Web Appendix

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HPV interaction model description

Model description

We built a deterministic compartmental dynamic model programmed in R using the deSolve package (1).

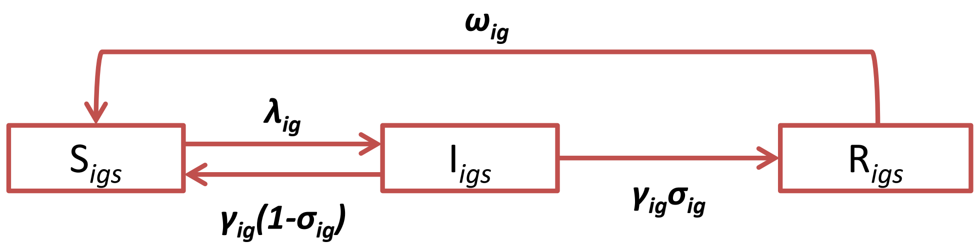
Demography

The model represents a sexually active population of heterosexual individuals. Individuals are stratified according to sex g {W = women, M = men} and sexual activity level s {L = low, H = high}. Age is not modeled. A proportion ϑ_{gs} of sex g enters the population with sexual activity level s. The rate of entry equals the rate of exit from the sexually active population (μ). The exit rate is assumed not to depend on an individual's health state, sex, or sexual activity. Individuals keep the same sexual activity level throughout their lives.

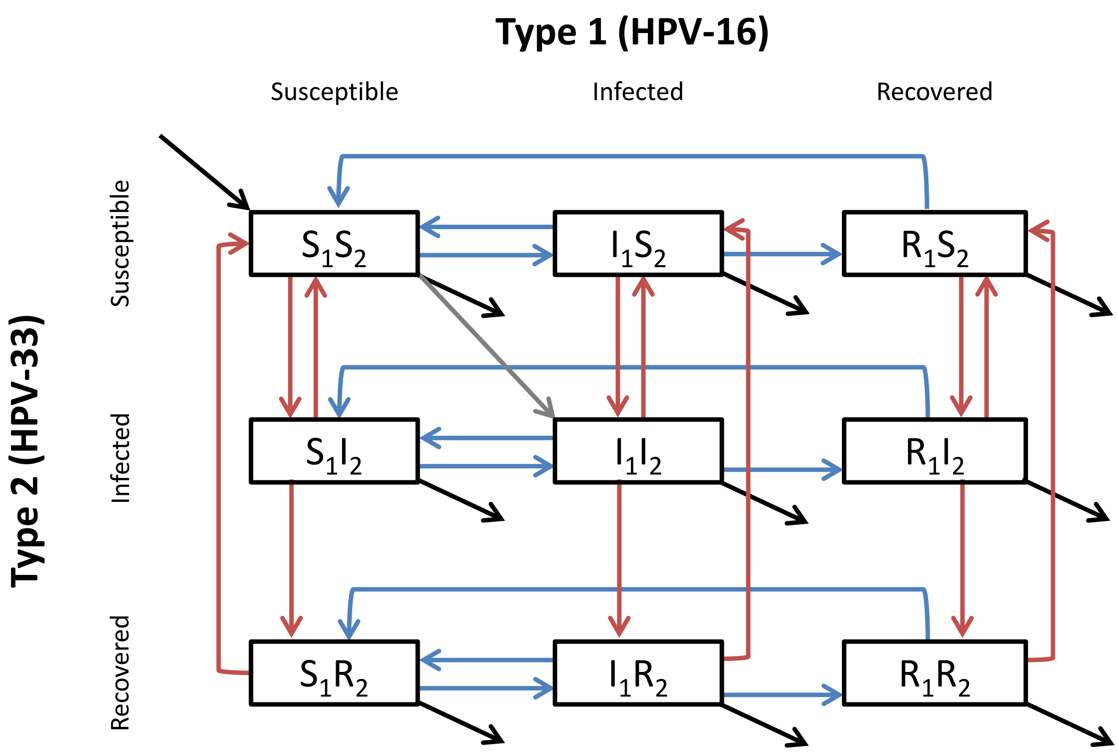
Natural history of HPV infection

We model the natural history of 2 types of human papillomavirus (HPV) using a *SIRS* natural history structure for both. The HPV type is designated with the subscript i {1 = type 1, 2 = type 2, 12 = types 1 and 2}. Individuals can be susceptible (S_i), infected (I_i), or recovered/immune (R_i) to infection with either type. Individuals may be in 9 mutually exclusive health state compartments ([S_1S_2], [S_1I_2], [S_1R_2], [I_1S_2], [I_1I_2], [I_1R_2], [R_1S_2], [R_1I_2], [R_1R_2]) representing the possible combinations of S_i, I_i, R_i states for the two HPV types.

Natural history transitions for the health states with a particular HPV type i are presented in Web Figure 1, while all possible transitions between the 9 potential health states with both HPV types are presented in Web Figure 2. Individuals enter the sexually active population susceptible to HPV types 1 and 2 (health state [S_1S_2]). The rate of infection of susceptible individuals λ_{ig} is detailed in the section Infection transmission on page 5. Infected health states are [I_1S_2], [I_1I_2], [I_1R_2] for type 1 and [S_1I_2], [I_1I_2], [R_1I_2] for type 2. Infected individuals clear infections at a type- and sex-specific rate γ_{ig}. Following infection clearance, each sex has a probability σ_{ig} of developing natural type-specific immunity while the rest (1-σ_{ig}) return to the susceptible health states. Individuals in immune health states have complete type-specific natural immunity. Natural immunity wanes at a type- and sex-specific rate ω_{ig}. Individuals whose type-specific immunity has waned become susceptible to infection again. Individuals who become susceptible again after clearing infection or losing their natural immunity have the same infection rate as individuals who have never been infected. The model can thus easily represent different natural histories such as SIS (σ_{ig} = 0), SIR (ω_{ig} = 0), or SI (γ_{ig} = 0).



Web Figure 1. Transition rates between health states for a single HPV type *i*. Individuals can be susceptible (S_i), infected (I_i), or recovered/immune (R_i) to infection. λ_{ig} = rate of infection. γ_{ig} = rate of infection clearance. σ_{ig} = probability of developing type-specific natural immunity. ω_{ig} = rate of waning of natural immunity.



Web Figure 2. Flow diagram of health states. Boxes represent mutually exclusive health states. Arrows represent possible transitions between health states, with black arrows representing population entries and exits, blue arrows representing changes in health status with HPV type 1, red arrows representing changes in health status with HPV type 2, and the grey arrows representing change in the health status of both HPV types 1 and 2. Individuals can be susceptible (S_i), infected (I_i), or recovered/immune (R_i) to infection with HPV type *i*.

Sexual partnership formation

Sexual partnership duration and dissolution are explicitly modeled. Individuals are stratified by whether they are single or in partnership, and by the sexual activity level and health state of their partner. This allows the model to keep track of the health state of both individuals in the partnership. In equations, the partners' characteristics are indicated with an apostrophe ('). Individuals enter the population as singles. Individuals will spend an average proportion of their sexually active lives in sexual partnerships (ζ) and the rest as singles (1-ζ).

Individuals have a sex- and sexual activity level-specific rate of new partner acquisition c_{gs}. The sexual partnership formation rate of singles (ν_{gs}) and the sexual partnership dissolution rates (τ_{gs}) are a function of the overall desired new partner acquisition rate (c_{gs}) and the proportion of time spent in sexual partnerships (ζ). In other words, the more time individuals spend in sexual partnerships, the higher the partnership formation rate must be once they are single to acquire the same number of new partners over time. Individuals in sexual partnerships have sex acts at a rate π. The two partners' dissolution rates (τ_{gs} and $\tau_{g's'}$) are averaged to determine the dissolution rate of their partnership. Sexual partnerships also end if one of the partners exits the population. Individuals from dissolved sexual partnerships return to the single strata.

The probability that an individual of sex *g* and sexual activity level *s* forms a sexual partnership with a member of the opposite sex *g'* of sexual activity level *s'* is $\rho_{gsg's'}$. This probability depends on the number of sexual partnerships available from singles in each sexual activity level and the assortativity parameter ε. The assortativity parameter ε is the proportion of sexual partnerships that are made exclusively with individuals of the same sexual activity level. The remaining sexual partnerships (1-ε) are made proportionately to the number of sexual partnerships available from each sexual activity level. Mixing does not depend on individuals' health states. We balanced the number of sexual partnerships between sexes and sexual activity levels using the method described by Garnett and Anderson (2). Men and women adjust their partner acquisition rates using a compromise parameter ξ, which determines whether women's (ξ = 1) or men's (ξ = 0) desires determine sexual partnership formation.

Infection transmission

Individuals susceptible to type *i* can only be infected when they are in a sexual partnership with a partner infected with type *i*. The rate of infection with type *i* during a partnership with an infected partner (λ_{ig}) depends on the rate of sexual acts in partnerships (π) and the type- and sex-specific probability of infection per-sex-act with an infected partner (δ_{ig}). Transmissions occur during sex acts within a partnership.

The model takes into account the probability of acquiring more than one HPV type from the same partner and from the same sex act if the individual is susceptible to both types [S_1S_2] and partnered with a coinfected partner [I_1I_2]. The overall rate of infection λ_{ig} in these individuals represents the sum of all type *i* transmissions, whether these occur as single-type transmission and or are cotransmitted simultaneously in the same sex act with the other HPV type. This overall rate of infection λ_{ig} is thus the sum of the rate of simultaneous cotransmissions with both HPVs (λ_{12g}) and the rate of single-type transmissions with *i* (λ_{ig}-λ_{12g}). The rate of cotransmission λ_{12g} depends on the rate of sexual acts in partnerships (π) and the joint probability of acquiring both infections during a sex act ($\delta_{1g}*\delta_{2g}$). The rate of single-type transmission (λ_{ig}-λ_{12g}) depends on the rate of sexual acts in partnerships (π) and the probability of acquiring only type *i* from a sex act with a coinfected partner ($\delta_{1g}(1-\delta_{2g})$ for type 1 and $(1-\delta_{1g})\delta_{2g}$ for type 2).

