Supplementary Information for "Multiplex social ecological network analysis reveals how social changes affect community robustness more than resource depletion"

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General Information on the Multiplex Networks

To assess the robustness of community networks, we focus on northern Alaska. We analyze household networks in three indigenous communities: two coastal Iñupiaq (Wainwright and Kaktovik) and one interior Gwich'in (Venetie) (see Figure 1). Communities represent mixed economies, in which a majority of households are employed, but continue to actively engage in subsistence hunting and fishing. The three communities are fairly isolated as no road networks lead to them; however, households do receive resources from other nearby communities and other institutional actors (i.e. oil companies, businesses etc.). Networks are inferred as explained in the supplementary material - Data. Nodes represent households, whaling crews, stores, and other entities operating or connected to households in one of the three communities. The analysis includes 218 nodes in Wainwright, 206 in Venetie and 164 in Kaktovik. Of these nodes, 69% in Wainwright 51% in Kaktovik and 45% in Venetie represent local households.

Households are connected to each other if they share, cooperate with, or contribute food and non-food supplies to each other. Each type of social relation (i.e. sharing, helping, contributing, lending equipment etc.) is paired to a specific resource (i.e. bowhead, caribou, moose, salmon, dall sheep, etc.). Each unique resource-relation edge type represent a layer of a multiplex network. Each multiplex network represent the sharing and cooperative network of a specific village ($n = 3$).

There are 36 total layers in Wainwright, 37 in Kaktovik and 43 in Venetie. Examples of different layers are: beluga-cooperative hunting, duck-trading, beluga-contribution, bowhead-contribution. Contributions include supplies such as gas, equipment, ammunition, processing and cooking labor and cash. Household often are connected via multiple-resource-relation edges. Table 1 describes the average number of layers in which households are active.

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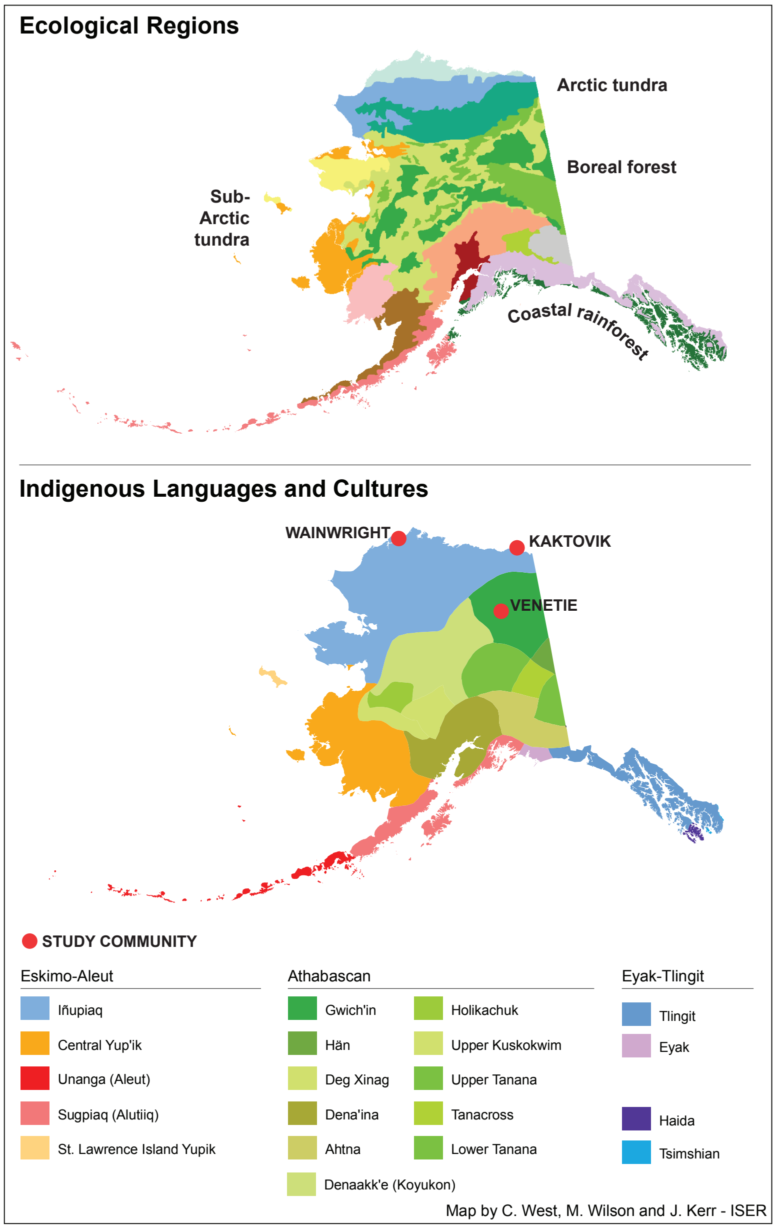

Fig. 1. Study Area: in Alaska, there is strong overlap between ecosystems and ethnolinguistic territories of indigenous groups. Iñupiaq territory is coastal, allowing access to diverse terrestrial, riverine and marine resources, while Gwich'in territory allows direct access only to terrestrial and riverine resources; Reprinted from Chapin III, F. Stuart, Kofinas, Gary P., Folke, Carl (2009) (Eds.) Principles of Ecosystem Stewardship Resilience-Based Natural Resource Management in a Changing World; Springer

(two connected nodes).

Table 1. Giving and Receiving: descriptive statistics of households' participation in layers

Village	NetType	Mean	Stdev	Min	Max	Median
All	Receiving	12.663	5.030	1	26	13
	Giving	10.417	6.525	1	25	10
Wainwright	Receiving	11.979	3.942	1	23	12
	Giving	10.730	5.644	1	22	11
Kaktovik	Receiving	14.110	5.571	1	26	15
	Giving	10.748	7.729	1	25	9
Venetie	Receiving	12.455	5.851	1	25	11
	Giving	9.671	6.551	1	21	9

Data. Data represent household networks for Wainwright, Kaktovik and Venetie. Edges are valued and directed and represent aggregated flows of food (Lbs.) and non-food resources (no. of different types of ties). All exchanges occurred during a 12- month period in 2009-10 - across 7 to 10 key resources (i.e. species) and 12 types of social relations.

A year of collaboration, focus groups, key informant interviews, literature review and research into previous harvest surveys guided choices of key resources and social relations (21). Key focal resources were selected to include species that contributed the most edible pounds to the community harvest and/or were used by most households (often the same species). Core species comprise 93-97% of total community harvests by edible weight (96% in Wainwright, 93% in Kaktovik and 97% in Venetie). For ecological and cultural reasons, key species varied by community. For example, moose, salmon and berries were key species only in Venetie: Arctic char was a key species only in Kaktovik, while smelt was a key species only in Wainwright. Venetie receives marine species (bowhead, beluga, bearded seal) only through sharing with coastal households. In a survey with household heads (couples when possible) respondents detailed for each key resource, by specific social relation, from whom and how much their household received, and what they contributed to others in trade or for shares by resource. Some relations are aggregated for the current analyses (i.e. purchasing and trading, and all non-food contributions and processing labor).

We used a name-generator question format for each species-relation question and input anonymous codes from an individual-based roster in which all individuals within the study communities were listed, including children. During extensive pre-testing of the survey instrument, respondents were able to recall previous interactions with other households in great detail, confirming resource-relation categories were culturally resonant. We asked respondents only about flows of food and supplies they received from other households, whaling crews and businesses in order to minimize potential cultural bias toward being generous. On average, interviews lasted 1 hour and 15 minutes, though many lasted more than 2 hours. Based on data-checking protocols, reported receiving under-estimated actual flows. In line with Alaska social science research protocols, respondents were compensated monetarily for their time and response rates were high across all communities (94% of households in Wainwright, 94% in Venetie and 82% in Kaktovik. Given these response rates, the complete dataset usually captured reciprocal flows for dyads (two connected nodes).

Detailing Types of Social Relations and Units of Flow. Pounds of flow:

- sharing;
- cooperative hunting share;
- shares for help (i.e. a household receives a share for contributing something to the hunting effort i.e. labor, equipment, groceries, gas, ammunition, cash, and so on);
- trading (including purchases);
- household share (only for beluga or bowhead whaling);
- whaling share (crew shares);
- feasts (Households receive small shares of a whale through whaling feasts: Nalukatuq and Captains Feasts).

Number of contributions:

- contributions (i.e. ammunition, gasoline, processing labor, equipment etc.); equipment exchange (including equipment lending and repairs);
- cash sharing

Weighting Edges. Relations are directed. All edges are weighted, however, food flows are weighted in pounds per year [within any layer], while non-food flows are weighted by the number of contribution types [within a layer]. For example, Household X gives Household Y 10 pounds of food as a share; and Household Y contributes to Household X a snow machine, a rifle and ammunition. In the first case, (HHX to HHY) = 10; while in the second case (HHY to HHX) = 3. To allow comparison and avoid biases towards food layers, we equate the average number of contributions (ammunition, gasoline, rifle, snow machine etc.) in a resource-social relation layer with the average pounds of the same resource given by households for a specific village.

