-x)^6.121770706997838`)) / (0.15929 + 0.59860) (*.15929 for fine veins*)

KMV[ψLi] - KMVi
```

0.

```
CL[x_] := If[x > -4, -.104 x^4 - .7588 x^3 - 1.6778 x^2 - 1.46 x - .0653, 13.9056 / x^2]
  (*mol m-2 MPa-1*)
```

Define perturbation

```
Δχ = .044;

Δt = 1;
```

Define time length

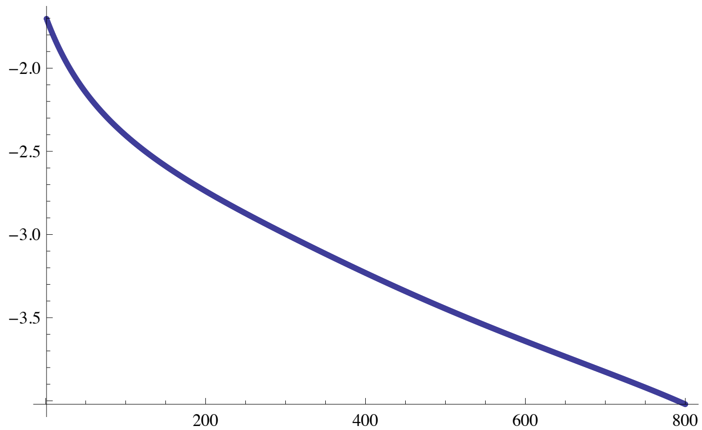
```
T = 800;

ψL = Table[0, {T}];

ψP = Table[0, {T}];

ψV = Table[0, {T}];

CAP = Table[0, {Length[ψL]}];

KVC = Table[0, {Length[ψL]}];

KVX = Table[KVi, {Length[ψL]}];
KMVX = Table[KMVi, {Length[ψL]}];
```

```
KEFF = Table[0, {Length[ψL]}];

ETX = Table[0, {Length[ψL]}];

ETC = Table[0, {Length[ψL]}];

ET = Δχ (1 / gs + 1 / 1.5968)^-1
0.0111123
```

Iterate local solution over all time steps: In this model driving vein K due to cav with wp of middle of vein - half way between stem and minor vein.

```
Do[CAP[[i]] = CL[ψLi]; (* log values of cap and KV lagged 1 step *)
 KVX[[i]] = KV[ψVi];
 (* calculate vein conductance from vulner curve cavitation*)
 KMVX[[i]] = KMV[ψLi]; (* get minor vein K *)
 KVC[[i]] = Min[KVX]; (* set cav K inc hysteresis effects *)
 ETX[[i]] = (-.1 - ψLi) (1/KS + 1/KVC[[i]] + 1/KMVX[[i]])^-1;
 (* flow from soil lagged 1 step,
 cond is collapse and cavitation effects in series *)
 KEFF[[i]] = (1/KVC[[i]] + 1/KMVX[[i]])^-1;
 (*record eff cond of veins due to cav and collapse*)
 (* Update water potentials for next step *)
 ETC[[i]] = ET - ETX[[i]];
 (* calculate discharge to meet E not met by lagged ETX *)
 ψL[[i]] = -(Δt (ETC[[i]])/CL[ψLi] - ψLi); (* new wpL set by discharge to satisfy ET*)
 ψP[[i]] = -.1 - (ETX[[i]] / KS); (* pet wp req to produce ETX from soil *)
 ψV[[i]] = -.1 - (ETX[[i]] / KS) - (ETX[[i]] / Min[KVX]); (* find new psi V*)
 ψPi = ψP[[i]];
 ψVi = ψV[[i]];
 ψLi = ψL[[i]], (* update water potential for calc cap,
 K, EX, ψP next time step *)
 {i, Length[ψL]}]
```

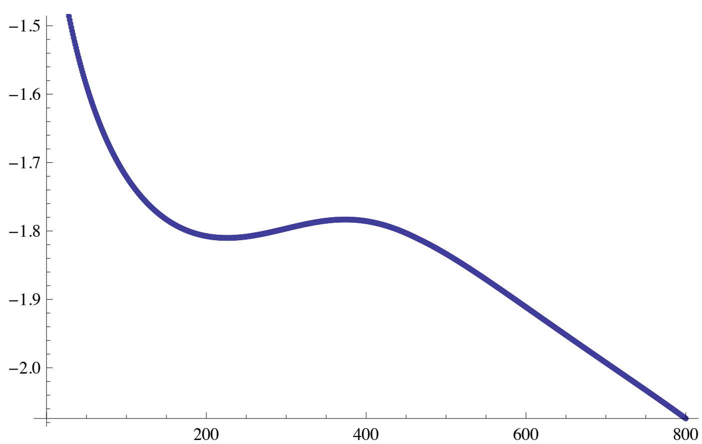
```
ListPlot[ψL, PlotRange → All]
```



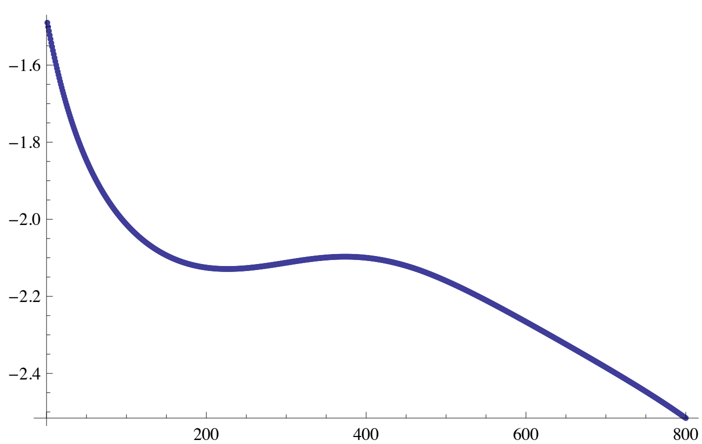