Individuals infected with different types in a sexual partnership ([S_1I_2] partnered with [I_1S_2]) may also simultaneously transmit both respective infections to each other upon a same sex act. This rate ($\lambda_{igi'g'}$) depends on the joint probability of each discordant partner acquiring the other's HPV type in a sex act ($\delta_{ig}*\delta_{i'g'}$).

Interactions between HPV types

The HPV types may interact by modifying the probability of transmission of the other HPV type from an infected to a susceptible partner. Individuals infected with a first HPV but susceptible to the other HPV type *i* ([S_1I_2] and [I_1S_2] states) have their rate of infection with HPV *i* multiplied by the type-specific factor χ_{ig}. This parameter represents any biological effect assumed to be caused by a current infection with a first HPV type which increases or reduces the probability a new HPV type can establish an infection after sexual contact with an infected partner (e.g. competition for host resources, creation of lesions vulnerable to other infections). Individuals immune to a first HPV but susceptible to the other HPV type *i* ([S_1R_2] and [R_1S_2] states) have their rate of infection with HPV *i* multiplied by a type-specific factor φ_{ig}. This parameter represents any biological effect assumed to be caused by the long-term immune response developed against one HPV type which increases or reduces the probability a new HPV type can establish an infection after sexual contact with an infected partner (e.g. cross-immunity).

Biological interactions are assumed to affect the probability of infection per-sex-act with an infected partner (δ_{ig}), the biological component of the infection rate. Because this probability must be between 0-100%, the interaction factors χ_{ig} and φ_{ig} have a lower bound of 0 and an upper bound of the inverse of the infection probability (δ_{ig}^{-1}). Values beyond this range do not make biological sense. If χ_{ig} and $\varphi_{ig} = 1$, then the health state with one HPV type does not affect the rate of infection with the other HPV type and there are no interactions. If χ_{ig} or $\varphi_{ig} <1$, infection or immunity with a first HPV type reduces the probability of infection with the other HPV type which corresponds to competitive interactions. If χ_{ig} or $\varphi_{ig} >1$, infection or immunity with a first HPV type increases the probability of infection with the other HPV type, which corresponds to facilitative interactions.

Note that our model assumes biological interactions operate on a multiplicative scale, though the epidemiologic literature generally considers biological interactions to operate on an additive scale (3). We chose to model interactions this way because we are modeling an ecological definition of interactions (which are beneficial and detrimental causal effects of organisms on each other's natural history) rather than an epidemiologic definition of interactions (exposures which participate in the same causal mechanism under a sufficient-cause model(3)). Epidemiological studies generally use relative ratios to measure associations between HPV types. This suggests that epidemiologists conceive infections with previous HPV types as potentially having a relative causal effect upon the probability of acquiring new HPV types.

Model variables and parameters

Web Table 1. Description of subscripts, variables, and parameters

Symbol	Description	Stratifications	Value(s) in Base Case Scenario	Reference
Subscripts				
g, g'	Gender, partner's sex	M=men, W=women	-	-
s, s'	Sexual activity level, partner's sexual activity level	L=low, H=high	-	-
i	HPV type	1=type 1, 2=type 2, 12=types1and2	-	-
Health and sexual partnership states				
$[S_1S_2]_{gs}^{0}$	Susceptible to both types, single	Gender, sexual activity level	Variable	-
$[S_1I_2]_{gs}^{0}$	Susceptible to type 1, infected with type 2, single	Gender, sexual activity level	Variable	-
$[S_1R_2]_{gs}^{0}$	Susceptible to type 1, immune to type 2, single	Gender, sexual activity level	Variable	-
$[I_1S_2]_{gs}^{0}$	Infected with type 1, susceptible to type 2, single	Gender, sexual activity level	Variable	-
$[I_1I_2]_{gs}^{0}$	Infected with both types, single	Gender, sexual activity level	Variable	-
$[I_1R_2]_{gs}^{0}$	Infected with type 1, immune to type 2, single	Gender, sexual activity level	Variable	-
$[R_1S_2]_{gs}^{0}$	Immune to type 1, susceptible to type 2, single	Gender, sexual activity level	Variable	-
$[R_1I_2]_{gs}^{0}$	Immune to type 1, infected with type 2, single	Gender, sexual activity level	Variable	-
$[R_1R_2]_{gs}^{0}$	Immune to both types, single	Gender, sexual activity level	Variable	-
$[S_1S_2]_{gs}^{[Y_1Y_2]_{g's'}}$	Susceptible to both types, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[S_1I_2]_{gs}^{[Y_1Y_2]_{g's'}}$	Susceptible to type 1, infected with type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[S_1R_2]_{gs}^{[Y_1Y_2]_{g's'}}$	Susceptible to type 1, immune to type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health	Variable	-

		state		
$[I_1S_2]_{gs}^{[Y_1Y_2]g's'}$	Infected with type 1, susceptible to type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[I_1I_2]_{gs}^{[Y_1Y_2]g's'}$	Infected with both types, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[I_1R_2]_{gs}^{[Y_1Y_2]g's'}$	Infected with type 1, immune to type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[R_1S_2]_{gs}^{[Y_1Y_2]g's'}$	Immune to type 1, susceptible to type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[R_1I_2]_{gs}^{[Y_1Y_2]g's'}$	Immune to type 1, infected with type 2, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
$[R_1R_2]_{gs}^{[Y_1Y_2]g's'}$	Immune to both types, in partnership with individual in health state $[Y_1Y_2]$	Gender, sexual activity level, partner's sexual activity level, partner's health state	Variable	-
Demography				
N_g	Population size of sex *g*	Gender	10 000	-
μ	Exit/entry rate	None	1/40.00 years	Assumed
ϑ_{gs}	Proportion of individuals entering the population in sexual activity level *s*	Gender, sexual activity level	*s*=L: 0.93 *s*=H: 0.07	(4)[a]
Natural history of infection				
γ_{ig}	Infection clearance rate	Type, sex	1/1.40 years	(5)[b]
σ_{ig}	Proportion of infections conferring natural immunity	Type, sex	0.60	(6)
ω_{ig}	Waning rate of natural immunity	Type, sex	1/10.00 years	(7)[c]
χ_{ig}	Relative probability of infection	Type, sex	1.00	Assumed

	with type *i* of individuals infected with other HPV type (interaction effect)			
φ_{ig}	Relative probability of infection with type *i* of individuals immune to other HPV type (interaction effect)	Type, sex	1.00	Assumed
Sexual behavior and infection transmission				
c_{gs}	New partner acquisition rate	Gender, sexual activity level	*s*=L: 0.80/year, *s*=H: 5.00/year	(4)[d]
ε	Proportion of sexual partnerships made exclusively with same sexual activity level	None	0.00	Assumed
$\rho_{gsg's'}$	Proportion of sexual partnerships formed with partners of opposite sex g' and sexual activity level *s'*. Depends on c_{gs} and ε.	Gender, sexual activity level, partner's sex, partner's sexual activity level	Variable	-
ξ	Compromise between women and men's desires for sexual partnerships (1=women choose, 0=men choose, 0-1= compromise)	None	1.00	Assumed
ζ	Proportion of time spent in sexual partnerships	None	0.75	(8)[e]
τ_{gs}	Sexual partnership dissolution rate. Depends on c_{gs}, and ζ.	Gender, sexual activity level	Variable	-
v_{gs}	Sexual partnership formation rate for singles. Depends on c_{gs}, and ζ.	Gender, sexual activity level	Variable	-
$v_{gsg's'}$	Effective partner formation rates with partners of opposite sex *g'* sexual activity level *s'*. Depends on v_{gs}, $\rho_{gsg's'}$, and ξ.	Gender, sexual activity level, partner's sex, partner's sexual activity level	Variable	-
δ_{ig}	Probability of acquiring infection with type *i* per-sex-act with an *i* infected partner	Type, sex	0.13	(9) [b,f]
π	Rate of sex acts in sexual partnerships	None	75.40/year	(10)[g]
λ_{ig}	Infection rate with type *i* of individuals susceptible to *i* who have a partner infected with *i*. Depends on δ_{ig} and π	Type, sex	Variable	-
λ_{12g}	Rate of simultaneous cotransmission of both HPV types from coinfected partners to individuals susceptible to both	Gender	Variable	-

	HPV types. Depends on δ_{1g}, δ_{2g}, and π.			
$\lambda_{1g2g'}, \lambda_{1g'2g}$	Rates of simultaneously transmitting one's HPV type and acquiring one's partner's HPV type for partners susceptible and infected with discordant HPV types (health states $[S_1I_2]_{gs}^{[I_1S_2]_{g's'}}$ and $[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}$). Depends on δ_{1g}, δ_{2g}, and π.	Gender	Variable	-

$[Y_1Y_2]$ = partner's health state, where Y_i can be S_i, I_i, or R_i. HPV = human papillomavirus.

[a] Based on the cumulative proportion of women who had ≤2 and >2 partners in the past year during 2006-2010.

[b] Averaged over all HPV types in order to obtain parameter for a generic HPV type.

[c] We assumed antibody seropositivity correlates with immunity, and that seropositivity wanes at a constant rate. We calculated the average duration of immunity which leads to 55% of women initially seropositive at baseline remaining seropositive after 6.4 years:

$$\frac{1}{\omega_{ig}} = \frac{6.4\ years}{\ln(1/0.55)} = 10.7\ years$$

[d] In the lower risk group, calculated as the cumulative distribution of women who have never had sex, those with no partners, 1, and 2 partners in the past year:

$$\frac{0.110(0) + 0.061(0) + 0.690(1) + 0.076(2)}{0.110 + 0.061 + 0.690 + 0.076} = 0.8$$

[e] Based on the proportion of women reporting being in a stable partnership.

