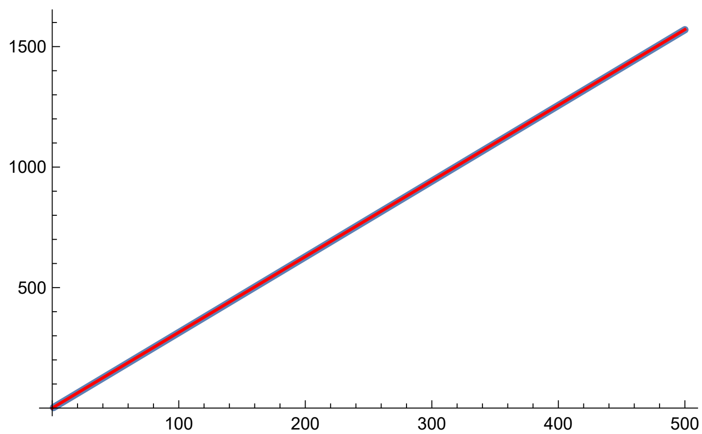
```
(*PART I: FLAT VELOCITY PROFILE *)

n[t] /. DSolve[{n''[t] + n'[t] / t + λ n[t] == 0}, n, t][[1]]

BesselJ[0, t √λ] C[1] + BesselY[0, t √λ] C[2]

(* BesselY[0,t √λ] diverges at t=0,
therefore the unnormalized eigenfunction is BesselJ[0,t √λ] *)

(*finding the eigenvalues from the boundary condition;
eigenvalues scale like (Pi*i)^2 *)

d = Table[BesselJZero[0, k], {k, 1., 500}];

Show[ListPlot[d], Plot[Pi x, {x, 0, 500}, PlotStyle → Red]]
```




```
(*eigenfunction normalization *)

Integrate[BesselJ[0, a t]^2 t, {t, 0, 1}]
```

$$\frac{1}{2}\left(\text{BesselJ}[0, a]^2 + \text{BesselJ}[1, a]^2\right)$$

```
norm = Table[Sqrt[1/2 (BesselJ[0, a]^2 + BesselJ[1, a]^2)], {a, d}];

(*finding a_i's*)

Simplify[Integrate[BesselJ[0, a t] t, {t, 0, 1}]]
```

$$\frac{\text{BesselJ}[1, a]}{a}$$

```
aiunnorm = Table[BesselJ[1, a] / a, {a, d}];

ai = aiunnorm / norm;

(* finding derivative at the boundary *)
```

```
D[BesselJ[0, a t], t]
```

```
-a BesselJ[1, a t]
```

```
deriv = Table[-a BesselJ[1, a ], {a, d}] / norm;
```

```
(*calculating b_i's; factor 2 is from the factor 1/(\int_0^1 t dt)*)
```

```
bi = -2 ai deriv / d^2 ;
```

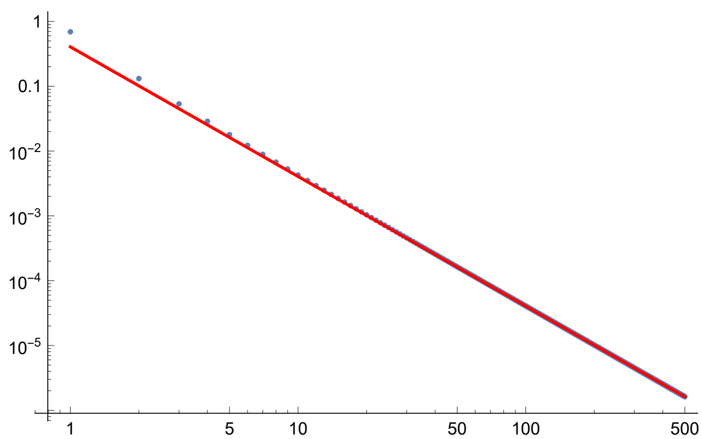
```
Simplify[- BesselJ[1, a]/a (-a BesselJ[1, a ]) / (1/2 (BesselJ[0, a]^2 + BesselJ[1, a]^2))]
```