```
ψL[[470]]
```

```
-3.38377
```

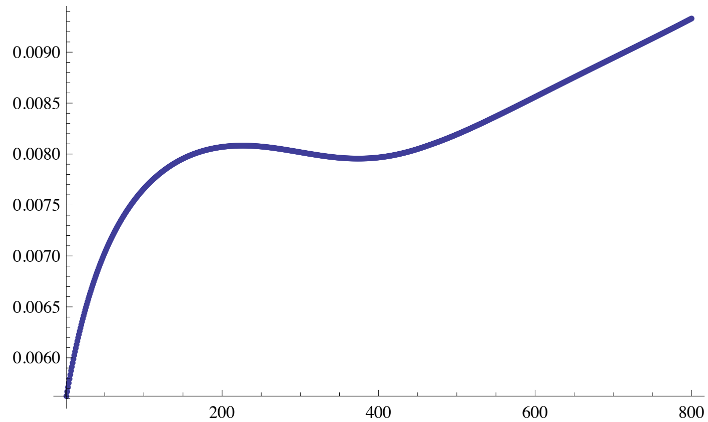
```
ListPlot[ψP]
```



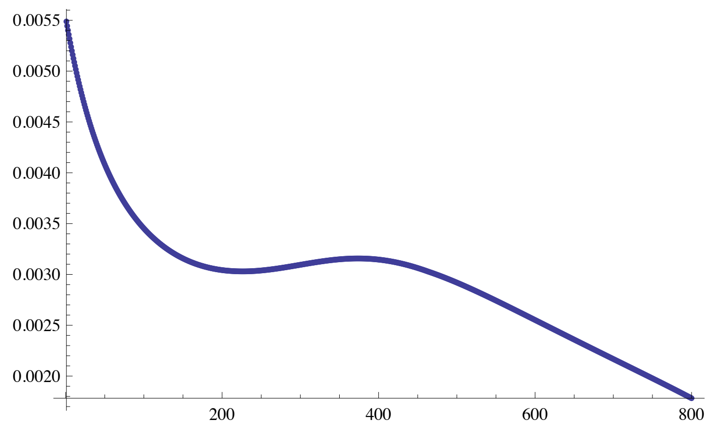

```
ListPlot[ψV]
```



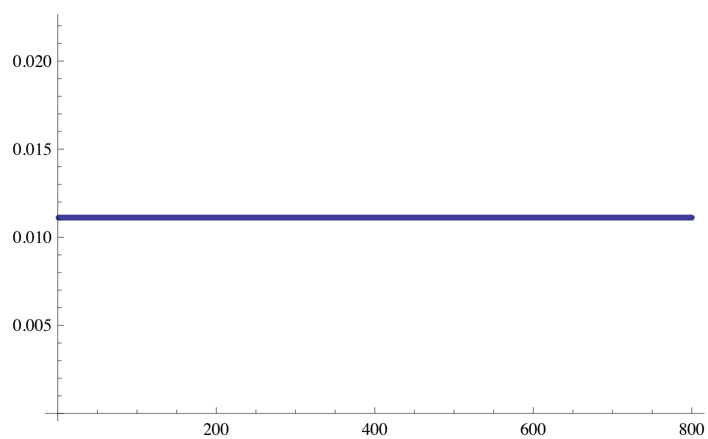
```
ListPlot[ETX, PlotRange → All]
```