[f] The probability per-sex-act with an infected partner is set so that the probability of acquisition over the course of an average sexual partnership in the high risk group (which has the shorter partnership duration) is equal to 0.79:

$$v = 1 - e^{\left(\frac{ln(1-\beta_{ig})}{\tau_s{}^{-1} * \pi}\right)} = 1 - e^{\left(\frac{ln(1-0.79)}{(5/0.75)^{-1} * 75.4}\right)} = 0.1289$$

[g] Based on frequency of sexual intercourse in the past 4 weeks in women 16-24 who had at least one male partner in the past year:

$$\frac{5.8 \text{ sex acts}}{4 \text{ partnership-weeks}} * \frac{52 \text{ weeks}}{\text{year}} = \frac{75.4 \text{ sex acts}}{\text{partnership-year}}$$

Differential equations

The model has 684 mutually exclusive compartments (2 sexes * 2 sexual activity levels * 9 health states *[1 single state + 1 partnered state * {2 partner sexual activity levels * 9 partner health states}]). Single individuals are indicated with the superscript 0 in $[X_1X_2]_{gs}^{0}$ (where $[X_1X_2]$ can be one of any of the 9 health states), while partnered individuals are indicated with the health state of their partner in superscript $[X_1X_2]_{gs}^{[Y_1Y_2]_{g's'}}$ (where $[Y_1Y_2]$ may also be one of any of the 9 health states). Compartments represent individuals, not partnerships, so there are equal numbers of women partnered with men as there are men partnered with women in any given health states ($[X_1X_2]_{g=\text{W},s}^{[Y_1Y_2]_{g'=\text{M},s'}} = [Y_1Y_2]_{g'=\text{M},s'}^{[X_1X_2]_{g=\text{W},s}}$). Individuals change compartments if their own health state changes, their partner's health state changes, if a partnership is created, or if a partnership dissolves.

Sums

The following represent sums of all single individuals (N_g^0), all partnered individuals ($N_g^{N_{g'}}$), all individuals of sexual activity level *s* with a partner of a given health state and sexual activity level *s'* ($N_{gs}^{[Y_1Y_2]_{g's'}}$), and all individuals with a given health state and sexual activity level *s* who have a partner of sexual activity level *s'* ($[X_1X_2]_{gs}^{N_{g's'}}$):

$$N_g^0 = \sum_s \left([S_1S_2]_{gs}^0 + [S_1I_2]_{gs}^0 + [S_1R_2]_{gs}^0 + [I_1S_2]_{gs}^0 + [I_1I_2]_{gs}^0 + [I_1R_2]_{gs}^0 + [R_1S_2]_{gs}^0 + [R_1I_2]_{gs}^0 + [R_1R_2]_{gs}^0 \right)$$

$$N_g^{N_{g'}} = \sum_s \left([S_1S_2]_{gs}^{N_{g'}} + [S_1I_2]_{gs}^{N_{g'}} + [S_1R_2]_{gs}^{N_{g'}} + [I_1S_2]_{gs}^{N_{g'}} + [I_1I_2]_{gs}^{N_{g'}} + [I_1R_2]_{gs}^{N_{g'}} + [R_1S_2]_{gs}^{N_{g'}} + [R_1I_2]_{gs}^{N_{g'}} + [R_1R_2]_{gs}^{N_{g'}} \right)$$

$$N_{gs}^{[Y_1Y_2]_{g's'}} = \begin{pmatrix} [S_1S_2]_{gs}^{[Y_1Y_2]_{g's'}} + [S_1I_2]_{gs}^{[Y_1Y_2]_{g's'}} + [S_1R_2]_{gs}^{[Y_1Y_2]_{g's'}} + [I_1S_2]_{gs}^{[Y_1Y_2]_{g's'}} + [I_1I_2]_{gs}^{[Y_1Y_2]_{g's'}} + [I_1R_2]_{gs}^{[Y_1Y_2]_{g's'}} \\ + [R_1S_2]_{gs}^{[Y_1Y_2]_{g's'}} + [R_1I_2]_{gs}^{[Y_1Y_2]_{g's'}} + [R_1R_2]_{gs}^{[Y_1Y_2]_{g's'}} \end{pmatrix}$$

$$[X_1X_2]_{gs}^{N_{g's'}} = \begin{pmatrix} [X_1X_2]_{gs}^{[S_1S_2]_{g's'}} + [X_1X_2]_{gs}^{[S_1I_2]_{g's'}} + [X_1X_2]_{gs}^{[S_1R_2]_{g's'}} + [X_1X_2]_{gs}^{[I_1S_2]_{g's'}} + [X_1X_2]_{gs}^{[I_1I_2]_{g's'}} + [X_1X_2]_{gs}^{[I_1R_2]_{g's'}} \\ + [X_1X_2]_{gs}^{[R_1S_2]_{g's'}} + [X_1X_2]_{gs}^{[R_1I_2]_{g's'}} + [X_1X_2]_{gs}^{[R_1R_2]_{g's'}} \end{pmatrix}$$

Equations for single individuals

The following are differential equations for singles:

$$\frac{d[S_1S_2]_{gs}^0}{dt} = (-\mu - \nu_{gs})[S_1S_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[S_1S_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[S_1S_2]_{gs}}\right) + \gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^0 + \gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^0$$
$$+\omega_{1g}[R_1S_2]_{gs}^0 + \omega_{2g}[S_1R_2]_{gs}^0 + \mu\theta_{gs}\left(N_g^0 + N_g^{N_{g'}}\right)$$

$$\frac{d[S_1I_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \gamma_{2g})[S_1I_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[S_1I_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[S_1I_2]_{gs}}\right) + \gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^0 + \omega_{1g}[R_1I_2]_{gs}^0$$

$$\frac{d[S_1R_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \omega_{2g})[S_1R_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[S_1R_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[S_1R_2]_{gs}}\right) + \gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^0$$
$$+\gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^0 + \omega_{1g}[R_1R_2]_{gs}^0$$

$$\frac{d[I_1S_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \gamma_{1g})[I_1S_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[I_1S_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[I_1S_2]_{gs}}\right) + \gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^0 + \omega_{2g}[I_1R_2]_{gs}^0$$

$$\frac{d[I_1I_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \gamma_{1g} - \gamma_{2g})[I_1I_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[I_1I_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[I_1I_2]_{gs}}\right)$$

$$\frac{d[I_1R_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \gamma_{1g} - \omega_{2g})[I_1R_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[I_1R_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[I_1R_2]_{gs}}\right) + \gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^0$$

$$\frac{d[R_1S_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \omega_{1g})[R_1S_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[R_1S_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[R_1S_2]_{gs}}\right) + \gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^0 + \gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^0$$
$$+\omega_{2g}[R_1R_2]_{gs}^0$$

$$\frac{d[R_1I_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \gamma_{2g} - \omega_{1g})[R_1I_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[R_1I_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[R_1I_2]_{gs}}\right) + \gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^0$$

$$\frac{d[R_1R_2]_{gs}^0}{dt} = (-\mu - \nu_{gs} - \omega_{1g} - \omega_{2g})[R_1R_2]_{gs}^0 + \sum_{s'}\left(\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)[R_1R_2]_{gs}^{N_{g's'}} + \mu N_{g's'}^{[R_1R_2]_{gs}}\right) + \gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^0$$
$$+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^0$$

Equations for individuals in partnerships

The following are differential equations for partnered individuals. Line 1 in each differential equation represents demographic and partnership changes (population exits, sexual partnership dissolution, and sexual partnership formation). Line 2 represents changes that occur to the individual's own health states. Lines 3-4 represent changes that occur to their partner's health states.

Individuals who have [S_1S_2] partners

$$\frac{d[S_1S_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[S_1S_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1S_2]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{1g}[R_1S_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{2g}[S_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[S_1S_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1S_2]_{gs}^{[S_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[S_1I_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1I_2]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\gamma_{2g}[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{1g}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[S_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{2g'}[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[S_1R_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[S_1R_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1R_2]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{1g}[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{2g}[S_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[S_1R_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[I_1S_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1S_2]_{gs}^{0}\\-\gamma_{1g}[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[I_1S_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[I_1S_2]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{1g'}[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[I_1I_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1I_2]_{gs}^{0}\\-\gamma_{1g}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[I_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[I_1I_2]_{gs}^{[S_1R_2]_{g's'}}\\-(\lambda_{1g'}-\lambda_{12g'})[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-(\lambda_{2g'}-\lambda_{12g'})[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\lambda_{12g'}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[I_1R_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1R_2]_{gs}^{0}\\-\gamma_{1g}[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\+\gamma_{1g'}(1-\sigma_{1g'})[I_1R_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1R_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[I_1R_2]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{1g'}[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[R_1S_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1S_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[S_1S_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\ +\gamma_{1g'}(1-\sigma_{1g'})[R_1S_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[R_1S_2]_{gs}^{[S_1R_2]_{g's'}}\end{pmatrix}$$

$$\frac{d[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[R_1I_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1I_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}\\ +\gamma_{1g'}(1-\sigma_{1g'})[R_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[R_1I_2]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{2g'}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}\end{pmatrix}$$

$$\frac{d[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}-\mu[S_1S_2]_{g's'}^{[R_1R_2]_{gs}}+v_{gsg's'}\left(\frac{[S_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1R_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[S_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[S_1S_2]_{g's'}}\\ +\gamma_{1g'}(1-\sigma_{1g'})[R_1R_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1R_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[R_1R_2]_{gs}^{[S_1R_2]_{g's'}}\end{pmatrix}$$