For example, assume that a household X contributes to caribou hunt by giving household Y contributions: ammunition and a snow machine. Caribou (summing all layers in which caribou is the resource) provides an average outflow of 3500 Lbs of meat. Further, there are 10 outflow contribution edges in the Caribou layers (on average). The 2 contributions from HHX to HHY are treated as $= 3500/10 * 2$ or 700. We scale contribution layers to reduce biases when analyzing how different layers explain giving and receiving patterns.

Rank 4 Tensor. Generally speaking, a tensor is a multidimensional array with specific multilinear properties. For instance, a matrix is a rank-2 tensor, i.e. a 2-dimensional array where one needs to specify two indices (i and j) to uniquely identify an entry.

A matrix is the algebraic representation of a complex network, therefore it is natural to use higher order tensors (i.e., with rank larger than 2) to represent multilayer networks. To uniquely identify a link in a multilayer network, one needs to specify four different indices, two to identify involved nodes and two to identify the involved layers.

For example, the link between node i in layer α and node j in layer β is uniquely identify in a rank-4 tensor, whose components are indicated as $M_{j\beta}^{i\alpha}$. Please see the following figure for a visual representation of multiplex analysis

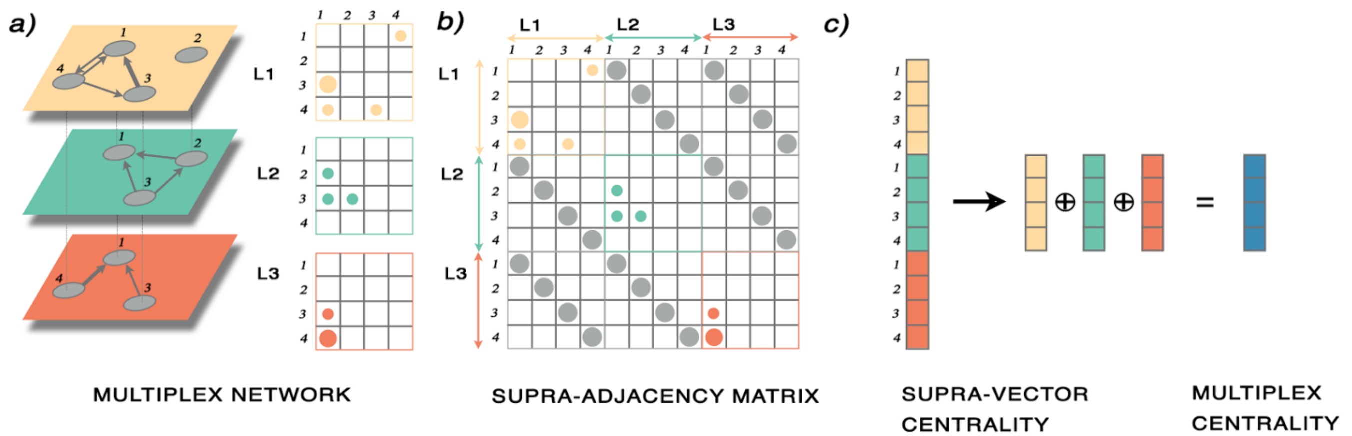

Fig. 2. Schematic illustration of multiplex representation and analysis. (A) Toy network with $N = 4$ nodes and $L = 3$ layers (note that not all nodes exist on all layers), building an interconnected multiplex system. Each layer is encoded by a different color. The network is directed (encoded by arrows) and weighted (encoded by edge thickness), as in our case study. The adjacency matrices, corresponding to each layer separately, are also shown. This system is mathematically represented by a rank-4 tensor $M^{i\alpha}_{j\beta}$, with $i, j = 1, 2, ..., N$ indicating nodes and $\alpha, \beta = 1, 2, ..., L$ indicating layers. (B) Matricization is applied to flatten the rank-4 tensor to a rank-2 supra-adjacency matrix, a matrix of matrices, without loss of topological information. In the supra-adjacency matrix, the adjacency matrices representing layers are placed as blocks on the main diagonal, whereas inter-layer connectivity is encoded into diagonal matrices that are placed on the off-diagonal blocks. (C) The multiplex centrality of each node in each layer is calculated from the supra-adjacency matrix, to obtain a supra-vector as a result (note that this is very different from calculating the centrality vector from each layer separately). The components of the supra-vector are $N \times L$: they are separated into L vectors and summed up entry-wise, as shown, to obtain the final multiplex centrality of each node. It is worth remarking that this approach takes into account, simultaneously, both intra- and inter-layer connectivity, and it has been shown to be a natural generalization of eigenvector centrality (as well as other centralities based on the calculation of the leading eigenvector of a certain matrix) to the multiplex domain (32, 35).

Clustering of Inter-Layer Correlation

Figures 3 through 14 show clustering of inter-layer correlation metrics by edge directionality. Figures 3- 4, 7- 8, and 11- 12 are giving and receiving figures repeated from the main text but enlarged to show layer details (labels) and cluster dendograms. Layer similarity is measured via Spearman correlation coefficient between weighted degree distributions. Euclidean distance and the complete linkage (maximum dissimilarity algorithm) are the basis for clustering of unique resources and social relations. Resources (species) and social relations are color-coded (y and top-x axes, respectively) to facilitate the visualization of correlations for which species and/or social relations are similar. Four images are presented for each community: giving, receiving, reciprocal and overall. While the giving and receiving visualizations highlight key unidirectional patterns across layers, the reciprocal correlogram illustrates layers through which exchanges occur (i.e. resources given in one layer are received in another).

Reciprocal patterns overall are less correlated in communities (see Figures 5, 9, and 13), so households that give a lot are not necessarily high receivers in that category or others. Reciprocity correlations are strongest in Kaktovik. The connection between households making contributions and receiving helper shares (of food) in return is clearest in this set of correlations (e.g. see BOW-HSH and BOW-CNT). Trading layers are often highly correlated, for example caribou in Wainwright and Kaktovik, and ducks in Venetie, illustrating that households active in trading for one species are active in other trading layers as well.

The overall figures emphasize that correlation patterns in Kaktovik are strongest across a broader set of network layers, followed by Venetie, then Wainwright (see Figures 6, 10, and 14. This points to a more distributed household participation pattern across layers in Wainwright. High inter-layer correlations can potentially indicate vulnerability to disturbances affecting individual households and is suggestive of areas of weakness related to species dependence patterns. The targeted and random node and layer removal taken here enables analysis of how observed structural characteristics may affect robustness of community networks to different potential disturbances.

Two-Part model and Shapely Value Decomposition. To assess contribution of specific resources and social relations to the variance of receiving and giving relations (i.e. receiving and giving patterns) we first fit a two-part model on in-flows (receiving) and out-flows (giving) separately, and then we decompose (via Shapley decomposition) the resulting R^2 by factors where each factor corresponds first to a specific resource, then to a specific social relation. We are not able to decompose R^2 taking all resource-relations into account at once because of severe multi-collinearity issues and computational limitations (indeed the number of calculations increases as N factorial where $N =$ number of factors).

A two-part model allows us to account for a high number of zeroes in the data, here corresponding to households that do not share or cooperate in a specific resource or social-relation. The two-part model fits first a binary choice model (i.e. logit or probit) for the probability of observing a value of the dependent variable > 0, then, conditional to the dependent variable being > 0, fits a specified regression model (standard OLS in our case) without making prior assumptions on the correlation between errors of the binary and continuous part of the model. Conceptually, zeros in the two-part model are for all intents and purposes true 0 (e.g. Heckman regression, on the other hand assumes that zeros are the result of censored data, rather than actual values).

Decomposition of the R^2 enables us to differentiate the between-species (between social-relations) differences in explaining the proportion of variance in receiving and giving patterns from the within-species (within-social relations). In our case, we denote R^2 as our aggregation measure denoting the relationship between N factors F_i with $i = 1, 2...n$ and receiving or giving patterns. Factors represent specific resources, specific social-relation or, in aggregate, all resources and all social-relations (see Table 2 in the main text). Let R^2 be the variance explained by all N factors, then the Shapley decomposition assesses the marginal effect of removing each factor in the $N!$ sequences possible, and averages such marginal contribution over all $N!$ sequences. In other words, the contribution of a single resource (species) or social relation, can be assessed as the difference between the overall R^2 of the underlying two-part model and the R^2 that is observed when that specific species or social-relation is removed from the overall set of factors determining the R^2. If there are N resources (or social relations) there are $N!$ combinations without a specific resource (or social-relation), meaning that the difference in R^2 due to a single species (social relation) needs to be averaged over the $N!$ permutation of the model without that specific resource (or social-relation).