$$\frac{2\,\text{BesselJ}[1,\,a]^2}{\text{BesselJ}[0,\,a]^2+\text{BesselJ}[1,\,a]^2}$$

```
(* 2 BesselJ[1,a]^2/(BesselJ[0,a]^2+BesselJ[1,a]^2) =2 for a=d; b= 4/d^2*)
```

```
(*checking the asymptotic scaling *)
```

```
Show[ListLogLogPlot[bi],
 LogLogPlot[4. / Pi^2 n^-2., {n, 1, 500}, PlotStyle → Red], PlotRange → All]
```




```
(*calculating the sequestration probability *)
```

```
ratio[z_] = Total[(1 - Exp[-d^2 / z]) bi];
```

```
Remainder[x_] := Sum[0.4 n^-2. (1 - Exp[- (Pi n)^2]), {n, 501, 1 000 000}];
```

```
(*sinusoid data for F*)
```

```
F = {1, 3, 6, 9}
```

```
{1, 3, 6, 9}
```

```
ratio[F] + Remainder[F]
```

```
{0.997859, 0.899362, 0.73536, 0.631763}
```

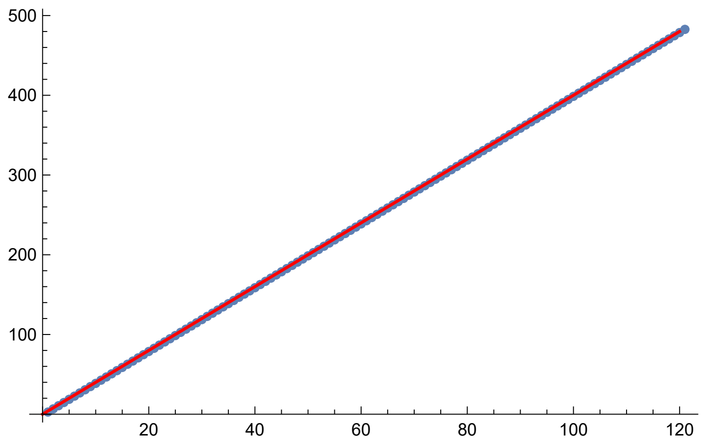
```
(*artery data *)
```

```
F = {18 431, 55 293, 110 585, 165 878}

{18 431, 55 293, 110 585, 165 878}

ratio[F] + Remainder[F]

{0.0165574, 0.00956786, 0.00676595, 0.00552364}

(*vein data *)

F = {165 361, 496 083, 992 165, 1 488 248}

{165 361, 496 083, 992 165, 1 488 248}

ratio[F] + Remainder[F]

{0.00553227, 0.00319118, 0.00226235, 0.00186529}

(*PARABOLIC FLOW PROFILE *)

(*finding the eigenvalues *)

Simplify[n[t] /. DSolve[{n''[t] + n'[t] / t + λ (1 - t^2) n[t] == 0}, n, t][[1]],
 Assumptions → t > 0]
```

$$\sqrt{2}\, e^{-\frac{1}{2} t^2 \sqrt{\lambda}} \left(C[1]\, \text{HypergeometricU}\left[\frac{1}{4}\left(2-\sqrt{\lambda}\right), 1, t^2\sqrt{\lambda}\right] + C[2]\, \text{LaguerreL}\left[\frac{1}{4}\left(-2+\sqrt{\lambda}\right), t^2\sqrt{\lambda}\right]\right)$$

```
(* HypergeometricU[1/4 (2-√λ),1,t^2 √λ] diverges at t=0 *)

(*finding the eigenvalues from the boundary condition a=√λ *)

d = Table[
   (a /. FindRoot[LaguerreL[1/4 (-2 + a), a] == 0, {a, 2.7 + 4 n}][[1]]), {n, 0, 120, 1}];

(*checking the eigenvalue scaling with i *)
```

```
Show[ListPlot[d], Plot[4 i, {i, 0, 120}, PlotStyle → Red ]]
```