Individuals who have [S_1I_2] partners

$$\frac{d[S_1S_2]_{gs}^{[S_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[S_1I_2]_{g's'}} - \mu[S_1I_2]_{g's'}^{[S_1S_2]_{gs}} + \nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1S_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + \gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g}[R_1S]_{gs}^{[S_1I_2]_{g's'}} + \omega_{2g}[S_1R_2]_{gs}^{[S_1I_2]_{g's'}} - \lambda_{2g}[S_1S_2]_{gs}^{[S_1I_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g'}[S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1I_2]_{gs}^{[S_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[S_1I_2]_{g's'}} - \mu[S_1I_2]_{g's'}^{[S_1I_2]_{gs}} + \nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1I_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[S_1I_2]_{g's'}} - \gamma_{2g}[S_1I_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g}[R_1I_2]_{gs}^{[S_1I_2]_{g's'}} + \lambda_{2g}[S_1S_2]_{gs}^{[S_1I_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1I_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g'}[S_1I_2]_{gs}^{[R_1I_2]_{g's'}} + \lambda_{2g'}[S_1I_2]_{gs}^{[S_1S_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1R_2]_{gs}^{[S_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[S_1I_2]_{g's'}} - \mu[S_1I_2]_{g's'}^{[S_1R_2]_{gs}} + \nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1R_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[S_1I_2]_{g's'}} + \gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g}[R_1R_2]_{gs}^{[S_1I_2]_{g's'}} - \omega_{2g}[S_1R_2]_{gs}^{[S_1I_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1R_2]_{gs}^{[S_1I_2]_{g's'}} + \omega_{1g'}[S_1R_2]_{gs}^{[R_1I_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1S_2]_{gs}^{0}\\&-\gamma_{1g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}-(\lambda_{2g}-\lambda_{1g'2g}\chi_{1g'})\chi_{2g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}-(\lambda_{1g'}-\lambda_{1g'2g}\chi_{2g})\chi_{1g'}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\\&-\lambda_{1g'2g}\chi_{1g'}\chi_{2g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1I_2]_{gs}^{0}\\&-\gamma_{1g}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+(\lambda_{2g}-\lambda_{1g'2g}\chi_{1g'})\chi_{2g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\lambda_{1g'}\chi_{1g'}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+(\lambda_{2g'}-\lambda_{12g'})[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1R_2]_{gs}^{0}\\&-\gamma_{1g}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\lambda_{1g'}\chi_{1g'}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1S_2]_{gs}^{0}\\&+\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}-\lambda_{2g}\varphi_{2g}[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1I_2]_{gs}^{0}\\&+\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\lambda_{2g}\varphi_{2g}[R_1S_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\lambda_{2g'}[R_1I_2]_{gs}^{[S_1S_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}-\mu[S_1I_2]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1R_2]_{gs}^{0}\\&+\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[S_1I_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}\end{aligned}\right)$$

Individuals who have [S_1R_2] partners

$$\frac{d[S_1S_2]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[S_1R_2]_{g's'}}-\mu[S_1R_2]_{g's'}^{[S_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1S_2]_{gs}^{0}\\&+\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}+\omega_{1g}[R_1S_2]_{gs}^{[S_1R_2]_{g's'}}+\omega_{2g}[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[S_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[S_1S_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1S_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[S_1S_2]_{gs}^{[S_1R_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}-\mu[S_1R_2]_{g's'}^{[S_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1I_2]_{gs}^{0}\\&+\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[S_1R_2]_{g's'}}-\gamma_{2g}[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}+\omega_{1g}[R_1I_2]_{gs}^{[S_1R_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[S_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[S_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\left(\begin{aligned}&\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}-\mu[S_1R_2]_{g's'}^{[S_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[S_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1R_2]_{gs}^{0}\\&+\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[S_1R_2]_{g's'}}+\omega_{1g}[R_1R_2]_{gs}^{[S_1R_2]_{g's'}}-\omega_{2g}[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}\\&+\gamma_{1g'}(1-\sigma_{1g'})[S_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[S_1R_2]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[S_1R_2]_{gs}^{[S_1R_2]_{g's'}}\end{aligned}\right)$$

$$\frac{d\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1S_2\right]_{gs}^{0} \\ -\gamma_{1g}\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}+\omega_{2g}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[I_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[I_1S_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[I_1S_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{1g'}\phi_{1g'}\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1I_2\right]_{gs}^{0} \\ -\gamma_{1g}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\gamma_{2g}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[I_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[I_1I_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[I_1I_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{1g'}\phi_{1g'}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1R_2\right]_{gs}^{0} \\ -\gamma_{1g}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\omega_{2g}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[I_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[I_1R_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[I_1R_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}-\lambda_{1g'}\phi_{1g'}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1S_2\right]_{gs}^{0} \\ +\gamma_{1g}\sigma_{1g}\left[I_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\omega_{1g}\left[R_1S_2\right]_{gs}^{[S_1R_2]_{g's'}}+\omega_{2g}\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[R_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[R_1S_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[R_1S_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[R_1S_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1I_2\right]_{gs}^{0} \\ +\gamma_{1g}\sigma_{1g}\left[I_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\gamma_{2g}\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\omega_{1g}\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[R_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[R_1I_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[R_1I_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}-\mu\left[S_1R_2\right]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[S_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1R_2\right]_{gs}^{0} \\ +\gamma_{1g}\sigma_{1g}\left[I_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[R_1I_2\right]_{gs}^{[S_1R_2]_{g's'}}-\omega_{1g}\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}}-\omega_{2g}\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \\ +\gamma_{1g'}(1-\sigma_{1g'})\left[R_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[R_1R_2\right]_{gs}^{[S_1I_2]_{g's'}}+\omega_{1g'}\left[R_1R_2\right]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}\left[R_1R_2\right]_{gs}^{[S_1R_2]_{g's'}} \end{pmatrix}$$

Individuals who have $[I_1S_2]$ partners

$$\frac{d[S_1S_2]_{gs}^{[I_1S_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[I_1S_2]_{g's'}} - \mu[I_1S_2]_{g's'}^{[S_1S_2]_{gs}} + \nu_{gsg's'}\left(\frac{[I_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1S_2]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[I_1S_2]_{g's'}} + \gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} + \omega_{1g}[R_1S_2]_{gs}^{[I_1S_2]_{g's'}} + \omega_{2g}[S_1R_2]_{gs}^{[I_1S_2]_{g's'}} - \lambda_{1g}[S_1S_2]_{gs}^{[I_1S_2]_{g's'}} \\ -\gamma_{1g'}[S_1S_2]_{gs}^{[I_1S_2]_{g's'}} + \gamma_{2g'}(1-\sigma_{2g'})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{2g'}[S_1S_2]_{gs}^{[I_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1I_2]_{gs}^{[I_1S_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} - \mu[I_1S_2]_{g's'}^{[S_1I_2]_{gs}} + \nu_{gsg's'}\left(\frac{[I_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1I_2]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[I_1S_2]_{g's'}} - \gamma_{2g}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} + \omega_{1g}[R_1I_2]_{gs}^{[I_1S_2]_{g's'}} - (\lambda_{1g} - \lambda_{1g2g'}\chi_{2g'})\chi_{1g}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} \\ -\gamma_{1g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} + \gamma_{2g'}(1-\sigma_{2g'})[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{2g'}[S_1I_2]_{gs}^{[I_1R_2]_{g's'}} - (\lambda_{2g'} - \lambda_{1g2g'}\chi_{1g})\chi_{2g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} \\ -\lambda_{1g2g'}\chi_{1g}\chi_{2g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1R_2]_{gs}^{[I_1S_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[I_1S_2]_{g's'}} - \mu[I_1S_2]_{g's'}^{[S_1R_2]_{gs}} + \nu_{gsg's'}\left(\frac{[I_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1R_2]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[I_1S_2]_{g's'}} + \gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} + \omega_{1g}[R_1R_2]_{gs}^{[I_1S_2]_{g's'}} - \omega_{2g}[S_1R_2]_{gs}^{[I_1S_2]_{g's'}} - \lambda_{1g}\phi_{1g}[S_1R_2]_{gs}^{[I_1S_2]_{g's'}} \\ -\gamma_{1g'}[S_1R_2]_{gs}^{[I_1S_2]_{g's'}} + \gamma_{2g'}(1-\sigma_{2g'})[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{2g'}[S_1R_2]_{gs}^{[I_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[I_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1S_2\right]_{gs}^{0}\\ -\gamma_{1g}\left[I_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}+\omega_{2g}\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\lambda_{1g}\left[S_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[I_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1S_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[I_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g'}\left[I_1S_2\right]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1I_2\right]_{gs}^{0}\\ -\gamma_{1g}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\gamma_{2g}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}+(\lambda_{1g}-\lambda_{1g2g'}\chi_{2g'})\chi_{1g}\left[S_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1I_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[I_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+(\lambda_{1g'}-\lambda_{12g'})\left[I_1I_2\right]_{gs}^{[S_1S_2]_{g's'}}-\lambda_{2g'}\chi_{2g'}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[I_1R_2\right]_{gs}^{0}\\ -\gamma_{1g}\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\omega_{2g}\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\lambda_{1g}\varphi_{1g}\left[S_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1R_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[I_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g'}\left[I_1R_2\right]_{gs}^{[S_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[R_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1S_2\right]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}\left[I_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\omega_{1g}\left[R_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}+\omega_{2g}\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[R_1S_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1S_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[R_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1I_2\right]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}\left[I_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\gamma_{2g}\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\omega_{1g}\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1I_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[R_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}-\lambda_{2g'}\chi_{2g'}\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}-\mu\left[I_1S_2\right]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1S_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[R_1R_2\right]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}\left[I_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[R_1I_2\right]_{gs}^{[I_1S_2]_{g's'}}-\omega_{1g}\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}-\omega_{2g}\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}\\ -\gamma_{1g'}\left[R_1R_2\right]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1R_2\right]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g'}\left[R_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}\end{array}\right)$$