Table 2. Lables - Symbols Dictionary: Correspondence between Figure Labels and Actual Resource-Relation layers

	Name	Symbol
Resources	Arctic Char	CHR
	Beluga	BLG
	Berries	BRR
	Bowhead Whale	BOW
	Caribou	CBU
	Dall Sheep	SHP
	Duck	DUC
	Geese	GES
	Grayling	GRY
	Moose	MOO
	Salmon	SAL
	Seal	SEA
	Smelt	SMT
Social Relations	Contribution	CNT
	Cooperative Hunting	COP
	Cash Sharing	CSH-SHR
	Equipment Exchange	EQP-EXC
	Feasts	FST
	Helper Shares	HSH
	Household Share	HHS
	Sharing	SHR
	Trade	TRD
	Whaling Shares	WSH

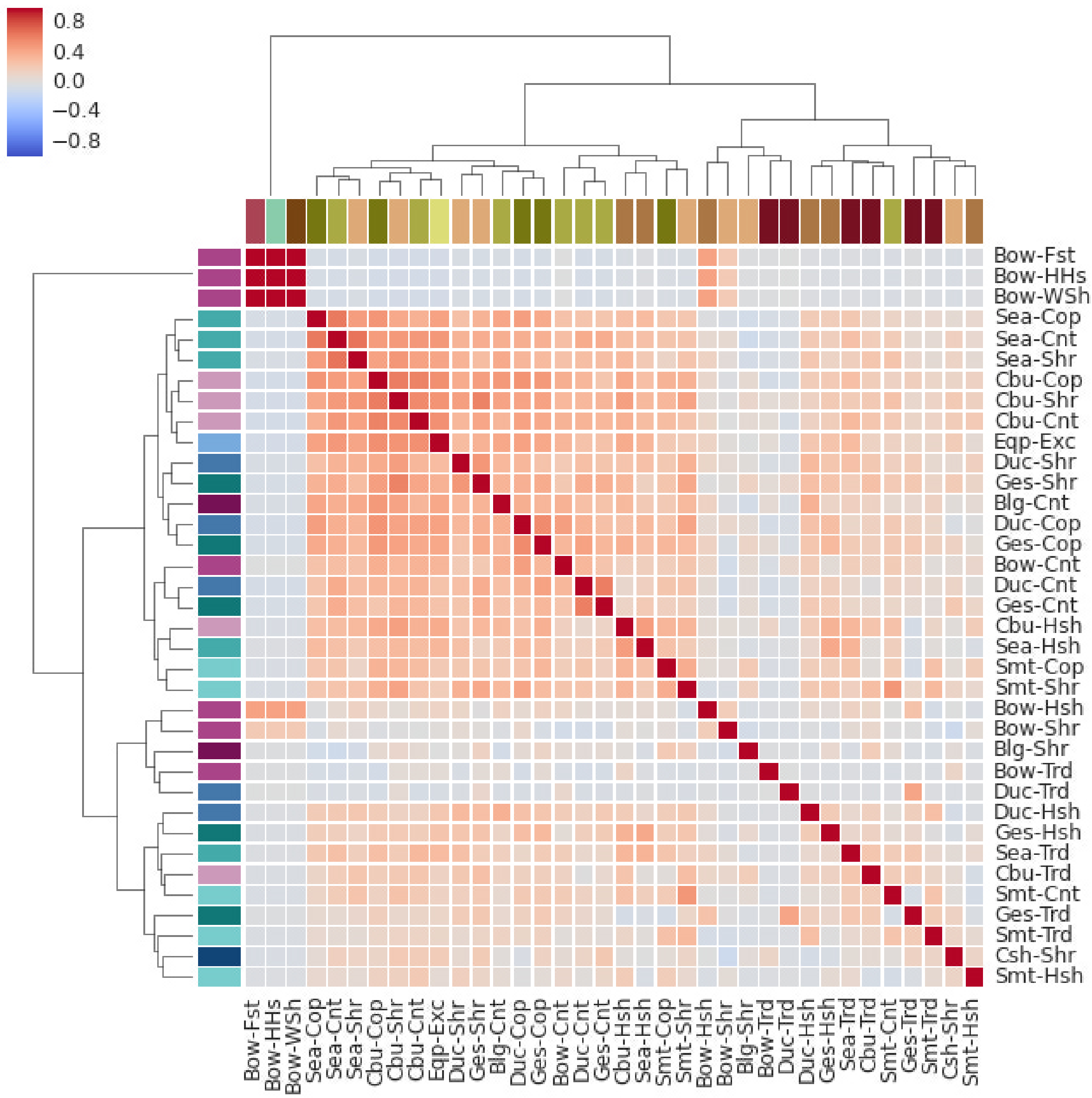

Fig. 3. Layer Similarity: clustered inter-layer correlation for Wainwright based on GIVING (out-weighted degree). Higher correlation indicates that it is more likely that the household that gives most in one layer also gives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

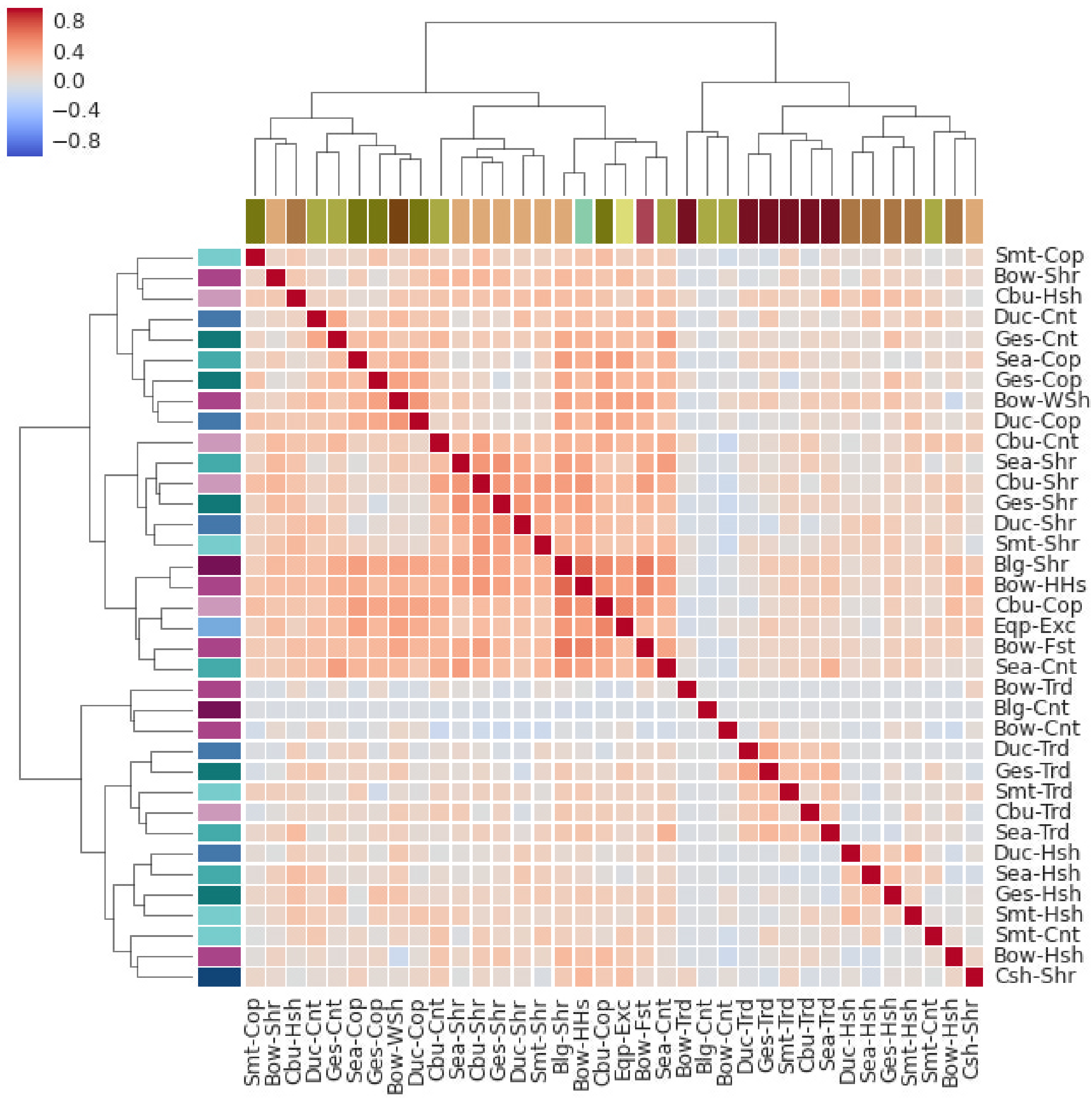

Fig. 4. Layer Similarity: clustered inter-layer correlation for Wainwright based on RECEIVING (in-weighted degree). Higher correlation indicates that it is more likely that the household that receives most in one layer also receives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

