Web-based Appendix for "Multivariate Bayesian Variable Selection Exploiting Dependence Structure Among Outcomes: Application to Air Pollution Effects on DNA Methylation"

Kyu Ha Lee
Epidemiology and Biostatistics Core, The Forsyth Institute, Cambridge, Massachusetts, U.S.A.
Department of Oral Health Policy and Epidemiology, Harvard School of Dental Medicine, Boston, Massachusetts, U.S.A.
klee@forsyth.org

Mahlet G. Tadesse
Department of Mathematics and Statistics, Georgetown University, Washington, DC, U.S.A.

Andrea A. Baccarelli
Department of Environmental Health,
Department of Epidemiology,
Harvard T. H. Chan School of Public Health, Boston, Massachusetts, U.S.A.

Joel Schwartz
Department of Environmental Health,
Department of Epidemiology,
Harvard T. H. Chan School of Public Health, Boston, Massachusetts, U.S.A.

Brent A. Coull
Department of Biostatistics,
Department of Environmental Health,
Harvard T. H. Chan School of Public Health, Boston, Massachusetts, U.S.A.

Introduction

We note that, to distinguish the two documents, alpha-numeric labels are used for sections, tables, figures, and equations in this Appendix while numeric labels are used in the main paper.

This document is organized as follows. In Section A, we provide a detailed description of Markov chain Monte Carlo (MCMC) algorithms to implement our proposed Bayesian framework. In Section B, we present the results from simulation studies under Scenarios III and IV as well as those not presented in the main paper for Scenarios I and II. We also provide the results from additional simulation studies where data are generated under a factor-analytic covariance structure. In Section C, we provide supplementary results and a visual assessment of convergence for the proposed MCMC scheme using trace plots for the application presented in the main paper.

A Computational Scheme

Let D denote a set of observations $\{D_i{=}(\boldsymbol{y}_i,\ \boldsymbol{x}_i){:}i{=}1,\ldots,n\}$.

A.1 Unstructured Covariance Model

A.1.1 Updating $\boldsymbol{\beta}_j$ and $\boldsymbol{\gamma}_j$, for $j = 1, \ldots, q$

At iteration t, we repeat the following random walk Metropolis-Hastings (MH) algorithm for each $j{=}1,\ldots,q$: Let $\boldsymbol{\beta}_j^{(t-1)}$ and $\boldsymbol{\gamma}_j^{(t-1)}$ denote the sample values from the previous iteration in an MCMC chain. Note that $\boldsymbol{\gamma}_j{=}(\gamma_{j,1},\ldots,\gamma_{j,p})^\top$ is the vector of latent variables corresponding to $\boldsymbol{\beta}_j$.

i. We determine a proposal $\boldsymbol{\gamma}_j^*$ by randomly selecting one of the following move types:

   - add/delete : randomly choose one of the p elements in $\boldsymbol{\gamma}_j^{(t-1)}$ and change its value from 1 to 0, or from 0 to 1.
   - swap (only when $p \geq 2$): randomly select a 1 and a 0 in $\boldsymbol{\gamma}_j^{(t-1)}$, then switch their values.

ii. Based on $\boldsymbol{\gamma}_j^*$, we generate (or determine) a proposal value $\boldsymbol{\beta}_j^*$ as follows:

   - If l-th element of $\boldsymbol{\gamma}_j^{(t-1)}$ is 'added', set $\boldsymbol{\beta}_j^*{=}\boldsymbol{\beta}_j^{(t-1)}$ except that $\beta_{j,l}^*$ is drawn from a Normal proposal distribution, $\mathcal{N}(\beta_{j,l}^{(t-1)},\ V_\beta)$, where V_β is the prespecified variance of the proposal density.
   - If l-th element of $\boldsymbol{\gamma}_j^{(t-1)}$ is 'deleted', set $\boldsymbol{\beta}_j^*{=}\boldsymbol{\beta}_j^{(t-1)}$ except that $\beta_{j,l}^*{=}0$.
   - If l-th element $\gamma_{j,l}^{(t-1)}{=}0$ is 'swapped' with m-th element $\gamma_{j,m}^{(t-1)}{=}1$, set $\boldsymbol{\beta}_j^*{=}\boldsymbol{\beta}_j^{(t-1)}$ except that $\beta_{j,l}^*$ is drawn from $\mathcal{N}(\beta_{j,l}^{(t-1)},\ V_\beta)$ and $\beta_{j,m}^*{=}0$.

iii. The acceptance probability for $(\boldsymbol{\beta}_j^*\ ,\ \boldsymbol{\gamma}_j^*)$ in the MH step is computed as the product of the likelihood ratio, prior ratio, proposal ratio. Let $B^{(t-1)}$ denote the $p{\times}q$ matrix whose columns are $\boldsymbol{\beta}_1^{(t-1)},\ldots,\boldsymbol{\beta}_q^{(t-1)}$. B^* is the matrix $B^{(t-1)}$ with the j-th column replaced by $\boldsymbol{\beta}_j^*$. The likelihood ratio is given by

$$\begin{aligned}
\text{likelihood ratio} &= \frac{\prod_{i=1}^n L(D_i|\boldsymbol{\beta}_0, B^*, \Sigma)}{\prod_{i=1}^n L(D_i|\boldsymbol{\beta}_0, B^{(t-1)}, \Sigma)} \\
&= \frac{\exp\left[-\frac{1}{2}\sum_{i=1}^n \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - (B^*)^\top \boldsymbol{x}_i\right\}^\top \Sigma^{-1} \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - (B^*)^\top \boldsymbol{x}_i\right\}\right]}{\exp\left[-\frac{1}{2}\sum_{i=1}^n \{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - (B^{(t-1)})^\top \boldsymbol{x}_i\}^\top \Sigma^{-1} \{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - (B^{(t-1)})^\top \boldsymbol{x}_i\}\right]}.
\end{aligned}$$

For "add" move ($\gamma_{j,l}^{(t-1)} = 0,\ \gamma_{j,l}^{*} = 1$), the prior ratio and the proposal ratio are given by

$$\begin{aligned}
\text{prior ratio} &= \frac{\pi(\beta_{j,l}^{*}|\gamma_{j,l}^{*}, \Sigma)}{\pi(\beta_{j,l}^{(t-1)}|\gamma_{j,l}^{(t-1)}, \Sigma)} \times \frac{\pi(\gamma_{j,l}^{*}|\boldsymbol{\gamma}_{(-j),l}, \Sigma)}{\pi(\gamma_{j,l}^{(t-1)}|\boldsymbol{\gamma}_{(-j),l}, \Sigma)} \\
&= \frac{\frac{1}{\sqrt{2\pi\Sigma_{(j,j)}\nu_j^2}} \exp\left\{-\frac{1}{2\Sigma_{(j,j)}\nu_j^2}(\beta_{j,l}^{*})^2\right\}}{1} \times \text{logit}^{-1}\left(\omega_l + \eta \sum_{r\neq j} |c_{r,l}|\gamma_{r,l}\right) \\
&\quad \times \left\{1 - \text{logit}^{-1}\left(\omega_l + \eta \sum_{r\neq j} |c_{r,l}|\gamma_{r,l}\right)\right\}^{-1},
\end{aligned}$$

$$\begin{aligned}
\text{proposal ratio} &= \frac{P(\text{delete}) \times 1/(g+1)}{P(\text{add}) \times 1/(q-g)} \times \frac{q_1(\boldsymbol{\beta}_j^{*} \to \boldsymbol{\beta}_j^{(t-1)}|\boldsymbol{\gamma}_j^{*})}{q_1(\boldsymbol{\beta}_j^{(t-1)} \to \boldsymbol{\beta}_j^{*}|\boldsymbol{\gamma}_j^{(t-1)})} \times \frac{q_2(\boldsymbol{\gamma}_j^{*} \to \boldsymbol{\gamma}_j^{(t-1)})}{q_2(\boldsymbol{\gamma}_j^{(t-1)} \to \boldsymbol{\gamma}_j^{*})} \\
&= \frac{1/(g+1)}{1/(q-g)} \times \frac{1}{\frac{1}{\sqrt{2\pi V_\beta}} \exp\left\{-\frac{1}{2V_\beta}(\beta_{j,l}^{*} - \beta_{j,l}^{(t-1)})^2\right\}} \times 1,
\end{aligned}$$

where g is the number of non-zero values in $\boldsymbol{\gamma}_j^{(t-1)}$. For "delete" move ($\gamma_{j,l}^{(t-1)} = 1,\ \gamma_{j,l}^{*} = 0$), the prior ratio and the proposal ratio are given by

$$\begin{aligned}
\text{prior ratio} &= \frac{1}{\sqrt{2\pi\Sigma_{(j,j)}\nu_j^2} \exp\left\{-\frac{1}{2\Sigma_{(j,j)}\nu_j^2}(\beta_{j,l}^{(t-1)})^2\right\}} \times \left\{\text{logit}^{-1}\left(\omega_l + \eta \sum_{r\neq j} |c_{r,l}|\gamma_{r,l}\right)\right\}^{-1} \\
&\quad \times \left\{1 - \text{logit}^{-1}\left(\omega_l + \eta \sum_{r\neq j} |c_{r,l}|\gamma_{r,l}\right)\right\},
\end{aligned}$$

$$\text{proposal ratio} = \frac{1/(q-g+1)}{1/g} \times \frac{\frac{1}{\sqrt{2\pi V_\beta}} \exp\left\{-\frac{1}{2V_\beta}(\beta_{j,l}^{(t-1)} - \beta_{j,l}^{*})^2\right\}}{1} \times 1.$$