Individuals who have $[I_1 I_2]$ partners

$$\frac{d[S_1S_2]_{gs}^{[I_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs} + \tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} - \mu[I_1I_2]_{g's'}^{[S_1S_2]_{gs}} + v_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1S_2]_{gs}^0 \\ + \gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + \gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{1g}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{2g}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} \\ -(\lambda_{1g} - \lambda_{12g})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} - (\lambda_{2g} - \lambda_{12g})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} - \lambda_{12g}[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} \\ -\gamma_{1g'}[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1I_2]_{gs}^{[I_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs} + \tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} - \mu[I_1I_2]_{g's'}^{[S_1I_2]_{gs}} + v_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1I_2]_{gs}^0 \\ + \gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{1g}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}} - \lambda_{1g}\chi_{1g}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + (\lambda_{2g} - \lambda_{12g})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}} \\ -\gamma_{1g'}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + (\lambda_{2g'} - \lambda_{1g2g'}\chi_{1g})\chi_{2g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[S_1R_2]_{gs}^{[I_1I_2]_{g's'}}}{dt} = \begin{pmatrix} \left(-\mu - \left(\frac{\tau_{gs} + \tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} - \mu[I_1I_2]_{g's'}^{[S_1R_2]_{gs}} + v_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1R_2]_{gs}^0 \\ + \gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[I_1I_2]_{g's'}} + \gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}} + \omega_{1g}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}} - \omega_{2g}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} - \lambda_{1g}\varphi_{1g}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} \\ -\gamma_{1g'}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} - \gamma_{2g'}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1S_2]_{gs}^{0}\\ -\gamma_{1g}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}+(\lambda_{1g}-\lambda_{12g})[S_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\lambda_{2g}\chi_{2g}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}+(\lambda_{1g'}-\lambda_{1g'2g}\chi_{2g})\chi_{1g'}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1I_2]_{gs}^{0}\\ -\gamma_{1g}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{1g}\chi_{1g}[S_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{2g}\chi_{2g}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{12g}[S_1S_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{1g'}\chi_{1g'}[I_1I_2]_{gs}^{[S_1I_2]_{g's'}}+\lambda_{2g'}\chi_{2g'}[I_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\lambda_{12g'}[I_1I_2]_{gs}^{[S_1S_2]_{g's'}}\\ +\lambda_{1g2g'}\chi_{1g}\chi_{2g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\lambda_{1g'2g}\chi_{1g'}\chi_{2g}[I_1S_2]_{gs}^{[S_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1R_2]_{gs}^{0}\\ -\gamma_{1g}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{1g}\phi_{1g}[S_1R_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{1g'}\chi_{1g'}[I_1R_2]_{gs}^{[S_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1S_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\lambda_{2g}\phi_{2g}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1I_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{2g}\phi_{2g}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}+\lambda_{2g'}\chi_{2g'}[R_1I_2]_{gs}^{[I_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\mu[I_1I_2]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1I_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1R_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}\\ -\gamma_{1g'}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}\end{array}\right)$$

Individuals who have [I1R2] partners

$$\frac{d\left[S_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}-\mu\left[I_1R_2\right]_{g's'}^{[S_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1S_2\right]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})\left[I_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\omega_{1g}\left[R_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\omega_{2g}\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}-\lambda_{1g}\left[S_1S_2\right]_{gs}^{[I_1R_2]_{g's'}} \\ -\gamma_{1g'}\left[S_1S_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[S_1S_2\right]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}\left[S_1S_2\right]_{gs}^{[I_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}-\mu\left[I_1R_2\right]_{g's'}^{[S_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1I_2\right]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})\left[I_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}-\gamma_{2g}\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\omega_{1g}\left[R_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}-\lambda_{1g}\chi_{1g}\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}} \\ -\gamma_{1g'}\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[S_1I_2\right]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}-\mu\left[I_1R_2\right]_{g's'}^{[S_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{\left[I_1R_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1R_2\right]_{gs}^{0} \\ +\gamma_{1g}(1-\sigma_{1g})\left[I_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[S_1I_2\right]_{gs}^{[I_1R_2]_{g's'}}+\omega_{1g}\left[R_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}-\omega_{2g}\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}-\lambda_{1g}\phi_{1g}\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}} \\ -\gamma_{1g'}\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}\left[S_1R_2\right]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}\left[S_1R_2\right]_{gs}^{[I_1R_2]_{g's'}} \end{pmatrix}$$

$$\frac{d[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1S_2]_{gs}^{0}\\ -\gamma_{1g}[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g}[S_1S_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g'}\phi_{1g'}[I_1S_2]_{gs}^{[S_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1I_2]_{gs}^{0}\\ -\gamma_{1g}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g}\chi_{1g}[S_1I_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g'}\phi_{1g'}[I_1I_2]_{gs}^{[S_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1R_2]_{gs}^{0}\\ -\gamma_{1g}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g}\phi_{1g}[S_1R_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\lambda_{1g'}\phi_{1g'}[I_1R_2]_{gs}^{[S_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1S_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1I_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}-\mu[I_1R_2]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[I_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1R_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}\\ -\gamma_{1g'}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\omega_{2g'}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}\end{array}\right)$$

Individuals who have $[R_1S_2]$ partners

$$\frac{d[S_1S_2]_{gs}^{[R_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[R_1S_2]_{g's'}}-\mu[R_1S_2]_{g's'}^{[S_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1S_2]_{gs}^{0}\\ +\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{1g}[R_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g}[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[S_1S_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[S_1S_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1S_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}-\mu[R_1S_2]_{g's'}^{[S_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1I_2]_{gs}^{0}\\ +\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[R_1S_2]_{g's'}}-\gamma_{2g}[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{1g}[R_1I_2]_{gs}^{[R_1S_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[S_1I_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\lambda_{2g'}\phi_{2g'}[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}-\mu[R_1S_2]_{g's'}^{[S_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1S_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[S_1R_2]_{gs}^{0}\\ +\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[R_1S_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{1g}[R_1R_2]_{gs}^{[R_1S_2]_{g's'}}-\omega_{2g}[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[S_1R_2]_{gs}^{[I_1S_2]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})[S_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[S_1R_2]_{gs}^{[R_1S_2]_{g's'}}+\omega_{2g'}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d\left[I_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[I_1S_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[I_1S_2\right]_{gs}^0 \\ -\gamma_{1g}\left[I_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g}\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[I_1S_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[I_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[I_1S_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[I_1I_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[I_1I_2\right]_{gs}^0 \\ -\gamma_{1g}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\gamma_{2g}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[I_1I_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[I_1I_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}}-\lambda_{2g'}\phi_{2g'}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[I_1R_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[I_1R_2\right]_{gs}^0 \\ -\gamma_{1g}\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\omega_{2g}\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[I_1R_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[I_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[I_1R_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[R_1S_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[R_1S_2\right]_{gs}^0 \\ +\gamma_{1g}\sigma_{1g}\left[I_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\omega_{1g}\left[R_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g}\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[R_1S_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[R_1S_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[R_1S_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[R_1I_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[R_1I_2\right]_{gs}^0 \\ +\gamma_{1g}\sigma_{1g}\left[I_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\gamma_{2g}\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\omega_{1g}\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[R_1I_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[R_1I_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}}-\lambda_{2g'}\phi_{2g'}\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \end{pmatrix}$$

$$\frac{d\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}}{dt}=\begin{pmatrix} \left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\mu\left[R_1S_2\right]_{g's'}^{\left[R_1R_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1S_2\right]_{g's'}^0}{N_{g's'}^0}\right)\left[R_1R_2\right]_{gs}^0 \\ +\gamma_{1g}\sigma_{1g}\left[I_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[R_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\omega_{1g}\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}-\omega_{2g}\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}\left[R_1R_2\right]_{gs}^{\left[I_1S_2\right]_{g's'}}+\gamma_{2g'}(1-\sigma_{2g'})\left[R_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[R_1R_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}+\omega_{2g'}\left[R_1R_2\right]_{gs}^{\left[R_1R_2\right]_{g's'}} \end{pmatrix}$$

Individuals who have [R_1I_2] partners

$$\frac{d\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\mu\left[R_1I_2\right]_{g's'}^{\left[S_1S_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1I_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1S_2\right]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\omega_{1g}\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\omega_{2g}\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\lambda_{2g}\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\gamma_{1g'}\sigma_{1g'}\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}-\gamma_{2g'}\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}$$

$$\frac{d\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\mu\left[R_1I_2\right]_{g's'}^{\left[S_1I_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1I_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1I_2\right]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})\left[I_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\gamma_{2g}\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\omega_{1g}\left[R_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\lambda_{2g}\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\gamma_{1g'}\sigma_{1g'}\left[S_1I_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}-\gamma_{2g'}\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\lambda_{2g'}\phi_{2g'}\left[S_1I_2\right]_{gs}^{\left[R_1S_2\right]_{g's'}}\end{pmatrix}$$

$$\frac{d\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}}{dt}=\begin{pmatrix}\left(-\mu-\left(\dfrac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\mu\left[R_1I_2\right]_{g's'}^{\left[S_1R_2\right]_{gs}}+\nu_{gsg's'}\left(\dfrac{\left[R_1I_2\right]_{g's'}^{0}}{N_{g's'}^{0}}\right)\left[S_1R_2\right]_{gs}^{0}\\+\gamma_{1g}(1-\sigma_{1g})\left[I_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\gamma_{2g}\sigma_{2g}\left[S_1I_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}+\omega_{1g}\left[R_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{2g}\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\gamma_{1g'}\sigma_{1g'}\left[S_1R_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}-\gamma_{2g'}\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}-\omega_{1g'}\left[S_1R_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}$$

$$\frac{d[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[I_1S_2]_{gs}^0\\ -\gamma_{1g}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\lambda_{2g}\chi_{2g}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[I_1I_2]_{gs}^0\\ -\gamma_{1g}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\lambda_{2g}\chi_{2g}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\lambda_{2g'}\phi_{2g'}[I_1I_2]_{gs}^{[R_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[I_1R_2]_{gs}^0\\ -\gamma_{1g}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[R_1S_2]_{gs}^0\\ +\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\lambda_{2g}\phi_{2g}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1S_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[R_1I_2]_{gs}^0\\ +\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\lambda_{2g}\phi_{2g}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1I_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}+\lambda_{2g'}\phi_{2g'}[R_1I_2]_{gs}^{[R_1S_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\mu[R_1I_2]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1I_2]_{g's'}^0}{N_{g's'}^0}\right)[R_1R_2]_{gs}^0\\ +\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1R_2]_{gs}^{[I_1I_2]_{g's'}}-\gamma_{2g'}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}\end{array}\right)$$