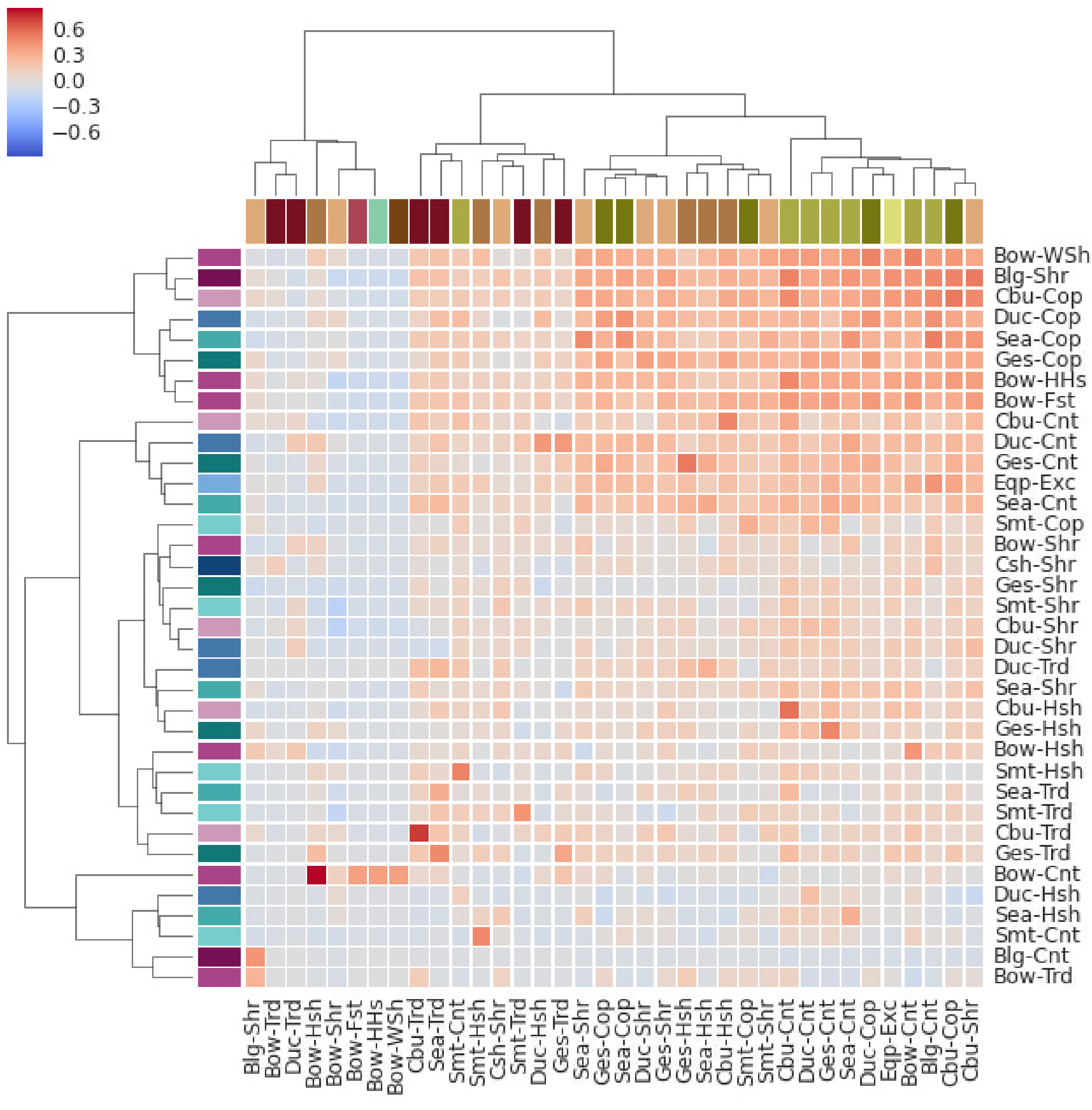

Fig. 5. Layer Similarity: clustered inter-layer correlation for Wainwright based on RECIPROCAL (in-out-weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one layer also gives/receive most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

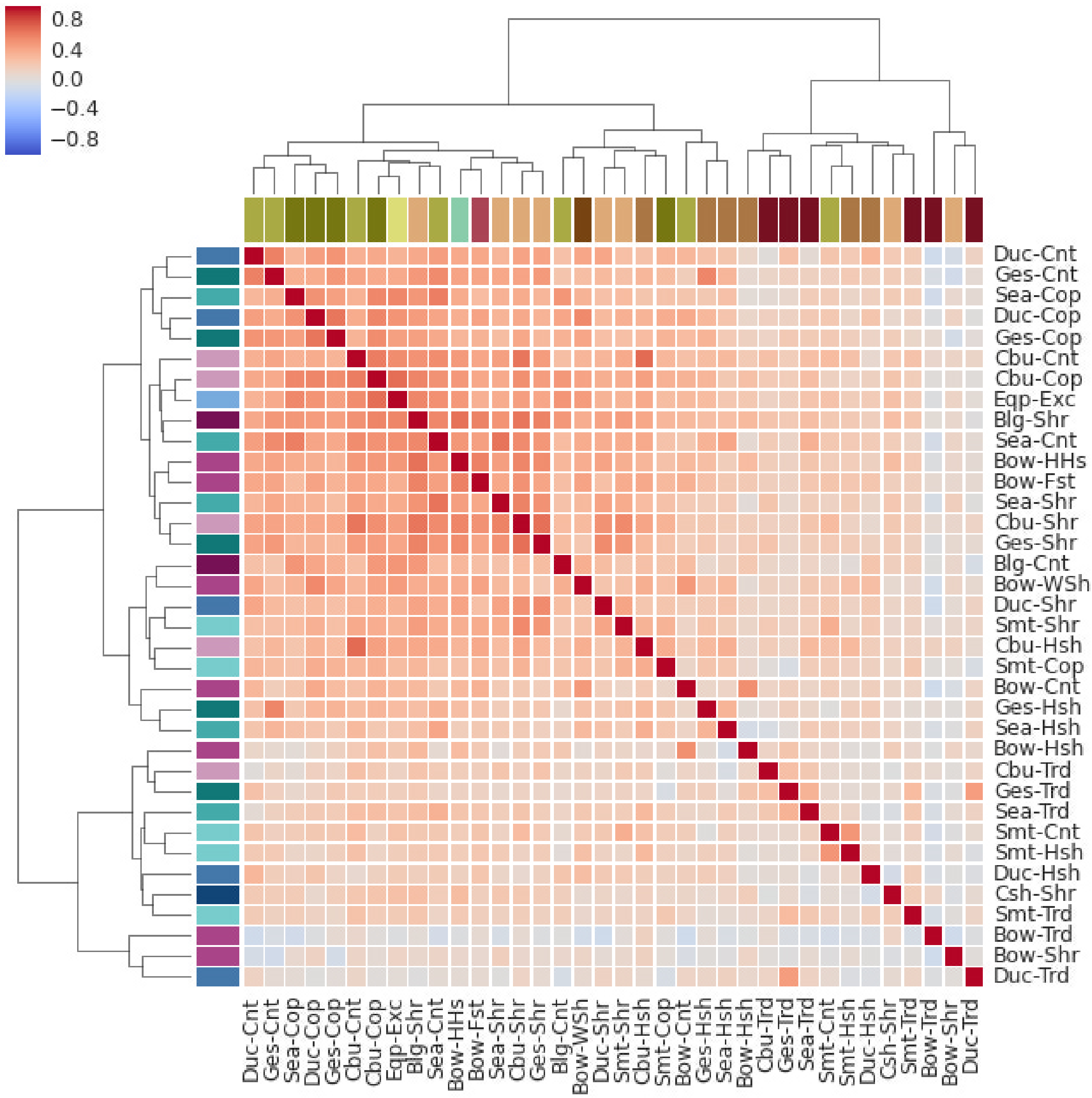

Fig. 6. Layer Similarity: clustered inter-layer correlation for Wainwright OVERALL (total weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one network also receives/give most in another network. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