For the "swap" move ($\gamma_{j,l}^{(t-1)}$=0, $\gamma_{j,m}^{(t-1)}$=1), the prior ratio and the proposal ratio are given by

$$\begin{aligned}
\text{prior ratio} &= \frac{\exp\left\{-\frac{1}{2\Sigma_{(j,j)}\nu_j^2}(\beta_{j,l}^*)^2\right\}\times 1}{1\times\exp\left\{-\frac{1}{2\Sigma_{(j,j)}\nu_j^2}(\beta_{j,m}^{(t-1)})^2\right\}}\times \text{logit}^{-1}\left(\omega_l+\eta\sum_{r\neq j}|c_{r,l}|\gamma_{r,l}\right)\\
&\times\left\{1-\text{logit}^{-1}\left(\omega_l+\eta\sum_{r\neq j}|c_{r,l}|\gamma_{r,l}\right)\right\}^{-1}\\
&\times\left\{\text{logit}^{-1}\left(\omega_m+\eta\sum_{r\neq j}|c_{r,m}|\gamma_{r,m}\right)\right\}^{-1}\\
&\times\left\{1-\text{logit}^{-1}\left(\omega_m+\eta\sum_{r\neq j}|c_{r,m}|\gamma_{r,m}\right)\right\},\\
\text{proposal ratio} &= \frac{\exp\left\{-\frac{1}{2V_\beta}(\beta_{j,m}^{(t-1)}-\beta_{j,m}^*)^2\right\}}{\exp\left\{-\frac{1}{2V_\beta}(\beta_{j,l}^*-\beta_{j,l}^{(t-1)})^2\right\}}\times 1.
\end{aligned}$$

A.1.2 Updating $\boldsymbol{\beta}_0$

The full conditional posterior distribution of $\boldsymbol{\beta}_0$ can be obtained by

$$\begin{aligned}
&\pi(\boldsymbol{\beta}_0|\boldsymbol{\beta}_1,\ldots,\boldsymbol{\beta}_q,\Sigma,\boldsymbol{\gamma}_1,\ldots,\boldsymbol{\gamma}_q,D)\\
\propto\ &\prod_{i=1}^n L(D_i|\boldsymbol{\beta}_0,\boldsymbol{\beta}_1,\ldots,\boldsymbol{\beta}_q,\Sigma)\pi(\boldsymbol{\beta}_0)\\
\propto\ &\exp\left\{-\frac{1}{2}\sum_{i=1}^n(\boldsymbol{y}_i-\boldsymbol{\beta}_0-B^\top\boldsymbol{x}_i)^\top\Sigma^{-1}(\boldsymbol{y}_i-\boldsymbol{\beta}_0-B^\top\boldsymbol{x}_i)\right\}\times\exp\left\{-\frac{1}{2h_0}(\boldsymbol{\beta}_0-\boldsymbol{\mu}_0)^\top(\boldsymbol{\beta}_0-\boldsymbol{\mu}_0)\right\}.
\end{aligned}$$

Therefore, we sample $\boldsymbol{\beta}_0$ from a multivariate Normal density given by

$$\mathcal{N}_q\left(\left(n\Sigma^{-1}+\frac{1}{h_0}I_q\right)^{-1}\left\{\Sigma^{-1}\left(\sum_{i=1}^n\boldsymbol{y}_i-B^\top\sum_{i=1}^n\boldsymbol{x}_i\right)+\frac{1}{h_0}\boldsymbol{\mu}_0\right\},\left(n\Sigma^{-1}+\frac{1}{h_0}I_q\right)^{-1}\right).$$

A.1.3 Updating Σ

While the inverse-Wishart is a conjugate prior for the variance-covariance matrix of a multivariate Normal distribution, our proposed framework cannot exploit the prior-posterior conjugacy because the Markov random field (MRF) prior for $\gamma_{j,k}$ involves Σ. Therefore, we update Σ using a random-walk MH step following Browne (2006). Letting $\Sigma^{(t-1)}$ denote the sample of Σ at the previous iteration $(t-1)$, we draw the proposal value Σ^* from

$$\text{inverse-Wishart}(\rho^*\Sigma^{(t-1)},\rho^*+q+1),$$

where ρ^* is a tuning constant of the proposal density. The acceptance probability for Σ^* is computed as the product of the following likelihood ratio, prior ratio, and proposal ratio:

$$\begin{aligned}\text{likelihood ratio} &= \frac{\prod_{i=1}^n L(D_i|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \Sigma^*)}{\prod_{i=1}^n L(D_i|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \Sigma^{(t-1)})} \\ &\propto \frac{|\Sigma^*|^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^n \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i\right\}^\top (\Sigma^*)^{-1} \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i\right\}\right]}{|\Sigma^{(t-1)}|^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^n \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i\right\}^\top (\Sigma^{(t-1)})^{-1} \left\{\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i\right\}\right]},\end{aligned}$$

$$\begin{aligned}\text{prior ratio} &= \frac{\text{inverse-Wishart}(\Sigma^*|\Psi_0, \rho_0)}{\text{inverse-Wishart}(\Sigma^{(t-1)}|\Psi_0, \rho_0)} \times \prod_{j=1}^q \prod_{k=1}^p \frac{\pi(\gamma_{j,k}|\boldsymbol{\gamma}_{(-j),k}, \Sigma^*)}{\pi(\gamma_{j,k}^{(t-1)}|\boldsymbol{\gamma}_{(-j),k}, \Sigma^{(t-1)})} \\ &\propto \frac{|\Sigma^*|^{-\frac{\rho_0+q+1}{2}}}{|\Sigma^{(t-1)}|^{-\frac{\rho_0+q+1}{2}}} \exp\left[-\frac{1}{2}\text{tr}\left\{\Psi_0(\Sigma^*)^{-1}\right\} + \frac{1}{2}\text{tr}\left\{\Psi_0(\Sigma^{(t-1)})^{-1}\right\}\right] \\ &\quad \times \prod_{j=1}^q \prod_{k=1}^p \left\{\frac{\text{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}|c_{r,k}|\gamma_{r,k}\right)}{\text{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}|c_{r,k}|\gamma_{r,k}\right)}\right\}^{\gamma_{j,k}} \\ &\quad \times \left\{\frac{1-\text{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}|c_{r,k}|\gamma_{r,k}\right)}{1-\text{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}|c_{r,k}|\gamma_{r,k}\right)}\right\}^{1-\gamma_{j,k}},\end{aligned}$$

$$\begin{aligned}\text{proposal ratio} &= \frac{\text{inverse-Wishart}(\Sigma^{(t-1)}|\rho^*\Sigma^*, \rho^* + q + 1)}{\text{inverse-Wishart}(\Sigma^*|\rho^*\Sigma^{(t-1)}, \rho^* + q + 1)} \\ &\propto \frac{|\Sigma^{(t-1)}|^{-\frac{2\rho^*+3q+3}{2}}}{|\Sigma^*|^{-\frac{2\rho^*+3q+3}{2}}} \exp\left[-\frac{\rho^*}{2}\text{tr}\left\{\Sigma^*(\Sigma^{(t-1)})^{-1}\right\} + \frac{\rho^*}{2}\text{tr}\left\{\Sigma^{(t-1)}(\Sigma^*)^{-1}\right\}\right].\end{aligned}$$

A.2 Factor-analytic Model

A.2.1 Updating $\boldsymbol{\lambda}$

The full conditional posterior distribution of $\boldsymbol{\lambda}$ is given by

$$
\begin{aligned}
& \pi(\boldsymbol{\lambda}|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \sigma^2, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_q, D) \\
\propto & \prod_{i=1}^{n} L(\mathrm{D}_i|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \boldsymbol{\lambda}, \sigma^2) \prod_{j=1}^{q}\prod_{k=1}^{p} \pi(\gamma_{j,k}|\boldsymbol{\gamma}_{(-j),k}, \boldsymbol{\lambda})\pi(\boldsymbol{\lambda}|\sigma^2) \\
\propto & |\Sigma_\lambda|^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i)^\top \Sigma_\lambda^{-1}(\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i)\right\} \\
& \times \prod_{j=1}^{q}\prod_{k=1}^{p}\left\{\mathrm{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}\frac{|\lambda_j\lambda_r|}{\sqrt{\lambda_j^2+1}\sqrt{\lambda_r^2+1}}\gamma_{r,k}\right)\right\}^{\gamma_{j,k}} \\
& \times \left\{1 - \mathrm{logit}^{-1}\left(\omega_k + \eta\sum_{r\neq j}\frac{|\lambda_j\lambda_r|}{\sqrt{\lambda_j^2+1}\sqrt{\lambda_r^2+1}}\gamma_{r,k}\right)\right\}^{1-\gamma_{j,k}} \\
& \times \exp\left\{-\frac{1}{2h_\lambda\sigma^2}(\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda)^\top(\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda)\right\},
\end{aligned}
$$

where Σ_λ=$(\boldsymbol{\lambda}\boldsymbol{\lambda}^\top$+$I_q)$. Since the full conditional does not have a standard form, we use the following random walk MH algorithm:

i. We first randomly select j from $\{1, \ldots, q\}$.