Individuals who have [R_1R_2] partners

$$\frac{d[S_1S_2]_{gs}^{[R_1R_2]_{g's'}}}{dt} = \left(\begin{array}{l} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1S_2]_{gs}^{[R_1R_2]_{g's'}} - \mu[R_1R_2]_{g's'}^{[S_1S_2]_{gs}} + \nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1S_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1S_2]_{gs}^{[R_1R_2]_{g's'}} + \gamma_{2g}(1-\sigma_{2g})[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} + \omega_{1g}[R_1S_2]_{gs}^{[R_1R_2]_{g's'}} + \omega_{2g}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}[S_1S_2]_{gs}^{[I_1R_2]_{g's'}} + \gamma_{2g'}\sigma_{2g'}[S_1S_2]_{gs}^{[R_1I_2]_{g's'}} - \omega_{1g'}[S_1S_2]_{gs}^{[R_1R_2]_{g's'}} - \omega_{2g'}[S_1S_2]_{gs}^{[R_1R_2]_{g's'}} \end{array} \right)$$

$$\frac{d[S_1I_2]_{gs}^{[R_1R_2]_{g's'}}}{dt} = \left(\begin{array}{l} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} - \mu[R_1R_2]_{g's'}^{[S_1I_2]_{gs}} + \nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1I_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1I_2]_{gs}^{[R_1R_2]_{g's'}} - \gamma_{2g}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} + \omega_{1g}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}[S_1I_2]_{gs}^{[I_1R_2]_{g's'}} + \gamma_{2g'}\sigma_{2g'}[S_1I_2]_{gs}^{[R_1I_2]_{g's'}} - \omega_{1g'}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} - \omega_{2g'}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} \end{array} \right)$$

$$\frac{d[S_1R_2]_{gs}^{[R_1R_2]_{g's'}}}{dt} = \left(\begin{array}{l} \left(-\mu - \left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[S_1R_2]_{gs}^{[R_1R_2]_{g's'}} - \mu[R_1R_2]_{g's'}^{[S_1R_2]_{gs}} + \nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^0}{N_{g's'}^0}\right)[S_1R_2]_{gs}^0 \\ +\gamma_{1g}(1-\sigma_{1g})[I_1R_2]_{gs}^{[R_1R_2]_{g's'}} + \gamma_{2g}\sigma_{2g}[S_1I_2]_{gs}^{[R_1R_2]_{g's'}} + \omega_{1g}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}} - \omega_{2g}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}} \\ +\gamma_{1g'}\sigma_{1g'}[S_1R_2]_{gs}^{[I_1R_2]_{g's'}} + \gamma_{2g'}\sigma_{2g'}[S_1R_2]_{gs}^{[R_1I_2]_{g's'}} - \omega_{1g'}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}} - \omega_{2g'}[S_1R_2]_{gs}^{[R_1R_2]_{g's'}} \end{array} \right)$$

$$\frac{d[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[I_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1S_2]_{gs}^{0}\\ -\gamma_{1g}[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}+\omega_{2g}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[I_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1I_2]_{gs}^{0}\\ -\gamma_{1g}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\gamma_{2g}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[I_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[I_1R_2]_{gs}^{0}\\ -\gamma_{1g}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[I_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[I_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1S_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1S_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[R_1S_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1S_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1S_2]_{gs}^{[R_1R_2]_{g's'}}+\gamma_{2g}(1-\sigma_{2g})[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{1g}[R_1S_2]_{gs}^{[R_1R_2]_{g's'}}+\omega_{2g}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1S_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1S_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1S_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[R_1S_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[R_1I_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1I_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\gamma_{2g}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{1g}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1I_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1I_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

$$\frac{d[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}}{dt}=\left(\begin{array}{l}\left(-\mu-\left(\frac{\tau_{gs}+\tau_{g's'}}{2}\right)\right)[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\mu[R_1R_2]_{g's'}^{[R_1R_2]_{gs}}+\nu_{gsg's'}\left(\frac{[R_1R_2]_{g's'}^{0}}{N_{g's'}^{0}}\right)[R_1R_2]_{gs}^{0}\\ +\gamma_{1g}\sigma_{1g}[I_1R_2]_{gs}^{[R_1R_2]_{g's'}}+\gamma_{2g}\sigma_{2g}[R_1I_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{1g}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}\\ +\gamma_{1g'}\sigma_{1g'}[R_1R_2]_{gs}^{[I_1R_2]_{g's'}}+\gamma_{2g'}\sigma_{2g'}[R_1R_2]_{gs}^{[R_1I_2]_{g's'}}-\omega_{1g'}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}-\omega_{2g'}[R_1R_2]_{gs}^{[R_1R_2]_{g's'}}\end{array}\right)$$

Equations for the rates of infection and partnering

$$\lambda_{1g} = \pi\delta_{1g}$$

$$\lambda_{2g} = \pi\delta_{2g}$$

$$\lambda_{12g} = \pi\delta_{1g}\delta_{2g}$$

$$\lambda_{1g2g'} = \pi\delta_{1g}\delta_{2g'}$$

$$\lambda_{1g'2g} = \pi\delta_{1g'}\delta_{2g}$$

If $s = s'$:

$$\rho_{gsg's'} = \varepsilon + (1-\varepsilon)(v_{g's'} \times N^{0}_{g's'}) \Big/ \sum_{s'} \left(v_{g's'} \times N^{0}_{g's'}\right)$$

If $s \neq s'$:

$$\rho_{gsg's'} = (1-\varepsilon)(v_{g's'} \times N^{0}_{g's'}) \Big/ \sum_{s'} \left(v_{g's'} \times N^{0}_{g's'}\right)$$

$$v_{gs} = \left(\frac{c_{gs}}{1-\zeta}\right)$$

$$\tau_{gs} = \left(\frac{c_{gs}}{\zeta}\right)$$

$$v_{g=\mathrm{W},s,g'=\mathrm{M},s'} = v_{g=\mathrm{W},s} \times \rho_{g=\mathrm{W},s,g'=\mathrm{M},s'} \times \left(\frac{v_{g=\mathrm{W},s,} \times \rho_{g=\mathrm{W},s,g'=\mathrm{M},s'} \times N_{g=\mathrm{W},s}}{v_{g'=\mathrm{M},s'} \times \rho_{g'=\mathrm{M},s',g=\mathrm{W},s} \times N_{g'=\mathrm{M},s'}}\right)^{-(1-\xi)}$$

$$v_{g=\mathrm{M},s,g'=\mathrm{W},s'} = v_{g=\mathrm{M},s} \times \rho_{g'=\mathrm{M},s',g=\mathrm{W},s} \times \left(\frac{v_{g'=\mathrm{W},s'} \times \rho_{g'=\mathrm{W},s',g=\mathrm{M},s} \times N_{g'=\mathrm{W},s'}}{v_{g=\mathrm{M},s} \times \rho_{g=\mathrm{M},s,g'=\mathrm{W},s'} \times N_{g=\mathrm{M},s}}\right)^{\xi}$$

Calculation of epidemiologic measures of association between HPV types (interaction estimates)

We assumed HPV types interact in our model by having causal effects on the probability of being infected with the other HPV type per-sex-act with an infected partner. These causal effects are estimated using measures of association in epidemiologic studies. Our relative measures of association use health states with HPV type 1 as the exposure and infection with HPV type 2 as the outcome. We present the calculation of both crude measures of association and of measures of association adjusted for sexual activity level (*adj(s)*).

Cross-sectional measures of association

Prevalence Odds Ratio (OR)

The OR is frequently used in cross-sectional studies of infection prevalence (11-15). The OR compares the odds of HPV type 2 prevalent infection in individuals infected with HPV type 1 relative to the odds of HPV type 2 prevalent infection in individuals uninfected with HPV type 1. ORs can be adjusted for potential confounders using multivariate logistic regression models or using Mantel-Haenszel estimators (shown below) which are a weighted average of stratum-specific estimates. The interaction effects χ_{ig} and φ_{ig} do not directly intervene in the calculation of the OR, but influence its value by affecting the relative distribution of $[S_1I_2]$, $[I_1I_2]$, and $[R_1I_2]$.

$$OR = \frac{\text{Odds of HPV type 2 prevalent infection in individuals infected with HPV type 1}}{\text{Odds of HPV type 2 prevalent infection in individuals uninfected with HPV type 1}}$$

$$OR = \frac{\left(\sum_{g,s}\left([I_1I_2]_{gs}^0 + \sum_{s'}[I_1I_2]_{gs}^{N_{g's'}}\right) \Big/ \sum_{g,s}\left([I_1S_2]_{gs}^0 + [I_1R_2]_{gs}^0 + \sum_{s'}\left([I_1S_2]_{gs}^{N_{g's'}} + [I_1R_2]_{gs}^{N_{g's'}}\right)\right)\right)}{\left(\sum_{g,s}\left([S_1I_2]_{gs}^0 + [R_1I_2]_{gs}^0 + \sum_{s'}\left([S_1I_2]_{gs}^{N_{g's'}} + [R_1I_2]_{gs}^{N_{g's'}}\right)\right) \Big/ \sum_{g,s}\left(\begin{array}{l}[S_1S_2]_{gs}^0 + [S_1R_2]_{gs}^0 + [R_1S_2]_{gs}^0 + [R_1R_2]_{gs}^0 \\ + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [S_1R_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} + [R_1R_2]_{gs}^{N_{g's'}}\right)\end{array}\right)\right)}$$

$$OR_{adj(s)} = \frac{\sum_s \left(\frac{\sum_g\left([I_1I_2]_{gs}^0 + \sum_{s'}[I_1I_2]_{gs}^{N_{g's'}}\right) \times \sum_g\left(\begin{array}{l}[S_1S_2]_{gs}^0 + [S_1R_2]_{gs}^0 + [R_1S_2]_{gs}^0 + [R_1R_2]_{gs}^0 \\ + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [S_1R_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} + [R_1R_2]_{gs}^{N_{g's'}}\right)\end{array}\right)}{\sum_g\left(N_{gs}^0 + \sum_{s'}N_{gs}^{N_{g's'}}\right)}\right)}{\sum_s \left(\frac{\sum_g\left([S_1I_2]_{gs}^0 + [R_1I_2]_{gs}^0 + \sum_{s'}\left([S_1I_2]_{gs}^{N_{g's'}} + [R_1I_2]_{gs}^{N_{g's'}}\right)\right) \times \sum_g\left([I_1S_2]_{gs}^0 + [I_1R_2]_{gs}^0 + \sum_{s'}\left([I_1S_2]_{gs}^{N_{g's'}} + [I_1R_2]_{gs}^{N_{g's'}}\right)\right)}{\sum_g\left(N_{gs}^0 + \sum_{s'}N_{gs}^{N_{g's'}}\right)}\right)}$$

Prospective measures of association

We use the incidence rate ratio as our prospective measure of association between HPV types. We assumed that follow-up is restricted to individuals who are susceptible to HPV type 2 ($[S_1S_2]$,$[I_1S_2]$,$[R_1S_2]$). This was done to remove the bias caused by immortal person-time from type-2 immune individuals (3) in order to isolate the bias due to correlation between times-at-risk for infection. In practice this may be hard to achieve as studies cannot generally determine which individuals are susceptible to HPV infection.