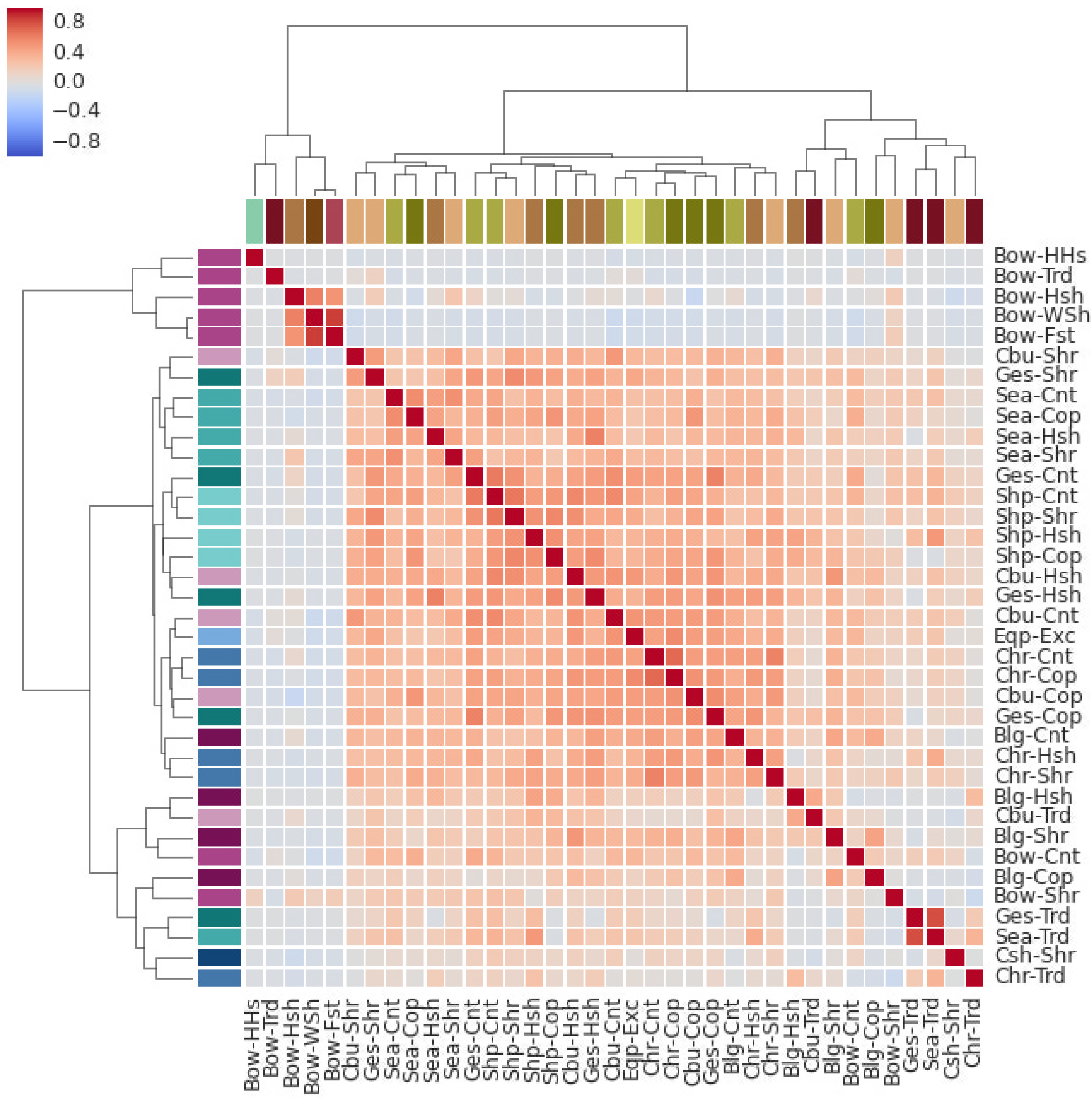

Fig. 7. Layer Similarity: clustered inter-layer correlation for Kaktovik based on GIVING (out-weighted degree). Higher correlation indicates that it is more likely that the household that gives most in one layer also gives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

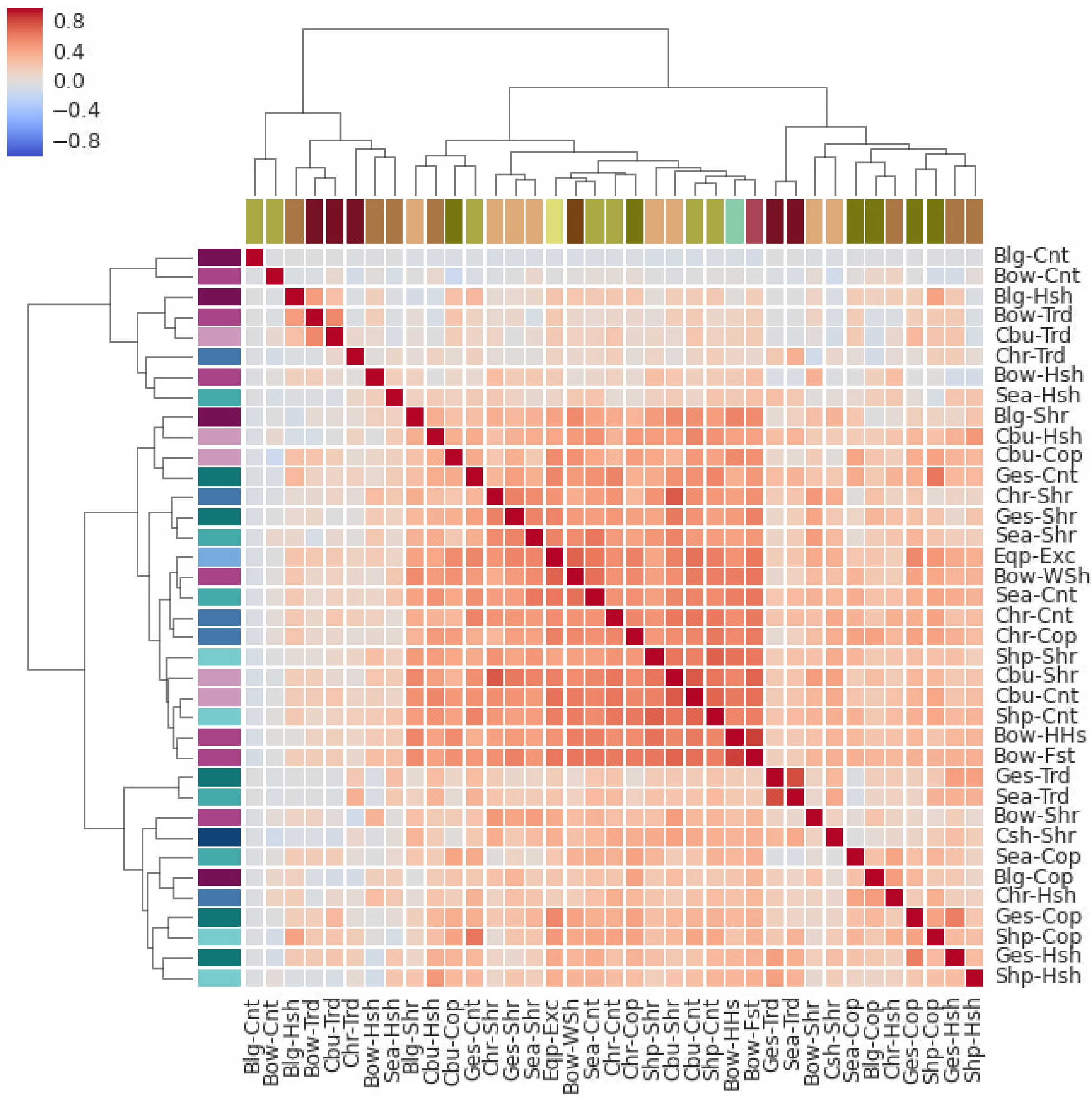

Fig. 8. Layer Similarity: clustered inter-layer correlation for Kaktovik based on RECEIVING (in-weighted degree). Higher correlation indicates that it is more likely that the household that receives most in one layer also receives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

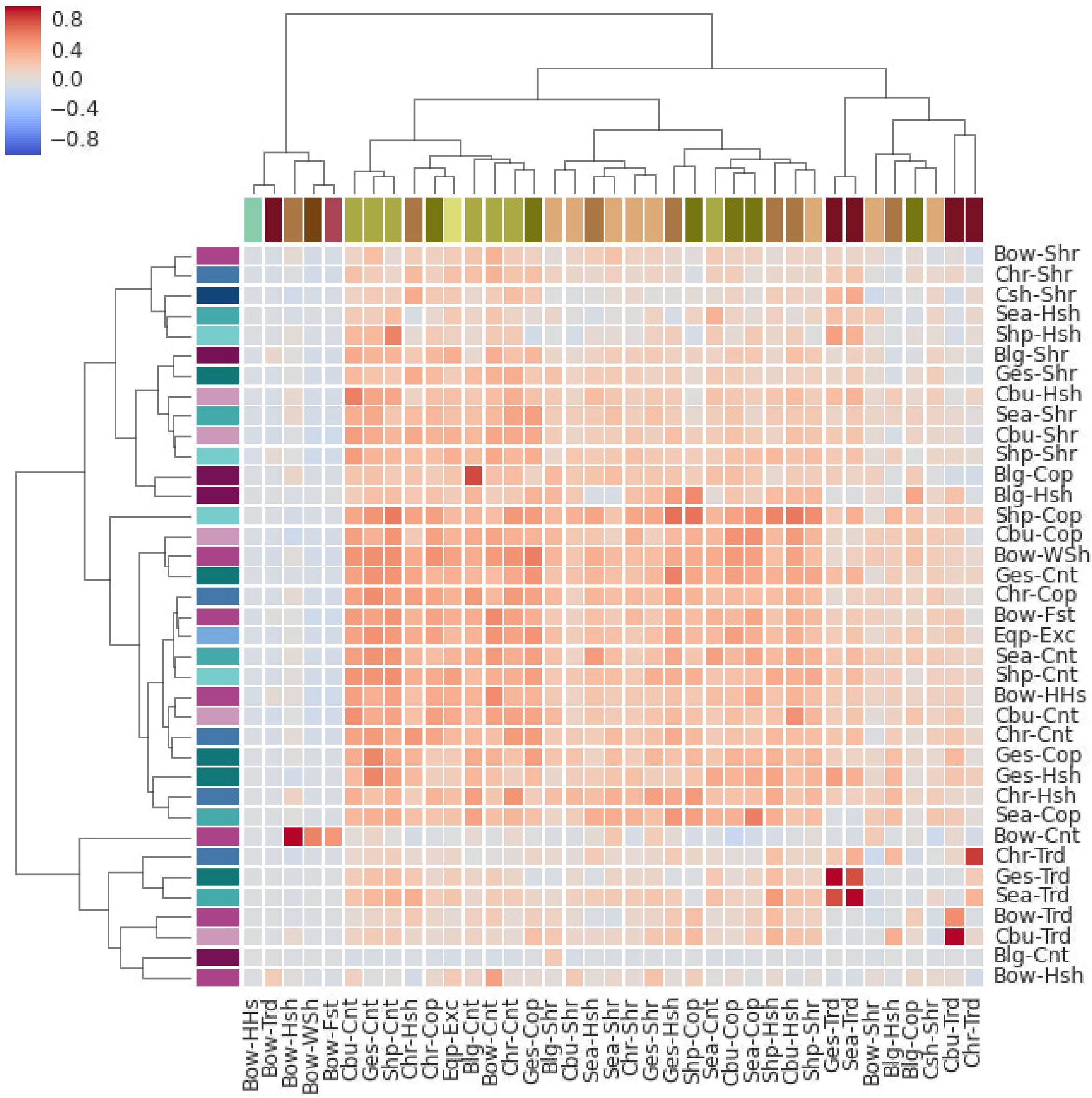

Fig. 9. Layer Similarity: clustered inter-layer correlation for Kaktovik based on RECIPROCAL (in-out-weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one layer also gives/receive most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