ii. The proposal value, λ_j^* is drawn from a Normal proposal distribution, $\mathcal{N}(\lambda_j^{(t-1)}, V_\lambda)$, where V_λ is the variance of the proposal density. Let $\boldsymbol{\lambda}^*$ be the $\boldsymbol{\lambda}^{(t-1)}$ with j-th element replaced by λ_j^*.

iii. we accept $\boldsymbol{\lambda}^*$ with probability $\min(1, P_{(\lambda)})$, where

$$
P_{(\lambda)} = \frac{\pi(\boldsymbol{\lambda}^*|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \sigma^2, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_q, D)}{\pi(\boldsymbol{\lambda}^{(t-1)}|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \sigma^2, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_q, D)}.
$$

A.2.2 Updating σ^2

The full conditional posterior distribution of σ^2 is given by

$$
\begin{aligned}
& \pi(\sigma^2|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \boldsymbol{\lambda}, \boldsymbol{\gamma}_1, \ldots, \boldsymbol{\gamma}_q, D) \\
\propto \; & \prod_{i=1}^{n} L(D_i|\boldsymbol{\beta}_0, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_q, \boldsymbol{\lambda}, \sigma^2) \prod_{j=1}^{q} \pi(\boldsymbol{\beta}_j|\boldsymbol{\gamma}_j, \sigma^2)\pi(\boldsymbol{\lambda}|\sigma^2)\pi(\sigma^2) \\
\propto \; & (\sigma^2)^{-\frac{nq}{2}} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i)^\top \Sigma_\lambda^{-1} (\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i) \right\} \\
& \times (\sigma^2)^{-\frac{pq}{2}} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{j=1}^{q} \frac{1}{v_j^2} \boldsymbol{\beta}_j^\top \boldsymbol{\beta}_j \right\} (\sigma^2)^{-\frac{q}{2}} \exp\left\{ -\frac{1}{2h_\lambda \sigma^2} (\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda)^\top (\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda) \right\} \\
& \times (\sigma^2)^{-\frac{\nu_0}{2}-1} \exp\left(-\frac{\nu_0 \sigma_0^2}{2\sigma^2} \right).
\end{aligned}
$$

Therefore, we sample σ^2 from an inverse-Gamma distribution given by

$$
\begin{aligned}
\mathcal{IG}\Bigg(\frac{nq + pq + q + \nu_0}{2}, \;\; \frac{1}{2} \Bigg\{ & \sum_{i=1}^{n} (\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i)^\top \Sigma_\lambda^{-1} (\boldsymbol{y}_i - \boldsymbol{\beta}_0 - B^\top \boldsymbol{x}_i) \\
& + \sum_{j=1}^{q} \frac{1}{v_j^2} \boldsymbol{\beta}_j^\top \boldsymbol{\beta}_j + \frac{1}{h_\lambda} (\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda)^\top (\boldsymbol{\lambda} - \boldsymbol{\mu}_\lambda) + \nu_0 \sigma_0^2 \Bigg\} \Bigg)
\end{aligned}
$$

A.2.3 Updating $\boldsymbol{\beta}_0$

The full conditional posterior distribution of $\boldsymbol{\beta}_0$ is the same as the one given in Section A.1.2, considering that $\Sigma = \sigma^2(\boldsymbol{\lambda}\boldsymbol{\lambda}^\top + I_q)$ for factor-analytic model.

A.2.4 Updating $\boldsymbol{\beta}_j$ and $\boldsymbol{\gamma}_j$

The random walk MH step to update $\boldsymbol{\beta}_j$ and $\boldsymbol{\gamma}_j$ is the same as the one given in Section A.1.1, considering that $\Sigma = \sigma^2(\boldsymbol{\lambda}\boldsymbol{\lambda}^\top + I_q)$ for factor-analytic model.

B Simulation Studies

B.1 Additional Results from Scenarios I and II

Table B.1 and Table B.2 provide the results related to covariate x_2 for our proposed Bayesian method under Scenario I and II when Cor(x_1, x_2)=0.3. The results are generally consistent with the conclusions we drew for x_1 in the main paper: our proposed MRF prior has a better capability to detect the true association by yielding higher inclusion probabilities without increasing the probability of false discoveries. We also provide the results when Cor(x_1, x_2)=0.6 in Tables B.3-B.6.

Table B.1: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I	1	0.05	0.04 (0.03)	0.02	0.02	0.18	0.05 (0.04)	0.20	0.08	0.60
	2	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.13	0.06	1.00
	3	0.15	0.15 (0.03)	0.99	0.13	0.80	0.15 (0.03)	0.99	0.14	0.85
	4	0.00	-0.01 (0.04)	0.01	0.01	1.00	-0.00 (0.04)	0.05	0.04	1.00
	5	0.25	0.23 (0.03)	1.00	0.14	0.87	0.24 (0.04)	1.00	0.15	0.90
	6	0.10	0.08 (0.05)	0.06	0.07	0.34	0.09 (0.05)	0.19	0.11	0.57
	7	0.20	0.19 (0.05)	0.93	0.20	0.81	0.19 (0.05)	0.97	0.20	0.88
	8	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.06	0.04	1.00
	9	0.00	-0.02 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.06	0.04	1.00
	10	0.00	-0.02 (0.05)	0.02	0.01	1.00	-0.01 (0.05)	0.06	0.04	1.00
II	1	0.00	0.00 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.08	0.04	1.00
	2	0.00	-0.00 (0.03)	0.01	0.01	1.00	0.01 (0.04)	0.08	0.04	1.00
	3	0.00	0.00 (0.03)	0.01	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.07	0.04	1.00
	5	0.00	-0.00 (0.03)	0.01	0.01	1.00	-0.01 (0.04)	0.08	0.04	1.00
	6	0.00	-0.00 (0.04)	0.02	0.01	1.00	0.00 (0.05)	0.04	0.02	1.00
	7	0.00	0.01 (0.04)	0.02	0.01	1.00	0.00 (0.05)	0.04	0.03	1.00
	8	0.00	-0.01 (0.04)	0.02	0.02	1.00	-0.00 (0.05)	0.05	0.03	1.00
	9	0.00	-0.00 (0.05)	0.02	0.01	1.00	-0.00 (0.05)	0.04	0.03	1.00
	10	0.00	0.00 (0.05)	0.02	0.01	1.00	0.01 (0.05)	0.05	0.03	1.00

Table B.2: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.02 (0.05)	0.02	0.01	0.06	0.04 (0.05)	0.12	0.08	0.54
	2	0.00	-0.02 (0.05)	0.02	0.01	1.00	-0.01 (0.05)	0.10	0.06	1.00
	3	0.15	0.16 (0.04)	0.66	0.18	0.78	0.16 (0.05)	0.74	0.18	0.81
	4	0.00	-0.01 (0.05)	0.01	0.01	1.00	-0.00 (0.05)	0.05	0.04	1.00
I	5	0.25	0.23 (0.05)	1.00	0.21	0.85	0.24 (0.05)	1.00	0.21	0.91
	6	0.10	0.08 (0.04)	0.10	0.08	0.43	0.08 (0.04)	0.23	0.11	0.60
	7	0.20	0.18 (0.04)	1.00	0.17	0.84	0.19 (0.04)	1.00	0.17	0.89
	8	0.00	-0.03 (0.04)	0.02	0.03	1.00	-0.02 (0.04)	0.08	0.05	1.00
	9	0.00	-0.03 (0.04)	0.02	0.03	1.00	-0.02 (0.04)	0.06	0.05	1.00
	10	0.00	-0.02 (0.04)	0.02	0.02	1.00	-0.01 (0.04)	0.06	0.04	1.00
	1	0.00	0.00 (0.04)	0.02	0.00	1.00	-0.00 (0.05)	0.06	0.03	1.00
	2	0.00	0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	3	0.00	0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	4	0.00	0.00 (0.04)	0.02	0.00	1.00	-0.00 (0.05)	0.06	0.03	1.00
II	5	0.00	0.00 (0.04)	0.02	0.00	1.00	-0.00 (0.05)	0.06	0.04	1.00
	6	0.00	0.00 (0.04)	0.02	0.02	1.00	0.00 (0.04)	0.04	0.03	1.00
	7	0.00	0.01 (0.04)	0.02	0.01	1.00	0.01 (0.04)	0.04	0.03	1.00
	8	0.00	-0.00 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.04	1.00
	9	0.00	0.00 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.03	1.00
	10	0.00	0.01 (0.04)	0.02	0.02	1.00	0.00 (0.04)	0.05	0.03	1.00