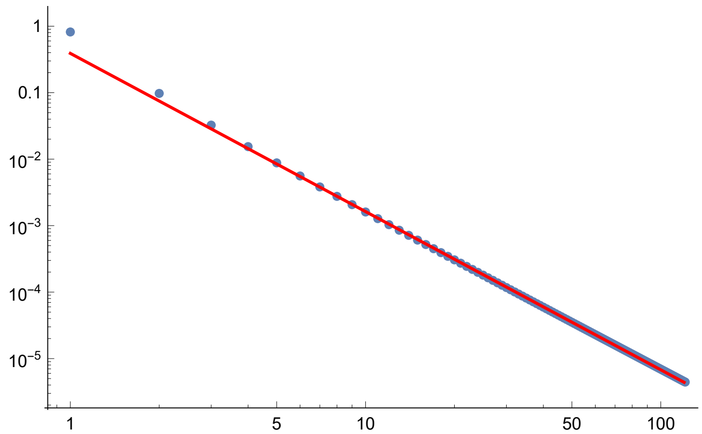



```
(*normalization constant for the eigenfunctions *)

norm = Table[
   Sqrt[NIntegrate[LaguerreL[1/4 (-2 + a), a t^2]^2 e^(-a t^2) t (1 - t^2), {t, 0, 1}]], {a, d}];

(* calcylating the a_i's *)

ai = Table[
    NIntegrate[LaguerreL[1/4 (-2 + a), a t^2] e^(-a t^2/2) t (1 - t^2), {t, 0, 1}], {a, d}] / norm;

(*calculating the derivative term at the boundary

Simplify[D[LaguerreL[1/4 (-2 + a), a t^2] e^(-a t^2/2), t]]
```

$$-a\, e^{-\frac{a t^2}{2}}\, t \left(\text{LaguerreL}\left[\frac{1}{4}(-2+a),\, a t^2\right] + 2\,\text{LaguerreL}\left[\frac{1}{4}(-6+a),\, 1,\, a t^2\right]\right)$$

```
deriv = Table[
    -a e^(-a/2) (LaguerreL[1/4 (-2 + a), a ] + 2 LaguerreL[1/4 (-6 + a), 1, a ]), {a, d}] / norm;

(* definining b_i's *)

bi = -4 ai / d^2 deriv;

(*checking scaling for b_i *)
```

```
Show[ListLogLogPlot[bi], LogLogPlot[0.39 / n^2.38, {n, 1, 120}, PlotStyle → Red]]
```




```
(*calculating the sequestration probability *)

ratio[z_] = Total[(1 - Exp[-d^2 / z]) bi];

remainder[x_] := Sum[0.39 / n^2.38 (1 - Exp[- (4 n)^2 / x]), {n, 121, 1 000 000}];

(*factor 3/2 comes from conversion of the mean to maximal flow velocity *)

(*sinusoid data for F*)
F = {1, 3, 6, 9} 3 / 2
```

$\left\{\frac{3}{2}, \frac{9}{2}, 9, \frac{27}{2}\right\}$

```
ratio[F] + remainder[F]
```

{0.993722, 0.838724, 0.635877, 0.519916}

```
(*arterial  data *)

F = {18 431, 55 293, 110 585, 165 878} 3 / 2
```

$\left\{\frac{55\,293}{2}, \frac{165\,879}{2}, \frac{331\,755}{2}, 248\,817\right\}$

```
ratio[F] + remainder[F]
```

{0.00433476, 0.00208179, 0.00130509, 0.000991623}

```
(* vein data *)

F = {165 361, 496 083, 992 165, 1 488 248} 3 / 2
```

$\left\{\frac{496\,083}{2}, \frac{1\,488\,249}{2}, \frac{2\,976\,495}{2}, 2\,232\,372\right\}$

```
ratio[F] + remainder[F]
```

{0.000993726, 0.00047012, 0.000292558, 0.000221568}