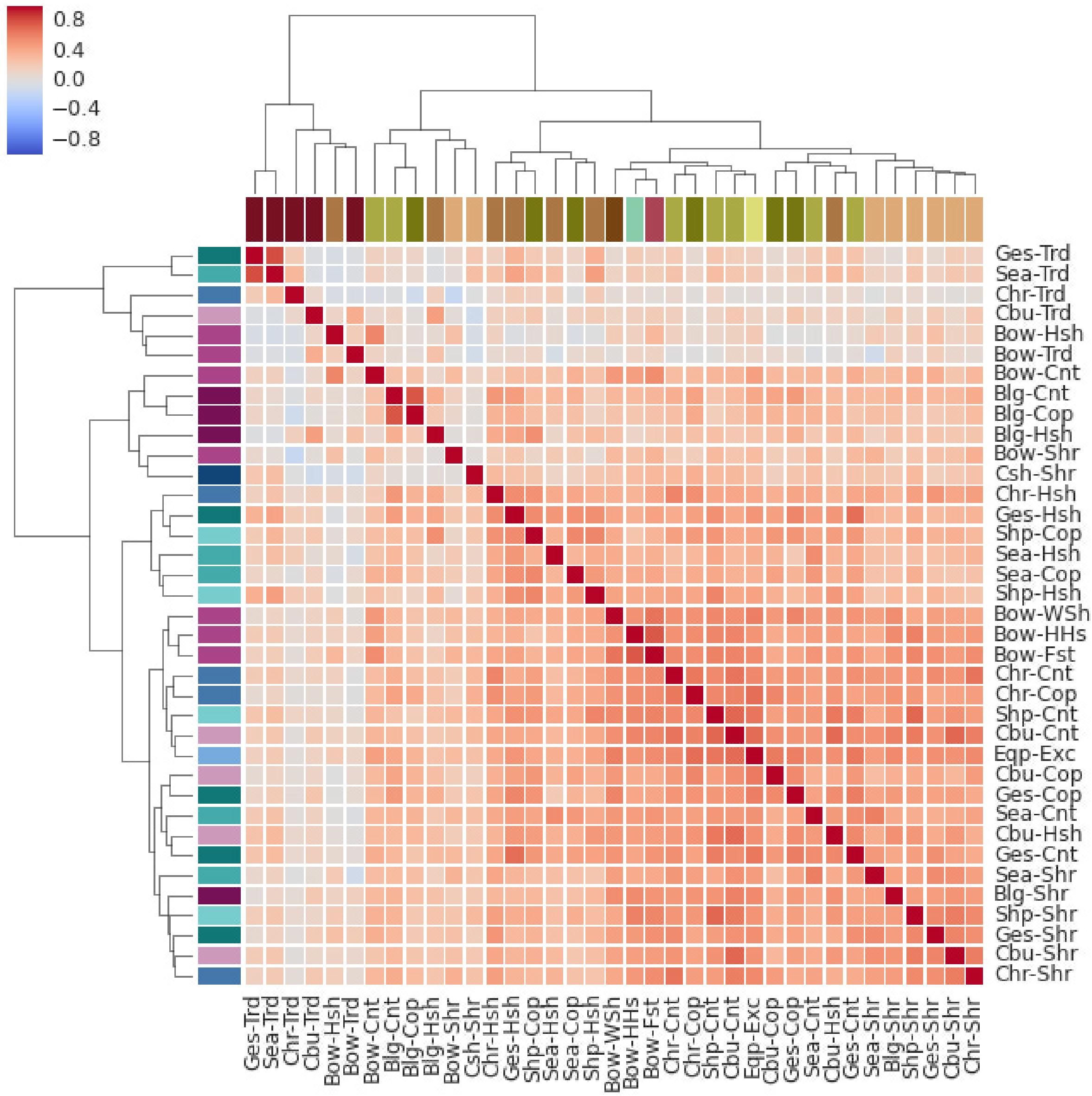

Fig. 10. Layer Similarity: clustered inter-layer correlation for Kaktovik OVERALL (total weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one network also receives/give most in another network. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

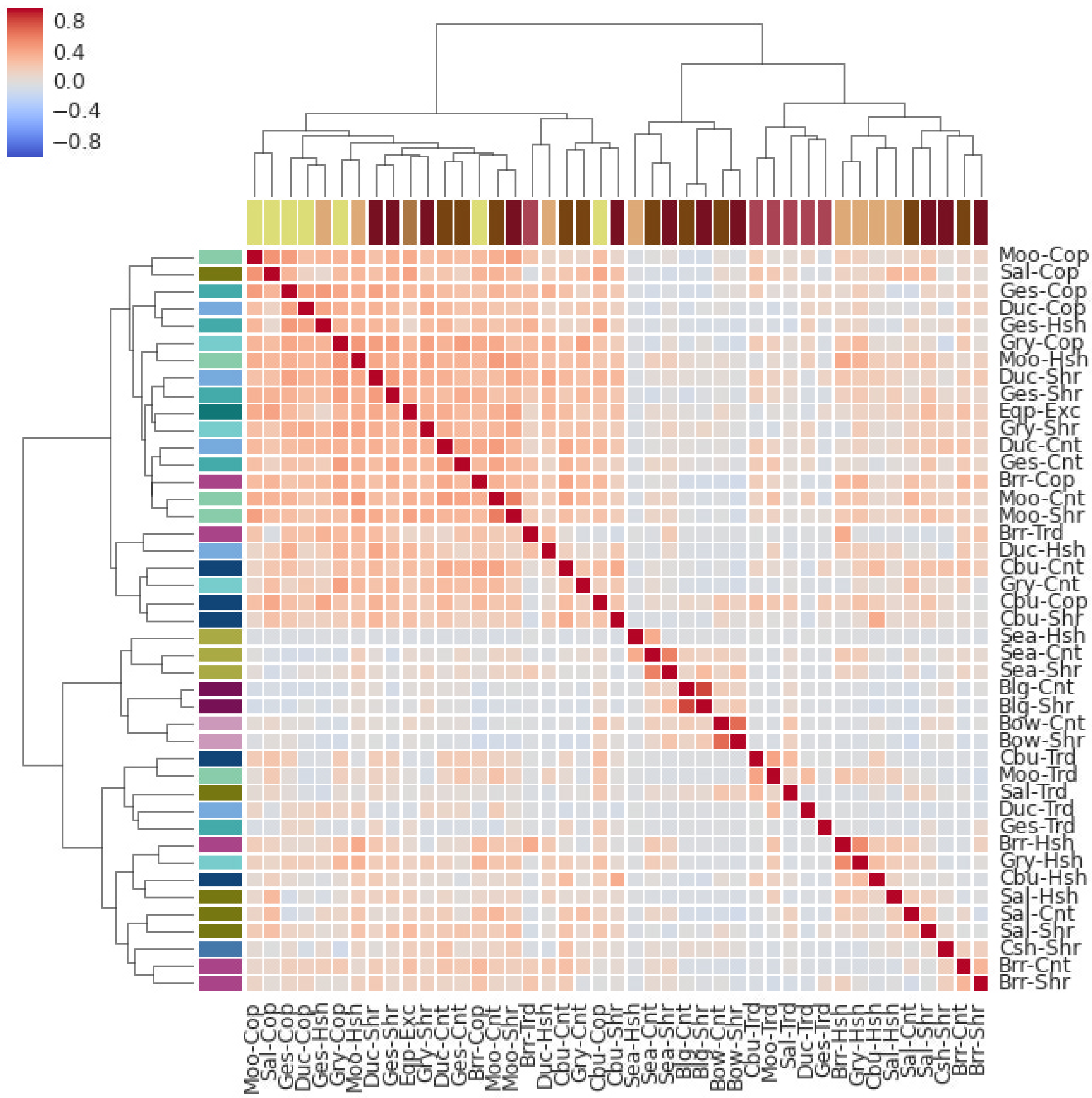

Fig. 11. Layer Similarity: clustered inter-layer correlation for Venetie based on GIVING (out-weighted degree)). Higher correlation indicates that it is more likely that the household that gives most in one layer also gives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