Table B.3: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I	1	0.05	-0.03 (0.04)	0.03	0.04	0.06	0.03 (0.04)	0.28	0.11	0.51
	2	0.10	0.02 (0.04)	0.02	0.04	0.15	0.08 (0.04)	0.36	0.12	0.61
	3	0.15	0.10 (0.04)	0.08	0.09	0.37	0.13 (0.04)	0.71	0.14	0.61
	4	0.20	0.13 (0.03)	0.86	0.13	0.38	0.16 (0.04)	0.99	0.17	0.70
	5	0.25	0.18 (0.04)	0.99	0.16	0.50	0.21 (0.04)	1.00	0.18	0.71
	6	0.00	0.01 (0.05)	0.02	0.01	1.00	0.02 (0.05)	0.06	0.03	1.00
	7	0.00	-0.02 (0.05)	0.01	0.01	1.00	-0.01 (0.05)	0.02	0.02	1.00
	8	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.06	0.05	1.00
	9	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.06	0.04	1.00
	10	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.07	0.05	1.00
II	1	0.00	0.00 (0.03)	0.01	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	2	0.00	0.00 (0.03)	0.01	0.01	1.00	0.01 (0.04)	0.07	0.04	1.00
	3	0.00	0.00 (0.03)	0.01	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	4	0.00	-0.00 (0.03)	0.01	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	5	0.00	0.01 (0.03)	0.01	0.00	1.00	0.00 (0.04)	0.07	0.04	1.00
	6	0.00	0.01 (0.04)	0.02	0.01	1.00	0.00 (0.04)	0.04	0.02	1.00
	7	0.00	-0.01 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.03	1.00
	8	0.00	0.01 (0.04)	0.02	0.01	1.00	0.02 (0.05)	0.05	0.03	1.00
	9	0.00	0.00 (0.05)	0.02	0.01	1.00	0.00 (0.05)	0.04	0.02	1.00
	10	0.00	-0.01 (0.05)	0.02	0.01	1.00	-0.01 (0.05)	0.04	0.02	1.00

Table B.4: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.01 (0.05)	0.03	0.02	0.13	0.04 (0.05)	0.12	0.09	0.52
	2	0.10	0.04 (0.05)	0.03	0.04	0.14	0.07 (0.05)	0.14	0.10	0.49
	3	0.15	0.17 (0.05)	0.26	0.16	0.67	0.18 (0.06)	0.42	0.18	0.69
	4	0.20	0.14 (0.05)	0.59	0.19	0.54	0.16 (0.05)	0.81	0.20	0.69
I	5	0.25	0.23 (0.06)	0.69	0.25	0.69	0.24 (0.06)	0.88	0.26	0.72
	6	0.00	0.01 (0.05)	0.02	0.01	1.00	0.02 (0.05)	0.06	0.04	1.00
	7	0.00	-0.01 (0.06)	0.01	0.02	1.00	-0.01 (0.06)	0.02	0.03	1.00
	8	0.00	-0.04 (0.04)	0.03	0.03	1.00	-0.03 (0.04)	0.07	0.05	1.00
	9	0.00	-0.04 (0.04)	0.03	0.03	1.00	-0.02 (0.04)	0.06	0.04	1.00
	10	0.00	-0.04 (0.04)	0.03	0.04	1.00	-0.03 (0.04)	0.06	0.05	1.00
	1	0.00	-0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	2	0.00	0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	3	0.00	0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	4	0.00	0.01 (0.04)	0.02	0.00	1.00	0.01 (0.05)	0.06	0.03	1.00
II	5	0.00	0.01 (0.04)	0.02	0.00	1.00	0.01 (0.05)	0.06	0.03	1.00
	6	0.00	0.00 (0.04)	0.02	0.01	1.00	0.01 (0.04)	0.05	0.03	1.00
	7	0.00	0.00 (0.04)	0.02	0.01	1.00	0.00 (0.04)	0.05	0.03	1.00
	8	0.00	0.01 (0.04)	0.02	0.02	1.00	0.01 (0.04)	0.05	0.03	1.00
	9	0.00	0.01 (0.04)	0.02	0.01	1.00	0.00 (0.04)	0.05	0.03	1.00
	10	0.00	-0.01 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.03	1.00

Table B.5: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I	1	0.05	0.04 (0.03)	0.02	0.02	0.18	0.05 (0.04)	0.20	0.08	0.60
	2	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.13	0.06	1.00
	3	0.15	0.15 (0.03)	0.99	0.13	0.80	0.15 (0.03)	0.99	0.14	0.85
	4	0.00	-0.01 (0.04)	0.01	0.01	1.00	-0.00 (0.04)	0.05	0.04	1.00
	5	0.25	0.23 (0.03)	1.00	0.14	0.87	0.24 (0.04)	1.00	0.15	0.90
	6	0.10	0.08 (0.05)	0.06	0.07	0.34	0.09 (0.05)	0.19	0.11	0.57
	7	0.20	0.19 (0.05)	0.93	0.20	0.81	0.19 (0.05)	0.97	0.20	0.88
	8	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.06	0.04	1.00
	9	0.00	-0.02 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.06	0.04	1.00
	10	0.00	-0.02 (0.05)	0.02	0.01	1.00	-0.01 (0.05)	0.06	0.04	1.00
II	1	0.00	0.00 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.08	0.04	1.00
	2	0.00	-0.00 (0.03)	0.01	0.01	1.00	0.01 (0.04)	0.08	0.04	1.00
	3	0.00	0.00 (0.03)	0.01	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.07	0.04	1.00
	5	0.00	-0.00 (0.03)	0.01	0.01	1.00	-0.01 (0.04)	0.08	0.04	1.00
	6	0.00	-0.00 (0.04)	0.02	0.01	1.00	0.00 (0.05)	0.04	0.02	1.00
	7	0.00	0.01 (0.04)	0.02	0.01	1.00	0.00 (0.05)	0.04	0.03	1.00
	8	0.00	-0.01 (0.04)	0.02	0.02	1.00	-0.00 (0.05)	0.05	0.03	1.00
	9	0.00	-0.00 (0.05)	0.02	0.01	1.00	-0.00 (0.05)	0.04	0.03	1.00
	10	0.00	0.00 (0.05)	0.02	0.01	1.00	0.01 (0.05)	0.05	0.03	1.00

Table B.6: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I and II where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I	1	0.05	0.02 (0.05)	0.02	0.02	0.11	0.05 (0.05)	0.11	0.08	0.58
	2	0.00	0.00 (0.05)	0.02	0.01	1.00	0.01 (0.05)	0.09	0.06	1.00
	3	0.15	0.18 (0.05)	0.62	0.19	0.78	0.18 (0.05)	0.62	0.19	0.80
	4	0.00	0.03 (0.06)	0.02	0.02	1.00	0.04 (0.06)	0.07	0.07	1.00
	5	0.25	0.27 (0.06)	0.97	0.26	0.78	0.26 (0.06)	0.98	0.26	0.80
	6	0.10	0.08 (0.04)	0.08	0.08	0.39	0.09 (0.04)	0.24	0.11	0.55
	7	0.20	0.17 (0.04)	0.98	0.18	0.82	0.18 (0.04)	1.00	0.18	0.88
	8	0.00	-0.04 (0.04)	0.03	0.04	1.00	-0.02 (0.04)	0.06	0.05	1.00
	9	0.00	-0.04 (0.04)	0.02	0.03	1.00	-0.02 (0.04)	0.07	0.04	1.00
	10	0.00	-0.03 (0.04)	0.02	0.03	1.00	-0.02 (0.04)	0.06	0.04	1.00
II	1	0.00	-0.00 (0.04)	0.02	0.00	1.00	-0.00 (0.05)	0.06	0.03	1.00
	2	0.00	0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	3	0.00	-0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	4	0.00	-0.00 (0.04)	0.02	0.00	1.00	0.00 (0.05)	0.06	0.03	1.00
	5	0.00	0.01 (0.04)	0.02	0.00	1.00	0.01 (0.05)	0.06	0.03	1.00
	6	0.00	0.00 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.04	1.00
	7	0.00	0.00 (0.04)	0.02	0.01	1.00	0.00 (0.04)	0.04	0.02	1.00
	8	0.00	0.01 (0.04)	0.02	0.02	1.00	0.01 (0.04)	0.05	0.03	1.00
	9	0.00	-0.00 (0.04)	0.02	0.01	1.00	-0.00 (0.04)	0.05	0.03	1.00
	10	0.00	-0.00 (0.04)	0.02	0.02	1.00	-0.00 (0.04)	0.05	0.03	1.00

B.2 Results from Scenarios III and IV

Tables B.7-B.10 provide the results for our proposed Bayesian method under Scenario III and IV when Cor(x_1, x_2)=0.3 and Tables B.11-B.14 present those when Cor(x_1, x_2)=0.6: in Scenario III, outcomes associated with x_1 are moderately correlated and the other outcomes are slightly correlated; in Scenario IV, outcomes associated with x_1 are weakly correlated while the remaining ones are highly correlated. For a more detailed description of the simulation settings, we refer to Section 3.1 of the main paper.

Since the results are almost the same between Cor(x_1, x_2)=0.3 and 0.6, we mainly discuss the results for covariate x_1 and for Cor(x_1, x_2)=0.3 presented in Table B.7 and Table B.8 in this document. First, we compare the unstructured model with independent Bernoulli (IB) prior to the unstructured model with MRF prior to examine the performance of the proposed MRF prior when the model is correctly specified. In Scenario III, we can see that the estimates of $\beta_{j,1}$, conditioning on $\gamma_{j,1}$ and the associated posterior uncertainties, measured by SD and WI are quite similar between IB and MRF priors. However, our proposed framework yields higher inclusion probabilities for smaller effects ($\hat{\gamma}_{j,1}$=0.15, 0.46, 0.88 for j=1,...,3) compared to IB prior ($\hat{\gamma}_{j,1}$=0.02, 0.10, 0.70 for j=1,...,3). In Scenario IV, the estimates of $\beta_{j,1}$ and the associated uncertainties appear to be similar between the two priors while we could see a slight increase in the estimated inclusion probabilities for the MRF prior (e.g. $\gamma_{3,1}$=0.46 for IB prior and 0.54 for the MRF prior). This is because in Scenario IV the correlation between outcomes relevant to x_1 is relatively weak (=0.20) so the MRF prior can borrow only little amount of strength to improve the power to detect subtle effects.