```
ListPlot[ETC, PlotRange → All]
```

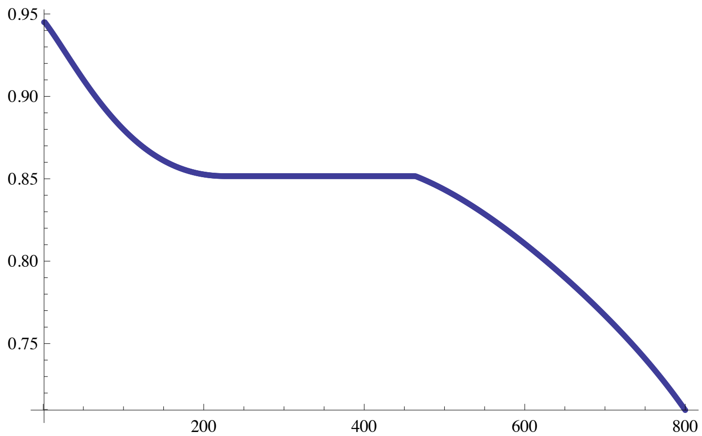



```
ListPlot[ETX + ETC]
```




```
KVN = KVC / KVMAX;
```

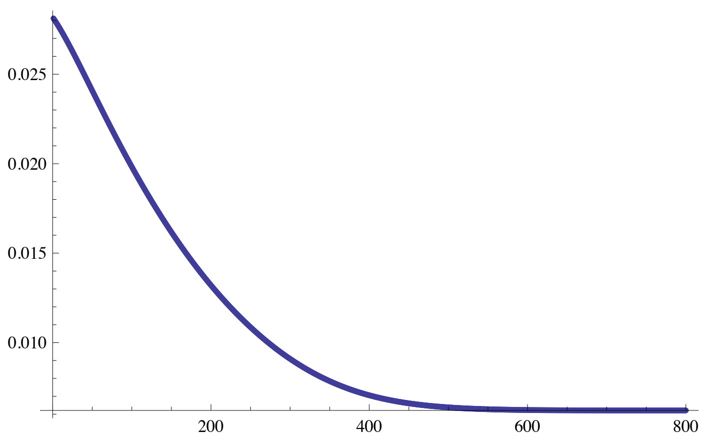
ListPlot[KVC / KVMAX, PlotRange → All]



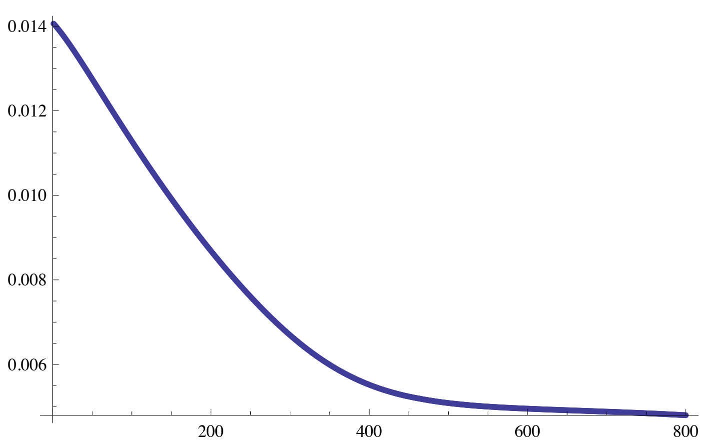
KVX[[470]] / KVMAX

0.85037

ListPlot[KMVX]



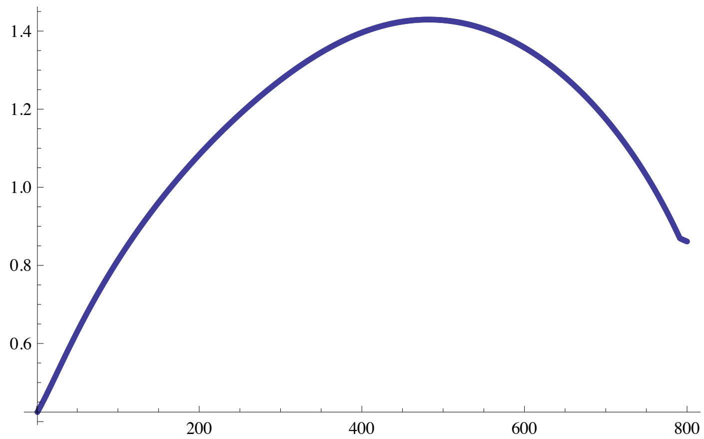
ListPlot[KEFF]



```
KEFF[[1]]
```

```
0.0140604
```

```
ListPlot[CAP, PlotRange → All]
```




```
SetDirectory[ToFileName[NotebookDirectory[]]]
```

```
/Users/Tony/Dropbox/John Taxus/Oak ms Literature/mathematica code/Collapse models
```

```
outfile = "collapse_"
```

```
collapse_
```

```
Export[outfile <> "PL.xls", ψL]
```

```
collapse_PL.xls
```

```
Export[outfile <> "PV.xls", ψV]
```

```
collapse_PV.xls
```

```
Export[outfile <> "PP.xls", ψP]
```

```
collapse_PP.xls
```

```
Export[outfile <> "KV.xls", KVC]
```

```
collapse_KV.xls
```

```
Export[outfile <> "KVN.xls", KVN]
```

```
collapse_KVN.xls
```

```
Export[outfile <> "KMV.xls", KMVX]
```

```
collapse_KMV.xls
```

```
Export[outfile <> "KEFF.xls", KEFF]
```

```
collapse_KEFF.xls
```

```
Export[outfile <> "EX.xls", ETX]
```

```
collapse_EX.xls
```

```
Export[outfile <> "EC.xls", ETC]
collapse_EC.xls
```