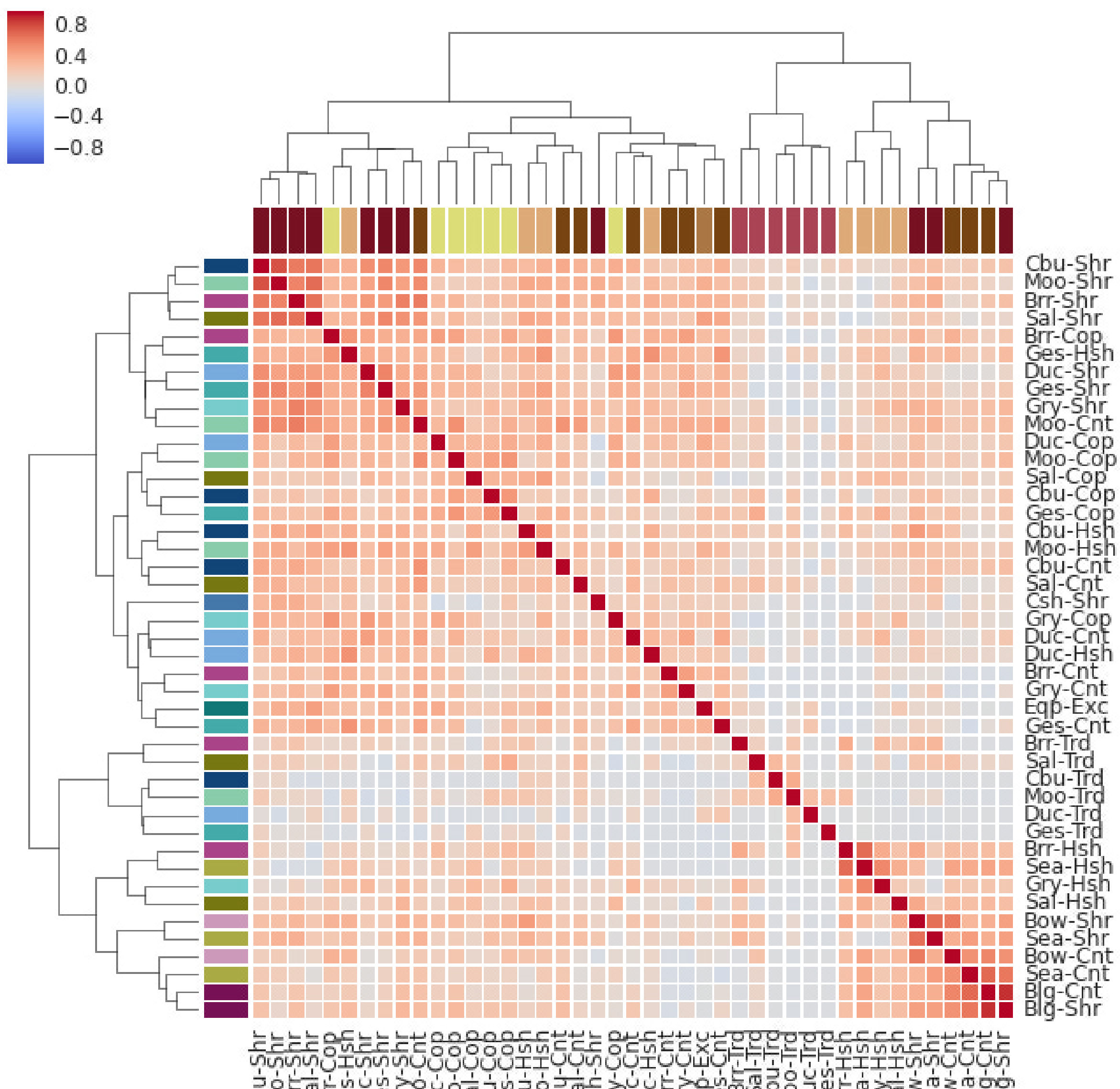

Fig. 12. Layer Similarity: clustered inter-layer correlation for Venetie based on RECEIVING (in-weighted degree). Higher correlation indicates that it is more likely that the household that receives most in one layer also receives most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

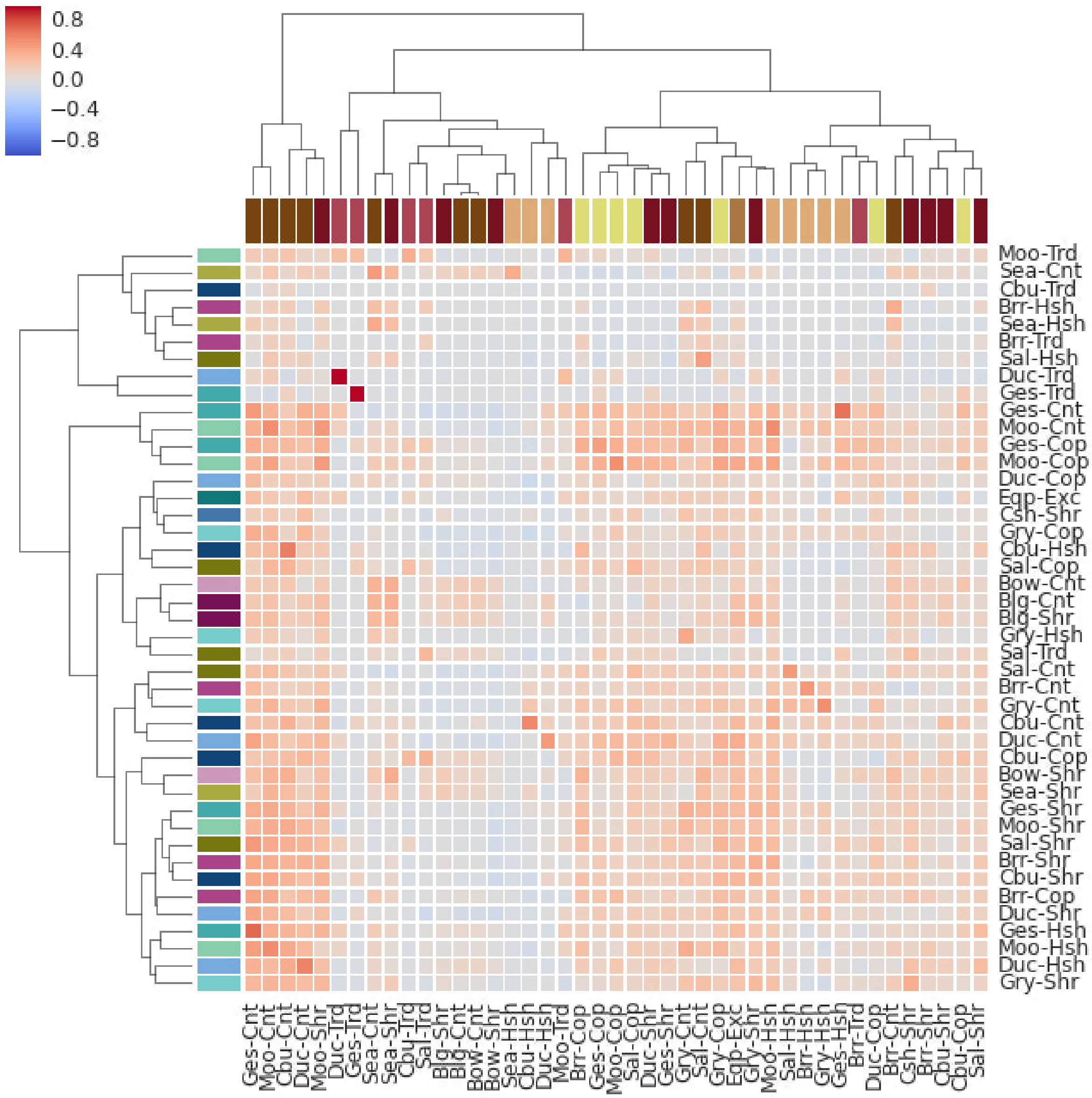

Fig. 13. Layer Similarity: clustered inter-layer correlation for Venetie based on RECIPROCAL (in-out-weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one layer also gives/receive most in another layer. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