The factor-analytic model with the MRF prior is compared with the unstructured model with the MRF prior to investigate if model misspecification could affect the performance of the MRF prior. In Scenario IV, we see that the estimated inclusion probabilities are generally a bit higher for the factor-analytic model. This indicates that although the model is misspecified, presumably the gain in parsimony outweighs the amount of misspecification for the factor-analytic model. In fact, the degree of misspecification is particularly small in Scenario III because the true correlation matrix for outcomes is very close to the correlation matrix from a factor-analytic model with $\boldsymbol{\lambda}$=(0.8, 0.8, 0.8, 0.8, 0.8, 0.2,..., 0.2)$^\top$.

From Scenarios I-IV, we can see that our proposed MRF prior shows better performance than the IB prior whether or not the covariance structure is correctly specified.

Table B.7: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.04 (0.04)	0.02	0.02	0.20	0.05 (0.04)	0.15	0.09	0.65
	2	0.10	0.08 (0.04)	0.10	0.08	0.45	0.09 (0.04)	0.46	0.13	0.70
	3	0.15	0.16 (0.04)	0.70	0.14	0.60	0.16 (0.04)	0.88	0.16	0.72
	4	0.20	0.17 (0.04)	0.98	0.17	0.75	0.18 (0.04)	1.00	0.18	0.85
III	5	0.25	0.25 (0.04)	1.00	0.19	0.86	0.25 (0.05)	1.00	0.19	0.91
	6	0.00	-0.02 (0.05)	0.02	0.02	1.00	-0.00 (0.05)	0.08	0.06	1.00
	7	0.00	-0.04 (0.05)	0.01	0.02	1.00	-0.02 (0.06)	0.03	0.03	1.00
	8	0.00	-0.06 (0.05)	0.03	0.05	1.00	-0.03 (0.05)	0.09	0.08	1.00
	9	0.00	-0.06 (0.05)	0.03	0.05	1.00	-0.02 (0.05)	0.08	0.08	1.00
	10	0.00	-0.07 (0.05)	0.03	0.06	1.00	-0.04 (0.05)	0.10	0.09	1.00
	1	0.05	0.05 (0.04)	0.02	0.03	0.24	0.05 (0.04)	0.05	0.05	0.36
	2	0.10	0.10 (0.04)	0.11	0.10	0.50	0.10 (0.04)	0.22	0.12	0.59
	3	0.15	0.16 (0.05)	0.46	0.15	0.57	0.16 (0.05)	0.54	0.16	0.57
	4	0.20	0.18 (0.05)	0.97	0.19	0.76	0.19 (0.05)	0.98	0.19	0.78
IV	5	0.25	0.26 (0.05)	1.00	0.22	0.86	0.25 (0.05)	1.00	0.21	0.88
	6	0.00	0.02 (0.03)	0.01	0.00	1.00	0.01 (0.04)	0.02	0.01	1.00
	7	0.00	-0.00 (0.03)	0.00	0.00	1.00	-0.00 (0.04)	0.01	0.00	1.00
	8	0.00	-0.01 (0.03)	0.01	0.00	1.00	-0.00 (0.03)	0.03	0.02	1.00
	9	0.00	-0.01 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.03	0.02	1.00
	10	0.00	-0.02 (0.03)	0.01	0.01	1.00	-0.01 (0.04)	0.04	0.02	1.00

Table B.8: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.01 (0.05)	0.02	0.01	0.07	0.04 (0.05)	0.12	0.09	0.59
	2	0.10	0.07 (0.05)	0.04	0.05	0.24	0.09 (0.05)	0.26	0.13	0.66
	3	0.15	0.14 (0.05)	0.20	0.14	0.54	0.16 (0.05)	0.50	0.17	0.68
	4	0.20	0.16 (0.05)	0.76	0.19	0.68	0.17 (0.05)	0.94	0.20	0.82
III	5	0.25	0.22 (0.05)	0.96	0.23	0.82	0.24 (0.05)	0.99	0.22	0.86
	6	0.00	-0.00 (0.05)	0.02	0.01	1.00	0.01 (0.05)	0.07	0.05	1.00
	7	0.00	-0.02 (0.05)	0.01	0.01	1.00	-0.02 (0.05)	0.02	0.03	1.00
	8	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.07	0.06	1.00
	9	0.00	-0.03 (0.04)	0.02	0.03	1.00	-0.01 (0.05)	0.08	0.06	1.00
	10	0.00	-0.04 (0.05)	0.03	0.03	1.00	-0.02 (0.05)	0.08	0.06	1.00
	1	0.05	0.06 (0.04)	0.04	0.05	0.36	0.06 (0.04)	0.07	0.07	0.46
	2	0.10	0.10 (0.04)	0.25	0.11	0.58	0.10 (0.04)	0.40	0.12	0.64
	3	0.15	0.16 (0.04)	0.74	0.14	0.59	0.16 (0.04)	0.76	0.15	0.58
	4	0.20	0.19 (0.04)	1.00	0.17	0.81	0.19 (0.04)	1.00	0.17	0.82
IV	5	0.25	0.26 (0.04)	1.00	0.17	0.83	0.26 (0.04)	1.00	0.17	0.84
	6	0.00	0.01 (0.04)	0.02	0.00	1.00	0.02 (0.05)	0.06	0.03	1.00
	7	0.00	-0.00 (0.05)	0.00	0.00	1.00	-0.00 (0.05)	0.02	0.01	1.00
	8	0.00	-0.02 (0.04)	0.02	0.00	1.00	-0.02 (0.04)	0.07	0.04	1.00
	9	0.00	-0.02 (0.04)	0.02	0.01	1.00	-0.02 (0.04)	0.07	0.04	1.00
	10	0.00	-0.03 (0.04)	0.02	0.01	1.00	-0.02 (0.04)	0.07	0.04	1.00

Table B.9: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.03 (0.04)	0.02	0.02	0.15	0.04 (0.04)	0.09	0.07	0.46
	2	0.00	-0.01 (0.04)	0.02	0.01	1.00	-0.01 (0.04)	0.06	0.04	1.00
	3	0.15	0.14 (0.04)	0.32	0.13	0.56	0.14 (0.04)	0.53	0.15	0.60
	4	0.00	-0.03 (0.04)	0.00	0.01	1.00	-0.02 (0.05)	0.02	0.02	1.00
III	5	0.25	0.22 (0.04)	0.99	0.20	0.76	0.22 (0.04)	0.99	0.20	0.80
	6	0.10	0.07 (0.05)	0.04	0.05	0.26	0.08 (0.05)	0.11	0.09	0.45
	7	0.20	0.18 (0.05)	0.91	0.20	0.78	0.19 (0.05)	0.96	0.20	0.84
	8	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.05	0.04	1.00
	9	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.05	0.04	1.00
	10	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.05	0.03	1.00
	1	0.05	0.06 (0.04)	0.03	0.04	0.26	0.06 (0.04)	0.12	0.08	0.59
	2	0.00	0.02 (0.05)	0.02	0.02	1.00	0.02 (0.05)	0.06	0.04	1.00
	3	0.15	0.16 (0.05)	0.57	0.16	0.55	0.17 (0.05)	0.70	0.17	0.62
	4	0.00	0.00 (0.05)	0.00	0.01	1.00	0.00 (0.05)	0.02	0.02	1.00
IV	5	0.25	0.24 (0.05)	0.98	0.22	0.82	0.24 (0.05)	1.00	0.22	0.82
	6	0.10	0.10 (0.03)	0.34	0.10	0.62	0.09 (0.04)	0.73	0.14	0.83
	7	0.20	0.19 (0.03)	1.00	0.14	0.96	0.19 (0.04)	1.00	0.15	0.95
	8	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.03 (0.04)	0.21	0.10	1.00
	9	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.03 (0.04)	0.20	0.10	1.00
	10	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.02 (0.04)	0.20	0.10	1.00

Table B.10: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.03 (0.05)	0.02	0.02	0.13	0.04 (0.05)	0.11	0.08	0.55
	2	0.00	-0.01 (0.05)	0.02	0.01	1.00	0.00 (0.05)	0.09	0.06	1.00
	3	0.15	0.16 (0.05)	0.64	0.17	0.70	0.16 (0.05)	0.64	0.18	0.68
	4	0.00	-0.01 (0.05)	0.01	0.01	1.00	0.00 (0.06)	0.04	0.03	1.00
III	5	0.25	0.23 (0.05)	0.98	0.22	0.81	0.23 (0.05)	0.99	0.22	0.85
	6	0.10	0.07 (0.04)	0.07	0.07	0.32	0.08 (0.05)	0.20	0.11	0.58
	7	0.20	0.18 (0.04)	0.98	0.19	0.84	0.19 (0.04)	1.00	0.19	0.88
	8	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.08	0.06	1.00
	9	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.02 (0.05)	0.06	0.05	1.00
	10	0.00	-0.02 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.06	0.04	1.00
	1	0.05	0.05 (0.04)	0.03	0.04	0.28	0.05 (0.04)	0.10	0.07	0.52
	2	0.00	0.02 (0.04)	0.02	0.02	1.00	0.02 (0.04)	0.06	0.04	1.00
	3	0.15	0.16 (0.04)	0.71	0.14	0.56	0.16 (0.04)	0.84	0.16	0.63
	4	0.00	-0.01 (0.04)	0.00	0.01	1.00	0.00 (0.04)	0.01	0.02	1.00
IV	5	0.25	0.23 (0.04)	1.00	0.18	0.81	0.23 (0.04)	1.00	0.18	0.85
	6	0.10	0.08 (0.04)	0.10	0.08	0.43	0.09 (0.05)	0.30	0.14	0.77
	7	0.20	0.18 (0.04)	0.98	0.19	0.93	0.18 (0.05)	0.99	0.19	0.93
	8	0.00	-0.04 (0.05)	0.02	0.03	1.00	-0.03 (0.05)	0.13	0.09	1.00
	9	0.00	-0.04 (0.05)	0.02	0.03	1.00	-0.03 (0.05)	0.12	0.08	1.00
	10	0.00	-0.04 (0.05)	0.03	0.02	1.00	-0.02 (0.05)	0.13	0.09	1.00