Incidence rate ratio (IRR) by infection status

Some studies evaluate prevalent infection with a first HPV (type 1) as the exposure and incident infection with the second HPV (type 2) as the outcome (14, 16-21). The IRR by infection status compares the incidence rate of HPV type 2 in individuals infected with HPV type 1 relative to the incidence rate of HPV type 2 in individuals uninfected with HPV type 1. We adjusted the IRR for the sexual activity level using Mantel-Haenszel estimators. Both interaction effects χ_{2g} and φ_{2g} affect the calculation of the IRR by infection status. This can be seen in the equations for the instantaneous partnership model in sections below: the IRR by infection status will only be equal to the interaction effect χ_{2g} caused by current infection when there is no cross-immune interaction (φ_{2g}=1) or alternatively when individuals immune to the HPV type 1 ($[R_1S_2]$) are excluded from calculations if $\varphi_{2g}\neq 1$. Cross-immune interactions ($\varphi_{2g}<1$) will tend to increase the value of the IRR by infection status because individuals immune to HPV type 1 contribute a lower rate of infection to the unexposed "uninfected" category.

$$IRR = \frac{\text{HPV type 2 infection incidence rate in HPV type 1 infected individuals}}{\text{HPV type 2 infection incidence rate in HPV type 1 uninfected individuals}}$$

Overall population

Studies of interaction most often do not measure sexual partnership characteristics, and measure the IRR in all recruited individuals regardless of partnership status. However, partnership durations and dissolutions create prospective correlation between the times-at-risk for HPV type 1 and type 2 infections. This correlation arises from the inclusion of person-time from individuals who are not at-risk of infection with HPV type 2 because they are either single or in a sexual partnership with an uninfected individual. This can be seen in the equations below, where singles and individuals partnered with uninfected individuals contribute person-time to denominators without contributing to the incidence of numerators. The IRR by infection status in the overall population will thus be a biased estimate of the interaction effect χ_{2g} of current infection.

$$IRR = \frac{\sum_{g,s,s'}\left(\lambda_{2g}\chi_{2g}\left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right)\right) \Big/ \sum_{g,s}\left([I_1S_2]_{gs}^{0} + \sum_{s'}[I_1S_2]_{gs}^{N_{g's'}}\right)}{\dfrac{\sum_{g,s,s'}\left(\lambda_{2g}\left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right) + \lambda_{2g}\varphi_{2g}\left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right)\right)}{\sum_{g,s}\left([S_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}}\right)\right)}}$$

$$IRR_{adj(s)} = \frac{\sum_{s}\left(\dfrac{\sum_{g,s'}\left(\lambda_{2g}\chi_{2g}\left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right)\right) \times \sum_{g}\left([S_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}}\right)\right)}{\sum_{g}\left([S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}}\right)\right)}\right)}{\sum_{s}\left(\dfrac{\sum_{g,s'}\left(\begin{array}{l}\lambda_{2g}\left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right)\\ + \lambda_{2g}\varphi_{2g}\left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}}\right)\end{array}\right) \times \sum_{g}\left([I_1S_2]_{gs}^{0} + \sum_{s'}[I_1S_2]_{gs}^{N_{g's'}}\right)}{\sum_{g}\left([S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + \sum_{s'}\left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}}\right)\right)}\right)}$$

Restricting to partnered individuals

The issue of correlation between times-at-risk for infections may be partly remedied by restricting analyses to individuals in sexual partnerships, which will remove the person-time not at-risk contributed by single individuals. However, many individuals will still contribute person-time not at-risk by being in sexual partnerships in which they are not at risk of acquiring the outcome HPV.

$$IRR = \frac{\sum_{g,s,s'} \left(\lambda_{2g}\chi_{2g}\left(\left[I_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\right) \Big/ \sum_{g,s,s'}\left(\left[I_1S_2\right]_{gs}^{N_{g's'}}\right)}{\dfrac{\sum_{g,s,s'}\left(\lambda_{2g}\left(\left[S_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right) + \lambda_{2g}\varphi_{2g}\left(\left[R_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\right)}{\sum_{g,s,s'}\left(\left[S_1S_2\right]_{gs}^{N_{g's'}} + \left[R_1S_2\right]_{gs}^{N_{g's'}}\right)}}$$

$$IRR_{adj(s)} = \frac{\sum_{s}\left(\dfrac{\sum_{g,s'}\left(\lambda_{2g}\chi_{2g}\left(\left[I_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\right) \times \sum_{g,s'}\left(\left[S_1S_2\right]_{gs}^{N_{g's'}} + \left[R_1S_2\right]_{gs}^{N_{g's'}}\right)}{\sum_{g,s'}\left(\left[S_1S_2\right]_{gs}^{N_{g's'}} + \left[I_1S_2\right]_{gs}^{N_{g's'}} + \left[R_1S_2\right]_{gs}^{N_{g's'}}\right)}\right)}{\sum_{s}\left(\dfrac{\sum_{g,s'}\begin{pmatrix}\lambda_{2g}\left(\left[S_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right) \\ + \lambda_{2g}\varphi_{2g}\left(\left[R_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\end{pmatrix} \times \sum_{g,s'}\left(\left[I_1S_2\right]_{gs}^{N_{g's'}}\right)}{\sum_{g,s'}\left(\left[S_1S_2\right]_{gs}^{N_{g's'}} + \left[I_1S_2\right]_{gs}^{N_{g's'}} + \left[R_1S_2\right]_{gs}^{N_{g's'}}\right)}\right)}$$

Restricting to at-risk partnered individuals

Ideally, analyses would be restricted to individuals who are at-risk of infection with the outcome HPV, that is, individuals susceptible to HPV type 2 in a sexual partnership with partners infected with HPV type 2 ($[X_1S_2]_{gs}^{[Y_1I_2]_{g's'}}$). This avoids the inclusion of person-time not at-risk from individuals who are not susceptible to infection or who are not in a sexual partnership where they are at-risk of infection. The IRR by infection status will be a valid estimate of the interaction effect of current infection χ_{2g} if analyses are restricted to at-risk individuals partnered with HPV type 2 infected partners, and if there is no cross-immune interaction (φ_{2g}=1).

$$IRR = \frac{\sum_{g,s,s'}\left(\lambda_{2g}\chi_{2g}\left(\left[I_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\right) \Big/ \sum_{g,s,s'}\left(\left[I_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[I_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)}{\left(\dfrac{\sum_{g,s,s'}\left(\lambda_{2g}\left(\left[S_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right) + \lambda_{2g}\varphi_{2g}\left(\left[R_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)\right)}{\sum_{g,s,s'}\left(\left[S_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[S_1S_2\right]_{gs}^{[R_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[S_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[I_1I_2]_{g's'}} + \left[R_1S_2\right]_{gs}^{[R_1I_2]_{g's'}}\right)}\right)}$$

$$
IRR_{adj(s)} = \frac{\displaystyle\sum_{s}\left(\frac{\displaystyle\sum_{g,s'}\left(\lambda_{2g}\chi_{2g}\begin{pmatrix}\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}\right)\times\sum_{g,s'}\begin{pmatrix}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}}{\displaystyle\sum_{g,s'}\begin{pmatrix}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}}\right)}{\displaystyle\sum_{s}\left(\frac{\displaystyle\sum_{g,s'}\begin{pmatrix}\lambda_{2g}\left(\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)\\+\lambda_{2g}\varphi_{2g}\left(\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)\end{pmatrix}\times\sum_{g,s'}\left(\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)}{\displaystyle\sum_{g,s'}\begin{pmatrix}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{pmatrix}}\right)}
$$

IRR of infection by immune status

Some studies evaluate seropositivity to a first HPV type as the exposure and incident infection with a second HPV type as the outcome (22, 23). The IRR by immune status compares the incidence of HPV type 2 individuals in individuals immune to HPV type 1 relative to the incidence rate of HPV type 2 in individuals not immune to HPV type 1. We adjusted the IRR for the sexual activity level using Mantel-Haenszel estimators. Both interaction effects χ_{2g} and φ_{2g} affect the calculation of the IRR by immune status. This can be seen in the equations in the sections below, where the IRR by immune status will only be equal to the parameter of cross-immune interactions φ_{2g} if there is no effect of current infection (χ_{2g}=1) or alternatively if individuals infected with HPV type 1 ([I_1S_2]) are excluded from calculations when $\chi_{2g}\neq1$. However, results in the main text suggest that interactions caused by current infection (χ_{2g}) do not strongly affect the value of the IRR by immune status. This is because in our base case scenario the duration of infection is much shorter than the duration of immunity.

$$
IRR = \frac{\text{HPV type 2 infection incidence rate in HPV type 1 immune individuals}}{\text{HPV type 2 infection incidence rate in HPV type 1 non-immune individuals}}
$$

