Dataset S1: Model Equations

Posterior parameter distributions, used in predictions, can be found in Table S1. Prior distributions are located in Table S4.

$$
\begin{aligned}
\frac{\mathrm{d}S_O}{\mathrm{d}t} &= -\mu S_O - \gamma_O S_O N_O - \beta I \frac{S_O}{K} + a_O N_O (1 - v_O + v_O(1-\epsilon)), \\
\frac{\mathrm{d}S_S}{\mathrm{d}t} &= -\mu S_S - \gamma_S N^2 \frac{S_S}{N_S} - \beta I \frac{S_S}{K} + a_S N_S (1 - v_S + v_S(1-\epsilon)) + \gamma_O S_O N_O \\
&\quad - \left[\phi h(f_T) \left\{ 1 - ♂(1-\epsilon) \right\} \right] \frac{S_S}{S_S + V_S}, \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= -\mu E - \gamma_S N^2 \frac{E}{N} - \sigma E + \beta I \frac{(S_O + S_S + X_S)}{K}, \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= -\mu I - \gamma_S N^2 \frac{I}{N} + \sigma E - \alpha I, \\
\frac{\mathrm{d}V_O}{\mathrm{d}t} &= -\mu V_O - \gamma_O V_O N_O + a_O N_O \nu_O \epsilon, \\
\frac{\mathrm{d}F_S}{\mathrm{d}t} &= -\mu F_S - \gamma_S N^2 \frac{F_S}{N_S} + (1 - ♂)\phi h(f_T)\epsilon, \\
\frac{\mathrm{d}V_S}{\mathrm{d}t} &= -\mu V_S - \gamma_S N^2 \frac{V_S}{N_S} + a_S N_S \nu_S \epsilon + \gamma_O V_O N_O - \left[\phi h(f_T) \left\{ 1 - ♂(1-\epsilon) \right\} \right] \frac{V_S}{S_S + V_S}, \\
\frac{\mathrm{d}M_S}{\mathrm{d}t} &= -\mu M_S - \gamma_S N^2 \frac{M_S}{N_S} + ♂\phi h(f_T)\epsilon, \\
\frac{\mathrm{d}X_S}{\mathrm{d}t} &= -\mu X_S - \gamma_S N^2 \frac{X_S}{N_S} + \phi h(f_T)(1 - ♂)(1-\epsilon) - \beta I \frac{X_S}{K},
\end{aligned}
$$

where a_0 and a_S are daily birth rate for owned and stray dogs respectively. $N_S = S_S + E + I + F_S + V_S + M_S + X_S$ is the stray dog population and $N_O = S_O + V_O$ is the owned dog population. Given the proportion of dogs which are female (♀),

$$
\begin{aligned}
a_O &= \rho\lambda \frac{♀}{365} \\
a_S &= \rho\lambda \frac{(♀ N_S - F_S - X_S)}{365 N_S},
\end{aligned}
$$

and

$$\gamma_O = \frac{(a_O - \mu)}{K_O},$$

$$\gamma_S = \frac{\left[\frac{(a_S N_S + a_0 N_0)}{N} - \mu\right]}{K}.$$

We used hill function of the form,

$$h(f_T) = \frac{2}{\left(1 + \frac{f_0 - f_T}{f_0}\right)^m - 1}$$

such that f_0 is the starting population of fertile females, f_T is the current population of fertile females. The constant m is set to 28, which hits the target of 90% efficiency when $f_T = 0.1 f_0$.