Table B.11: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III	1	0.05	0.04 (0.04)	0.02	0.02	0.20	0.05 (0.04)	0.15	0.09	0.65
	2	0.10	0.08 (0.04)	0.10	0.08	0.45	0.09 (0.04)	0.46	0.13	0.70
	3	0.15	0.16 (0.04)	0.70	0.14	0.60	0.16 (0.04)	0.88	0.16	0.72
	4	0.20	0.17 (0.04)	0.98	0.17	0.75	0.18 (0.04)	1.00	0.18	0.85
	5	0.25	0.25 (0.04)	1.00	0.19	0.86	0.25 (0.05)	1.00	0.19	0.91
	6	0.00	-0.02 (0.05)	0.02	0.02	1.00	-0.00 (0.05)	0.08	0.06	1.00
	7	0.00	-0.04 (0.05)	0.01	0.02	1.00	-0.02 (0.06)	0.03	0.03	1.00
	8	0.00	-0.06 (0.05)	0.03	0.05	1.00	-0.03 (0.05)	0.09	0.08	1.00
	9	0.00	-0.06 (0.05)	0.03	0.05	1.00	-0.02 (0.05)	0.08	0.08	1.00
	10	0.00	-0.07 (0.05)	0.03	0.06	1.00	-0.04 (0.05)	0.10	0.09	1.00
IV	1	0.05	0.05 (0.04)	0.02	0.03	0.24	0.05 (0.04)	0.05	0.05	0.36
	2	0.10	0.10 (0.04)	0.11	0.10	0.50	0.10 (0.04)	0.22	0.12	0.59
	3	0.15	0.16 (0.05)	0.46	0.15	0.57	0.16 (0.05)	0.54	0.16	0.57
	4	0.20	0.18 (0.05)	0.97	0.19	0.76	0.19 (0.05)	0.98	0.19	0.78
	5	0.25	0.26 (0.05)	1.00	0.22	0.86	0.25 (0.05)	1.00	0.21	0.88
	6	0.00	0.02 (0.03)	0.01	0.00	1.00	0.01 (0.04)	0.02	0.01	1.00
	7	0.00	-0.00 (0.03)	0.00	0.00	1.00	-0.00 (0.04)	0.01	0.00	1.00
	8	0.00	-0.01 (0.03)	0.01	0.00	1.00	-0.00 (0.03)	0.03	0.02	1.00
	9	0.00	-0.01 (0.03)	0.01	0.01	1.00	-0.00 (0.04)	0.03	0.02	1.00
	10	0.00	-0.02 (0.03)	0.01	0.01	1.00	-0.01 (0.04)	0.04	0.02	1.00

Table B.12: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III	1	0.05	0.04 (0.05)	0.03	0.02	0.13	0.06 (0.05)	0.15	0.10	0.61
	2	0.10	0.07 (0.05)	0.04	0.04	0.22	0.08 (0.05)	0.17	0.12	0.54
	3	0.15	0.18 (0.05)	0.42	0.18	0.66	0.20 (0.06)	0.49	0.19	0.65
	4	0.20	0.16 (0.05)	0.74	0.20	0.68	0.18 (0.05)	0.88	0.21	0.80
	5	0.25	0.26 (0.06)	0.82	0.26	0.71	0.26 (0.06)	0.95	0.27	0.77
	6	0.00	0.02 (0.05)	0.02	0.01	1.00	0.03 (0.05)	0.07	0.05	1.00
	7	0.00	0.01 (0.06)	0.01	0.02	1.00	0.02 (0.06)	0.02	0.04	1.00
	8	0.00	-0.03 (0.05)	0.03	0.03	1.00	-0.02 (0.05)	0.07	0.06	1.00
	9	0.00	-0.03 (0.05)	0.03	0.02	1.00	-0.02 (0.05)	0.07	0.05	1.00
	10	0.00	-0.04 (0.05)	0.02	0.04	1.00	-0.03 (0.05)	0.07	0.05	1.00
IV	1	0.05	0.06 (0.04)	0.04	0.06	0.42	0.06 (0.04)	0.09	0.08	0.50
	2	0.10	0.09 (0.04)	0.14	0.10	0.51	0.09 (0.04)	0.26	0.11	0.61
	3	0.15	0.20 (0.05)	0.72	0.16	0.52	0.20 (0.05)	0.74	0.17	0.52
	4	0.20	0.20 (0.04)	1.00	0.17	0.82	0.20 (0.04)	1.00	0.17	0.83
	5	0.25	0.27 (0.05)	1.00	0.21	0.71	0.27 (0.05)	1.00	0.21	0.74
	6	0.00	0.04 (0.04)	0.02	0.01	1.00	0.03 (0.05)	0.06	0.04	1.00
	7	0.00	0.03 (0.06)	0.01	0.01	1.00	0.03 (0.06)	0.02	0.02	1.00
	8	0.00	-0.03 (0.04)	0.02	0.01	1.00	-0.02 (0.05)	0.07	0.04	1.00
	9	0.00	-0.03 (0.04)	0.02	0.01	1.00	-0.02 (0.05)	0.07	0.04	1.00
	10	0.00	-0.03 (0.04)	0.02	0.01	1.00	-0.02 (0.05)	0.07	0.04	1.00

Table B.13: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.03 (0.04)	0.02	0.02	0.15	0.04 (0.04)	0.09	0.07	0.46
	2	0.00	-0.01 (0.04)	0.02	0.01	1.00	-0.01 (0.04)	0.06	0.04	1.00
	3	0.15	0.14 (0.04)	0.32	0.13	0.56	0.14 (0.04)	0.53	0.15	0.60
	4	0.00	-0.03 (0.04)	0.00	0.01	1.00	-0.02 (0.05)	0.02	0.02	1.00
III	5	0.25	0.22 (0.04)	0.99	0.20	0.76	0.22 (0.04)	0.99	0.20	0.80
	6	0.10	0.07 (0.05)	0.04	0.05	0.26	0.08 (0.05)	0.11	0.09	0.45
	7	0.20	0.18 (0.05)	0.91	0.20	0.78	0.19 (0.05)	0.96	0.20	0.84
	8	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.05	0.04	1.00
	9	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.05	0.04	1.00
	10	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.05	0.03	1.00
	1	0.05	0.06 (0.04)	0.03	0.04	0.26	0.06 (0.04)	0.12	0.08	0.59
	2	0.00	0.02 (0.05)	0.02	0.02	1.00	0.02 (0.05)	0.06	0.04	1.00
	3	0.15	0.16 (0.05)	0.57	0.16	0.55	0.17 (0.05)	0.70	0.17	0.62
	4	0.00	0.00 (0.05)	0.00	0.01	1.00	0.00 (0.05)	0.02	0.02	1.00
IV	5	0.25	0.24 (0.05)	0.98	0.22	0.82	0.24 (0.05)	1.00	0.22	0.82
	6	0.10	0.10 (0.03)	0.34	0.10	0.62	0.09 (0.04)	0.73	0.14	0.83
	7	0.20	0.19 (0.03)	1.00	0.14	0.96	0.19 (0.04)	1.00	0.15	0.95
	8	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.03 (0.04)	0.21	0.10	1.00
	9	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.03 (0.04)	0.20	0.10	1.00
	10	0.00	-0.03 (0.04)	0.02	0.02	1.00	-0.02 (0.04)	0.20	0.10	1.00

Table B.14: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III and IV where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.04 (0.05)	0.02	0.02	0.14	0.05 (0.05)	0.11	0.08	0.56
	2	0.00	0.02 (0.05)	0.02	0.01	1.00	0.03 (0.05)	0.08	0.06	1.00
	3	0.15	0.18 (0.05)	0.52	0.18	0.69	0.19 (0.06)	0.57	0.18	0.73
	4	0.00	0.03 (0.06)	0.02	0.02	1.00	0.04 (0.07)	0.05	0.06	1.00
III	5	0.25	0.27 (0.06)	0.96	0.25	0.66	0.26 (0.06)	0.97	0.26	0.71
	6	0.10	0.08 (0.05)	0.07	0.08	0.44	0.10 (0.05)	0.22	0.11	0.56
	7	0.20	0.18 (0.05)	0.94	0.19	0.79	0.18 (0.05)	0.98	0.19	0.86
	8	0.00	-0.04 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.06	0.05	1.00
	9	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.06	0.04	1.00
	10	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.06	0.04	1.00
	1	0.05	0.06 (0.04)	0.04	0.05	0.33	0.06 (0.04)	0.10	0.08	0.55
	2	0.00	0.04 (0.04)	0.02	0.03	1.00	0.04 (0.04)	0.07	0.05	1.00
	3	0.15	0.19 (0.05)	0.41	0.15	0.50	0.19 (0.05)	0.59	0.16	0.53
	4	0.00	-0.01 (0.05)	0.00	0.02	1.00	0.00 (0.05)	0.02	0.03	1.00
IV	5	0.25	0.25 (0.05)	1.00	0.22	0.72	0.25 (0.05)	1.00	0.22	0.73
	6	0.10	0.09 (0.04)	0.11	0.09	0.48	0.09 (0.05)	0.32	0.13	0.71
	7	0.20	0.18 (0.04)	0.96	0.20	0.90	0.18 (0.05)	0.98	0.20	0.93
	8	0.00	-0.04 (0.05)	0.03	0.03	1.00	-0.03 (0.05)	0.11	0.08	1.00
	9	0.00	-0.04 (0.05)	0.03	0.03	1.00	-0.03 (0.05)	0.11	0.08	1.00
	10	0.00	-0.04 (0.05)	0.03	0.03	1.00	-0.03 (0.05)	0.10	0.08	1.00