Overall population

The correlation between the times-at-risk for HPV type 1 and type 2 infections also affects the IRR by immune status. This can be seen in the equations below, where singles and individuals partnered with uninfected individuals contribute person-time to denominators without contributing to the incidence of numerators. The IRR by immune status in the overall will thus be a biased estimate of cross-immune interactions φ_{2g}, though this bias does not appear to be very strong in our analysis.

$$IRR = \frac{\sum_{g,s,s'} \left(\lambda_{2g} \varphi_{2g} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \right) \Big/ \sum_{g,s,s'} \left([R_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s,s'} \begin{pmatrix} \lambda_{2g} \left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \\ + \lambda_{2g} \chi_{2g} \left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \end{pmatrix} \Big/ \sum_{g,s,s'} \begin{pmatrix} [S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} \\ + [S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} \end{pmatrix}}$$

$$IRR_{adj(s)} = \frac{\sum_{s} \left(\frac{\sum_{g,s'} \left(\lambda_{2g} \varphi_{2g} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \right) \times \sum_{g,s'} \left([S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} + [S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s'} \left([S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + [S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} \right)} \right)}{\sum_{s} \left(\frac{\sum_{g,s'} \begin{pmatrix} \lambda_{2g} \left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \\ + \lambda_{2g} \chi_{2g} \left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \end{pmatrix} \times \sum_{g,s'} \left([R_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s'} \left([S_1S_2]_{gs}^{0} + [I_1S_2]_{gs}^{0} + [R_1S_2]_{gs}^{0} + [S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} \right)} \right)}$$

Restricting to partnered individuals

$$IRR = \frac{\sum_{g,s,s'} \left(\lambda_{2g} \varphi_{2g} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \right) \Big/ \sum_{g,s,s'} \left([R_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s,s'} \begin{pmatrix} \lambda_{2g} \left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \\ + \lambda_{2g} \chi_{2g} \left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \end{pmatrix} \Big/ \sum_{g,s,s'} \left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} \right)}$$

$$IRR_{adj(s)} = \frac{\sum_{s} \left(\frac{\sum_{g,s'} \left(\lambda_{2g} \varphi_{2g} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \right) \times \sum_{g,s'} \left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s'} \left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} \right)} \right)}{\sum_{s} \left(\frac{\sum_{g,s'} \begin{pmatrix} \lambda_{2g} \left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \\ + \lambda_{2g} \chi_{2g} \left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \end{pmatrix} \times \sum_{g,s'} \left([R_1S_2]_{gs}^{N_{g's'}} \right)}{\sum_{g,s'} \left([S_1S_2]_{gs}^{N_{g's'}} + [I_1S_2]_{gs}^{N_{g's'}} + [R_1S_2]_{gs}^{N_{g's'}} \right)} \right)}$$

Restricting to at-risk individuals

The IRR by immune status will be a valid estimate of the interaction effect of immunity φ_{2g} if analyses are restricted to at-risk individuals partnered with HPV type 2 infected partners, and if there is no effect of current infection ($\chi_{2g} = 1$).

$$IRR = \frac{\sum_{g,s,s'} \left(\lambda_{2g} \varphi_{2g} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \right) \Big/ \sum_{g,s,s'} \left([R_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [R_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right)}{\sum_{g,s,s'} \begin{pmatrix} \lambda_{2g} \left([S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \\ + \lambda_{2g} \chi_{2g} \left([I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \right) \end{pmatrix} \Big/ \sum_{g,s,s'} \begin{pmatrix} [S_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [S_1S_2]_{gs}^{[R_1I_2]_{g's'}} \\ + [I_1S_2]_{gs}^{[S_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[I_1I_2]_{g's'}} + [I_1S_2]_{gs}^{[R_1I_2]_{g's'}} \end{pmatrix}}$$

$$
IRR_{adj(s)} = \frac{\displaystyle\sum_{s}\left(\frac{\displaystyle\sum_{g,s'}\left(\lambda_{2g}\varphi_{2g}\left(\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)\right)\times\sum_{g,s'}\left(\begin{array}{l}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{array}\right)}{\displaystyle\sum_{g,s'}\left(\begin{array}{l}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{array}\right)}\right)}{\displaystyle\sum_{s}\left(\frac{\displaystyle\sum_{g,s'}\left(\begin{array}{l}\lambda_{2g}\left(\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)\\+\lambda_{2g}\chi_{2g}\left(\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)\end{array}\right)\times\sum_{g,s'}\left(\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\right)}{\displaystyle\sum_{g,s'}\left(\begin{array}{l}\left[S_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[S_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[I_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[I_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\\+\left[R_1S_2\right]_{gs}^{\left[S_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[I_1I_2\right]_{g's'}}+\left[R_1S_2\right]_{gs}^{\left[R_1I_2\right]_{g's'}}\end{array}\right)}\right)}
$$

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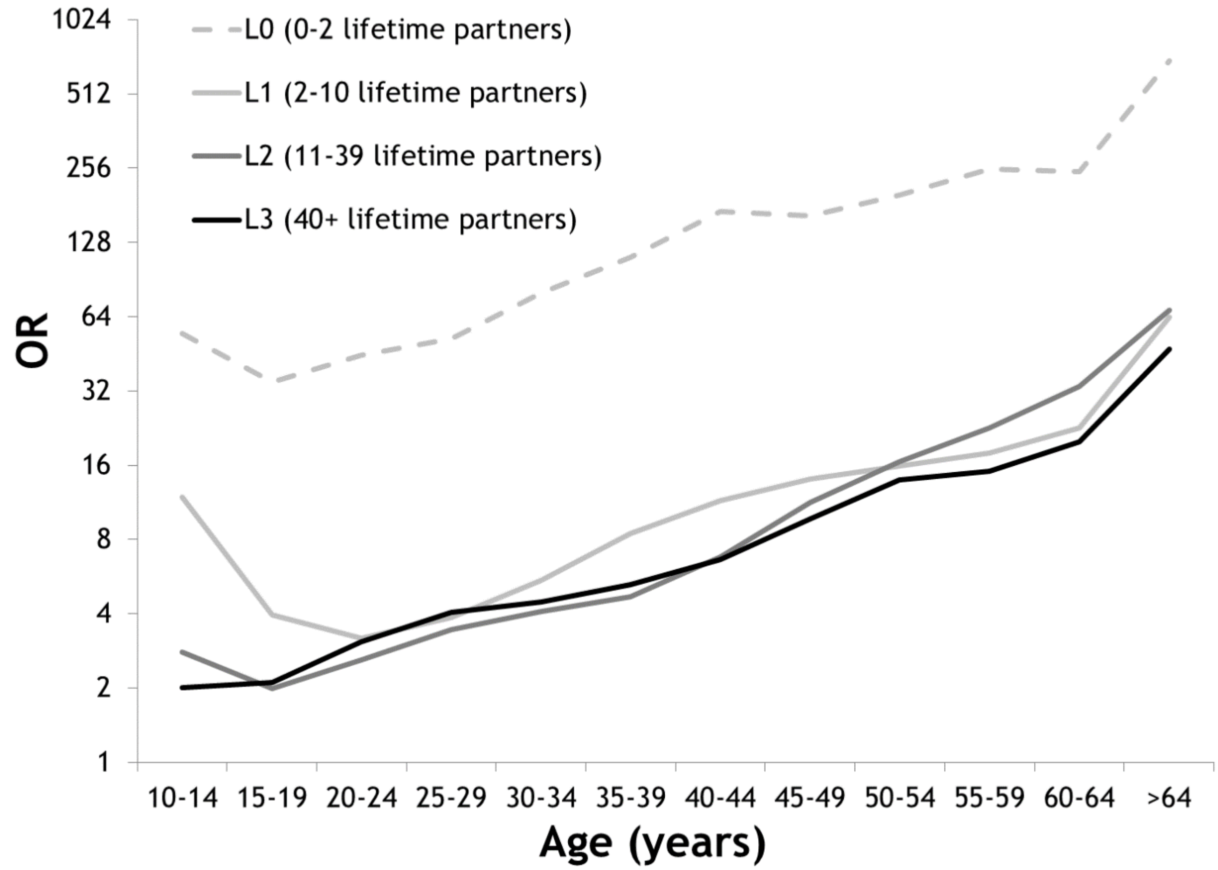
Web Table 2. Measures of association between HPV types 1 and 2 in the base case scenario with no modeled interactions

	Single and Partnered Individuals				Partnered Individuals (Adjusted[a])	At-Risk Partnered Individuals (Adjusted[a])
	Overall (Crude)	Overall (Adjusted[a])	Low Sexual Activity Level	High Sexual Activity Level		
OR	1.39	1.31	1.36	1.12	1.29	1.10
IRR by infection status	1.37	1.27	1.32	1.07	1.22	1.00[b]
IRR by immune status	1.09	1.00	1.00	1.00	1.00	1.00[b]

HPV = human papillomavirus; OR = prevalence odds ratio of HPV type 2 infection prevalence odds in HPV type 1 infected vs. uninfected individuals; IRR by infection status = incidence rate ratio of HPV type 2 infection in HPV type 1 infected vs. uninfected individuals; IRR by immune status = incidence rate ratio of HPV type 2 infection in HPV type 1 immune vs. non-immune individuals.

[a] Adjusted for sexual activity level using Mantel-Haenszel estimators.

[b] Equal to exactly 1 (no bias).



Web Figure 3. Prevalence odds ratio of HPV-33 infection in HPV-16 infected vs. uninfected sexually active women in HPV-ADVISE, by age and sexual activity level. HPV types have no interactions in HPV-ADVISE. Sexual activity levels L0, L1, L2, and L3 represent categories of women with increasing sexual risk behaviors, with earlier ages of sexual onset, a higher proportion of casual partnerships, higher partner acquisition rates, and shorter partnership durations. Women in sexual activity level L0, L1, L2, and L3 will have on average 0-2, 2-10, 11-39, and 40+ lifetime sexual partners respectively by the end of their life. Sexual behavior also varies over age, with lower partner acquisition rates and longer partnership durations as women age. The y-axis uses a base-2 log scale. OR = prevalence odds ratio.