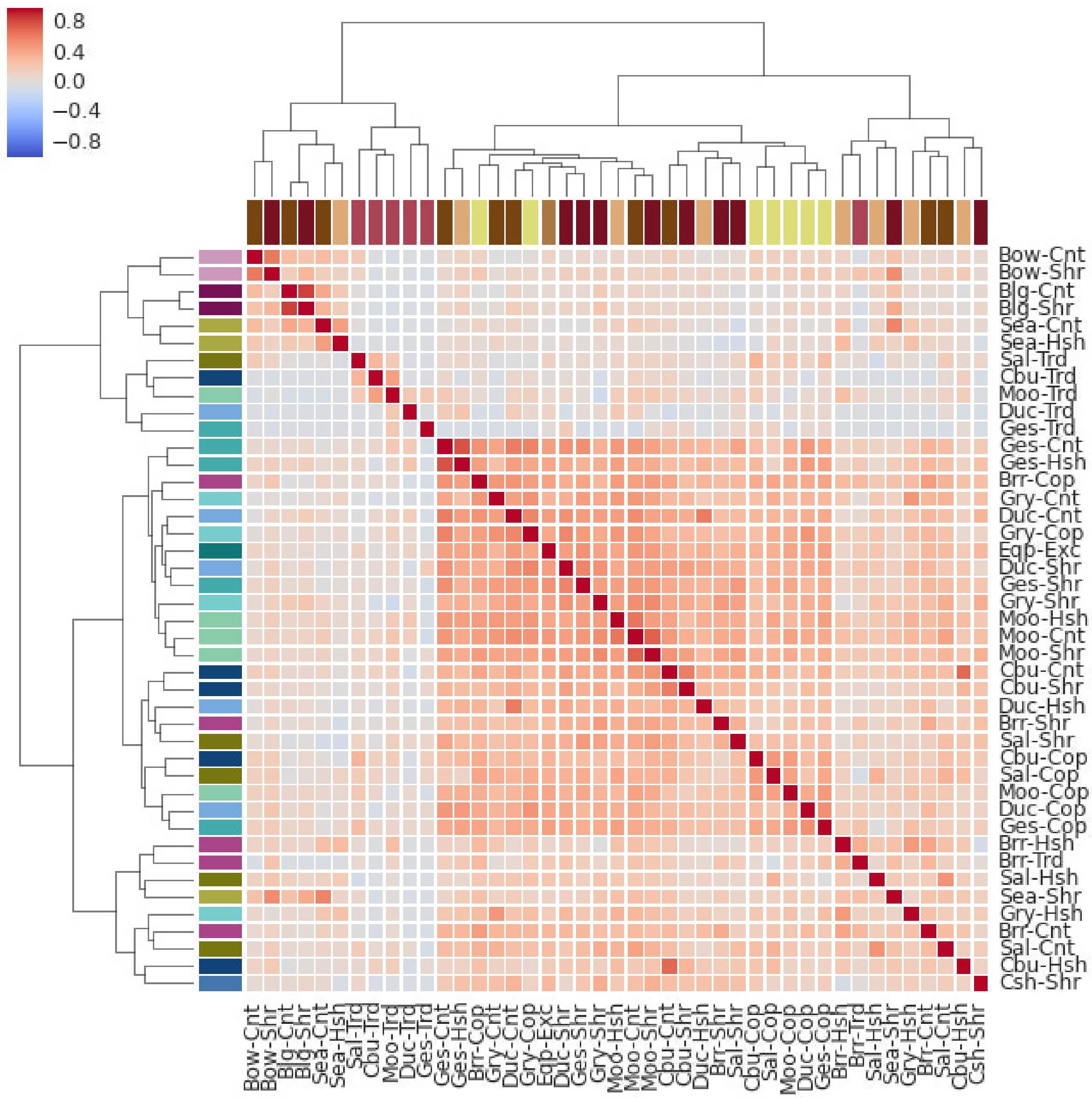

Fig. 14. Layer Similarity: clustered inter-layer correlation for Venetie OVERALL (total weighted degree). Higher correlation indicates that it is more likely that the household that receives/give most in one network also receives/give most in another network. Axis color codes indicate species and social relations. Color code on the left of each graph indicates whether layers relating to the same species (i.e. beluga-sharing, beluga-helper shares etc.) are clustered together, while the color code on the top of each graph indicates whether layers relating to the same social relation are clustered together (i.e. caribou-contributions, moose-contributions etc.).

Importance of layers

Table 3 depicts calculated layer importance. Importance of a resource-relation layer is calculated based on the sum of the nodes (thus households, crews, etc.) participating in specific resource-relation layers. We use the sum of nodes criteria as it best exemplifies the importance of a specific layer within a community. Averaging over the number of layers would benefit sparse and not very connected layers such as trading, or species that belong to fewer layers, such as beluga and bowhead in Venetie. Calculated layer importance varies by social relation, species or species complexes (terrestrial, riverine, marine).

Table 3. Ranking Layers depending on household's participation

Village	Social Relation	Resource Species	Resource Type	Rank
Wainwright	Sharing	Bowhead	Marine	1
	Contribution	Caribou	Terrestrial	2
	Cooperative	Seal	Marine	3
	Helper	Geese	Terrestrial	4
	HHShare	Duck	Terrestrial	5
	SmallFeast	Smelt	River	6
	Equipment Lending	Beluga	Marine	7
	Trading			8
	Cash Sharing			9
Kaktovik	Sharing	Bowhead	Marine	1
	Contribution	Caribou	Terrestrial	2
	Cooperative	Char	River	3
	Helper	Seal	Marine	4
	Equipment Lending	Sheep	Terrestrial	5
	SmallFeast	Geese	Terrestrial	6
	HHShare	Beluga		7
	Cash Sharing			8
	Trading			9
Venetie	Sharing	Caribou	Terrestrial	1
	Contribution	Moose	Terrestrial	2
	Cooperative	Salmon	River	3
	Helper	Geese	Terrestrial	4
	Equipment Lending	Duck	Terrestrial	5
	Trading	Berries	Terrestrial	6
	Cash Sharing	Grayling	River	7
		Bowhead	Marine	8
		Seal	Marine	9
		Beluga	Marine	10

Relationship between Loss of Productive Households and Layers. Figures 14, 15 and 16 unpack the mechanism by which removing individual nodes reduces connectedness of the multiplex. With 20% of households removed, we calculated the sum of weights represented by their removal from individual layers, and then represented the percentage decline of weighted flows by layer. Figures compare these declines by community. On average, 63%, 73% and 66% of weighted flows of food and resources are lost when 20% of households drop out of community networks. Specific resources and all associated social relations - are affected, particularly Dall sheep and seal in Kaktovik, and marine species in Venetie. Bowhead in contrast declines less, given its community-wide distribution patterns contingent on crew structures.

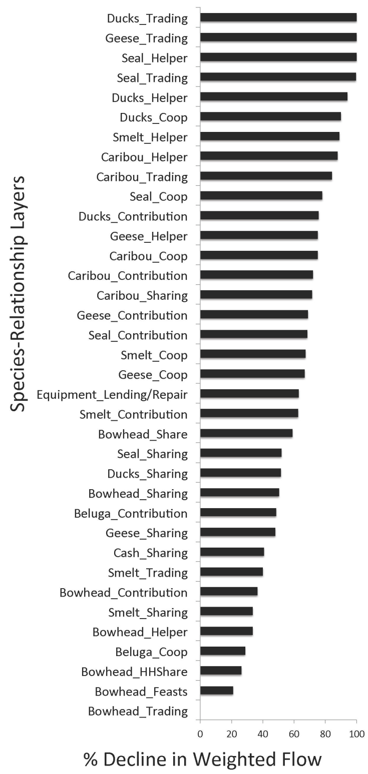

Fig. 15. Effect of removal of the most productive 20% of Households on flows within layers in Wainwright. The average decrease of flow per layer is 63%.

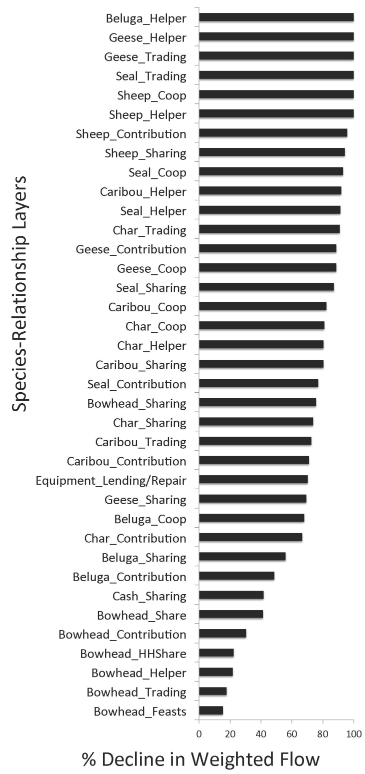

Fig. 16. Effect of removal of the most productive 20% of Households on flows within layers in Kaktovik. The average decrease of flow per layer is 73%.

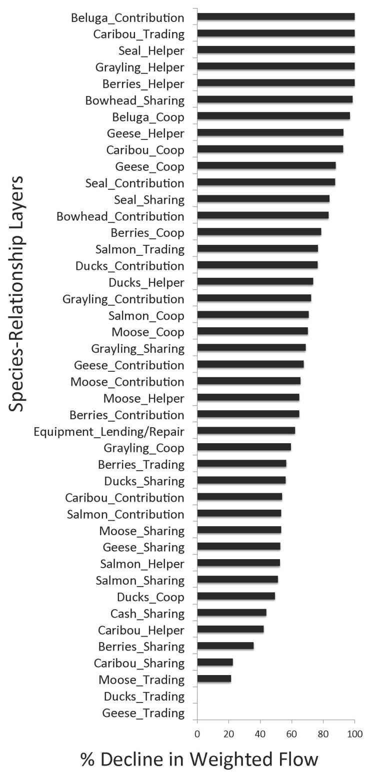

Fig. 17. Effect of removal of the most productive 20% of Households on flows within layers in Venetie. The average decrease of flow per layer is 66%.