B.3 Additional Simulation Studies

We conduct additional simulation studies where data are generated under a factor-analytic covariance structure. The outline of the correlation between outcomes in Scenarios I$'$-IV$'$ is the same as that of Scenarios I-IV: in Scenario I$'$ the relevant outcomes associated with the covariate x_1 are highly correlated but others are moderately correlated; in Scenario II$'$ the outcomes have the same correlation as in Scenario I$'$ but none of them are associated with any of the covariates; in Scenario III$'$ the relevant outcomes associated with the covariate x_1 are moderately correlated and irrelevant outcomes are slightly correlated; in Scenario IV$'$ the relevant outcomes associated with x_1 are moderately correlated while irrelevant ones are highly correlated. Two continuous covariates are generated from $\mathcal{N}(0, 4)$ assuming Cor(x_1, x_2)=0.3 and 0.6, and the intercepts $\boldsymbol{\beta}_0$ are set to values in the range of (-1, 1). In Scenarios I$'$, III$'$ and IV$'$, the effects of the covariate on the responses are assumed to be

$$B = \begin{pmatrix} 0.05 & 0.10 & 0.15 & 0.20 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0.05 & 0 & 0.15 & 0 & 0.25 & 0.10 & 0.20 & 0 & 0 & 0 \end{pmatrix}$$

while Scenario II$'$ considers a emphnull case by setting all elements of B to zero. Samples of size n=100 with q=10 responses and p=2 covariates are generated under the factor-analytic model with σ^2=0.3 and

I$'$. $\boldsymbol{\lambda}$=(1.5, 1.5, 1.5, 1.5, 1.5, 0.5,. . . , 0.5)$^\top$,

II$'$. $\boldsymbol{\lambda}$=(1.5, 1.5, 1.5, 1.5, 1.5, 0.5,. . . , 0.5)$^\top$,

III$'$. $\boldsymbol{\lambda}$=(0.8, 0.8, 0.8, 0.8, 0.8, 0.2,. . . , 0.2)$^\top$,

IV$'$. $\boldsymbol{\lambda}$=(0.5, 0.5, 0.5, 0.5, 0.5, 1.0,. . . , 1.0)$^\top$.

Then the resulting correlation matrix can be presented as follows:

$$\begin{pmatrix} 1 & c_1 & c_1 & c_1 & c_1 & c_3 & c_3 & c_3 & c_3 & c_3 \\ c_1 & 1 & c_1 & c_1 & c_1 & c_3 & c_3 & c_3 & c_3 & c_3 \\ c_1 & c_1 & 1 & c_1 & c_1 & c_3 & c_3 & c_3 & c_3 & c_3 \\ c_1 & c_1 & c_1 & 1 & c_1 & c_3 & c_3 & c_3 & c_3 & c_3 \\ c_1 & c_1 & c_1 & c_1 & 1 & c_3 & c_3 & c_3 & c_3 & c_3 \\ c_3 & c_3 & c_3 & c_3 & c_3 & 1 & c_2 & c_2 & c_2 & c_2 \\ c_3 & c_3 & c_3 & c_3 & c_3 & c_2 & 1 & c_2 & c_2 & c_2 \\ c_3 & c_3 & c_3 & c_3 & c_3 & c_2 & c_2 & 1 & c_2 & c_2 \\ c_3 & c_3 & c_3 & c_3 & c_3 & c_2 & c_2 & c_2 & 1 & c_2 \\ c_3 & c_3 & c_3 & c_3 & c_3 & c_2 & c_2 & c_2 & c_2 & 1 \end{pmatrix},$$

where the pairwise correlations (c_1, c_2, c_3) are (0.69, 0.20, 0.37) in Scenarios I$'$ and II$'$ and (0.39, 0.04, 0.12) and (0.20, 0.50, 0.32) in Scenario III$'$ and IV$'$, respectively.

In order to investigate the performance of the MRF prior, we implemented a factor-analytic model with the conventional IB prior as well as the model with MRF prior and present the results in Tables B.15-B.30. We also provide the results from the unstructured model with MRF prior. The results from Scenarios I$'$-IV$'$ shown in Tables B.15-B.30 are generally consistent with the conclusions we drew in Scenarios I-IV: contrary to the conventional IB prior, our proposed MRF prior not only has more power to detect real effects by yielding higher inclusion probabilities but also avoids the inclusion of non-relevant associations.

Table B.15: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	-0.00 (0.03)	0.02	0.02	0.08	0.03 (0.04)	0.15	0.08	0.50
	2	0.10	0.05 (0.03)	0.04	0.05	0.37	0.08 (0.04)	0.38	0.10	0.53
	3	0.15	0.12 (0.03)	0.14	0.09	0.42	0.13 (0.04)	0.74	0.12	0.61
	4	0.20	0.14 (0.03)	0.97	0.14	0.50	0.16 (0.04)	1.00	0.15	0.74
	5	0.25	0.18 (0.04)	1.00	0.15	0.55	0.20 (0.04)	1.00	0.16	0.71
	6	0.00	-0.01 (0.03)	0.01	0.00	1.00	0.01 (0.03)	0.03	0.01	1.00
	7	0.00	-0.03 (0.03)	0.00	0.00	1.00	-0.01 (0.03)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.07	0.03	1.00
	9	0.00	-0.03 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.07	0.03	1.00
	10	0.00	-0.03 (0.03)	0.02	0.01	1.00	-0.02 (0.03)	0.07	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.01 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	3	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	4	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	6	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.01	1.00
	7	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.05	0.02	1.00
	8	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	10	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.04	0.01	1.00

Table B.16: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	-0.01 (0.04)	0.04	0.03	0.13	0.02 (0.04)	0.19	0.09	0.55
	2	0.10	0.05 (0.04)	0.06	0.06	0.32	0.07 (0.04)	0.41	0.12	0.66
	3	0.15	0.12 (0.04)	0.26	0.11	0.50	0.13 (0.04)	0.79	0.14	0.63
	4	0.20	0.14 (0.04)	0.98	0.15	0.63	0.16 (0.04)	1.00	0.16	0.79
	5	0.25	0.19 (0.04)	1.00	0.16	0.55	0.22 (0.04)	1.00	0.16	0.76
	6	0.00	-0.01 (0.03)	0.01	0.00	1.00	0.01 (0.03)	0.02	0.01	1.00
	7	0.00	-0.02 (0.03)	0.00	0.00	1.00	-0.01 (0.03)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.07	0.03	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.06	0.03	1.00
	10	0.00	-0.02 (0.03)	0.03	0.00	1.00	-0.02 (0.03)	0.06	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.01 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	3	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	4	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	6	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.01	1.00
	7	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.05	0.02	1.00
	8	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	10	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.04	0.01	1.00

Table B.17: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	0.04 (0.03)	0.03	0.03	0.21	0.05 (0.03)	0.29	0.09	0.68
	2	0.00	-0.00 (0.03)	0.01	0.00	1.00	0.01 (0.04)	0.11	0.05	1.00
	3	0.15	0.15 (0.03)	0.99	0.13	0.76	0.16 (0.03)	1.00	0.13	0.79
	4	0.00	-0.02 (0.03)	0.00	0.00	1.00	-0.00 (0.04)	0.05	0.02	1.00
	5	0.25	0.23 (0.03)	1.00	0.14	0.89	0.24 (0.04)	1.00	0.14	0.97
	6	0.10	0.09 (0.03)	0.60	0.09	0.61	0.09 (0.03)	0.91	0.11	0.71
	7	0.20	0.20 (0.03)	1.00	0.11	0.89	0.20 (0.03)	1.00	0.11	0.92
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.06	0.02	1.00
	9	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.07	0.02	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.06	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.09	0.05	1.00
	3	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	6	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.03)	0.06	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.04	0.01	1.00

Table B.18: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	0.05 (0.03)	0.05	0.04	0.26	0.06 (0.04)	0.30	0.10	0.79
	2	0.00	-0.02 (0.04)	0.03	0.00	1.00	0.00 (0.04)	0.16	0.07	1.00
	3	0.15	0.16 (0.03)	0.98	0.14	0.82	0.16 (0.04)	1.00	0.14	0.82
	4	0.00	-0.03 (0.04)	0.00	0.00	1.00	-0.01 (0.04)	0.06	0.03	1.00
	5	0.25	0.23 (0.03)	1.00	0.14	0.95	0.24 (0.04)	1.00	0.14	0.97
	6	0.10	0.10 (0.03)	0.78	0.10	0.63	0.10 (0.03)	0.91	0.12	0.74
	7	0.20	0.20 (0.03)	1.00	0.11	0.92	0.20 (0.03)	1.00	0.11	0.89
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.07	0.02	1.00
	9	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.07	0.02	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.06	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.09	0.05	1.00
	3	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	6	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.03)	0.06	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.04	0.01	1.00

Table B.19: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.04 (0.04)	0.03	0.05	0.36	0.06 (0.03)	0.43	0.09	0.64
	2	0.10	0.09 (0.03)	0.41	0.09	0.54	0.10 (0.03)	0.95	0.12	0.82
	3	0.15	0.14 (0.03)	0.80	0.14	0.77	0.16 (0.03)	1.00	0.13	0.87
	4	0.20	0.17 (0.03)	1.00	0.14	0.79	0.19 (0.03)	1.00	0.13	0.92
	5	0.25	0.22 (0.03)	1.00	0.14	0.79	0.24 (0.03)	1.00	0.13	0.85
	6	0.00	-0.01 (0.03)	0.01	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	7	0.00	-0.01 (0.03)	0.00	0.00	1.00	-0.00 (0.03)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.07	0.03	1.00
	9	0.00	-0.01 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.10	0.04	1.00
	10	0.00	-0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.07	0.03	1.00
IV$'$	1	0.05	0.06 (0.03)	0.08	0.06	0.47	0.06 (0.03)	0.13	0.07	0.58
	2	0.10	0.11 (0.03)	0.94	0.12	0.85	0.11 (0.03)	0.95	0.12	0.88
	3	0.15	0.15 (0.03)	1.00	0.13	0.87	0.15 (0.03)	1.00	0.13	0.85
	4	0.20	0.19 (0.03)	1.00	0.11	0.90	0.19 (0.03)	1.00	0.11	0.90
	5	0.25	0.25 (0.03)	1.00	0.12	0.93	0.25 (0.03)	1.00	0.12	0.93
	6	0.00	-0.00 (0.03)	0.01	0.00	1.00	-0.00 (0.04)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.00	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	8	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.10	0.04	1.00
	9	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.10	0.04	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.10	0.04	1.00

Table B.20: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1} \mid \gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.03 (0.04)	0.03	0.04	0.36	0.06 (0.04)	0.69	0.12	0.79
	2	0.10	0.08 (0.04)	0.16	0.09	0.51	0.10 (0.04)	0.96	0.14	0.87
	3	0.15	0.14 (0.04)	0.66	0.14	0.72	0.15 (0.04)	1.00	0.14	0.87
	4	0.20	0.16 (0.04)	0.99	0.15	0.79	0.19 (0.04)	1.00	0.14	0.92
	5	0.25	0.21 (0.04)	1.00	0.15	0.82	0.24 (0.04)	1.00	0.14	0.92
	6	0.00	-0.00 (0.03)	0.01	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	7	0.00	-0.01 (0.03)	0.00	0.00	1.00	0.00 (0.03)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.08	0.03	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.08	0.03	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.05	0.02	1.00
IV$'$	1	0.05	0.06 (0.03)	0.08	0.06	0.47	0.06 (0.03)	0.13	0.07	0.58
	2	0.10	0.11 (0.03)	0.94	0.12	0.85	0.11 (0.03)	0.95	0.12	0.88
	3	0.15	0.15 (0.03)	1.00	0.13	0.87	0.15 (0.03)	1.00	0.13	0.85
	4	0.20	0.19 (0.03)	1.00	0.11	0.90	0.19 (0.03)	1.00	0.11	0.90
	5	0.25	0.25 (0.03)	1.00	0.12	0.93	0.25 (0.03)	1.00	0.12	0.93
	6	0.00	-0.00 (0.03)	0.01	0.00	1.00	-0.00 (0.04)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.00	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	8	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.10	0.04	1.00
	9	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.10	0.04	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.10	0.04	1.00

Table B.21: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.06 (0.03)	0.07	0.04	0.33	0.06 (0.03)	0.28	0.08	0.56
	2	0.00	0.00 (0.03)	0.01	0.00	1.00	0.01 (0.03)	0.08	0.03	1.00
	3	0.15	0.16 (0.03)	1.00	0.13	0.85	0.16 (0.03)	1.00	0.12	0.87
	4	0.00	-0.01 (0.03)	0.00	0.00	1.00	-0.00 (0.03)	0.04	0.01	1.00
	5	0.25	0.25 (0.03)	1.00	0.12	0.92	0.26 (0.03)	1.00	0.12	0.92
	6	0.10	0.10 (0.03)	0.89	0.10	0.72	0.10 (0.03)	0.98	0.11	0.74
	7	0.20	0.19 (0.03)	1.00	0.11	0.92	0.20 (0.03)	1.00	0.11	0.92
	8	0.00	0.02 (0.03)	0.02	0.01	1.00	0.02 (0.03)	0.09	0.04	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.07	0.03	1.00
	10	0.00	-0.00 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.06	0.02	1.00
IV$'$	1	0.05	0.07 (0.03)	0.10	0.06	0.48	0.07 (0.03)	0.28	0.08	0.62
	2	0.00	0.01 (0.03)	0.01	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	3	0.15	0.15 (0.03)	1.00	0.13	0.85	0.16 (0.03)	1.00	0.12	0.88
	4	0.00	-0.01 (0.03)	0.00	0.00	1.00	-0.00 (0.03)	0.01	0.00	1.00
	5	0.25	0.25 (0.03)	1.00	0.12	0.98	0.25 (0.03)	1.00	0.12	0.97
	6	0.10	0.10 (0.03)	0.83	0.11	0.75	0.10 (0.03)	0.96	0.13	0.82
	7	0.20	0.20 (0.03)	1.00	0.12	0.92	0.20 (0.03)	1.00	0.13	0.93
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.18	0.08	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.20	0.08	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.18	0.07	1.00

Table B.22: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.06 (0.03)	0.08	0.05	0.49	0.06 (0.03)	0.35	0.10	0.82
	2	0.00	0.00 (0.03)	0.02	0.00	1.00	0.01 (0.04)	0.19	0.07	1.00
	3	0.15	0.16 (0.03)	1.00	0.14	0.87	0.16 (0.03)	1.00	0.14	0.92
	4	0.00	-0.01 (0.04)	0.00	0.00	1.00	-0.00 (0.04)	0.12	0.06	1.00
	5	0.25	0.24 (0.03)	1.00	0.13	0.95	0.25 (0.03)	1.00	0.13	0.97
	6	0.10	0.10 (0.03)	0.86	0.10	0.72	0.10 (0.03)	0.96	0.11	0.77
	7	0.20	0.19 (0.03)	1.00	0.11	0.90	0.20 (0.03)	1.00	0.11	0.90
	8	0.00	0.02 (0.03)	0.02	0.01	1.00	0.02 (0.03)	0.06	0.03	1.00
	9	0.00	-0.01 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.06	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.04	0.01	1.00
IV$'$	1	0.05	0.07 (0.03)	0.10	0.06	0.48	0.07 (0.03)	0.28	0.08	0.62
	2	0.00	0.01 (0.03)	0.01	0.00	1.00	0.00 (0.03)	0.03	0.01	1.00
	3	0.15	0.15 (0.03)	1.00	0.13	0.85	0.16 (0.03)	1.00	0.12	0.88
	4	0.00	-0.01 (0.03)	0.00	0.00	1.00	-0.00 (0.03)	0.01	0.00	1.00
	5	0.25	0.25 (0.03)	1.00	0.12	0.98	0.25 (0.03)	1.00	0.12	0.97
	6	0.10	0.10 (0.03)	0.83	0.11	0.75	0.10 (0.03)	0.96	0.13	0.82
	7	0.20	0.20 (0.03)	1.00	0.12	0.92	0.20 (0.03)	1.00	0.13	0.93
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.18	0.08	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.20	0.08	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.18	0.07	1.00

Table B.23: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	-0.01 (0.04)	0.02	0.03	0.08	0.03 (0.04)	0.16	0.09	0.46
	2	0.10	0.04 (0.04)	0.03	0.03	0.14	0.06 (0.04)	0.18	0.10	0.46
	3	0.15	0.12 (0.04)	0.11	0.10	0.43	0.13 (0.04)	0.34	0.14	0.57
	4	0.20	0.14 (0.03)	0.93	0.14	0.35	0.16 (0.04)	0.98	0.16	0.70
	5	0.25	0.18 (0.04)	0.94	0.17	0.51	0.21 (0.04)	1.00	0.18	0.73
	6	0.00	0.02 (0.04)	0.02	0.01	1.00	0.03 (0.04)	0.04	0.03	1.00
	7	0.00	-0.03 (0.04)	0.00	0.00	1.00	-0.01 (0.04)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.06	0.02	1.00
	9	0.00	-0.03 (0.03)	0.02	0.01	1.00	-0.02 (0.03)	0.07	0.03	1.00
	10	0.00	-0.03 (0.03)	0.02	0.01	1.00	-0.02 (0.03)	0.06	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.07	0.04	1.00
	2	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.04)	0.08	0.04	1.00
	3	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	4	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.04)	0.08	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	6	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.01	1.00
	7	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.05	0.02	1.00
	10	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.05	0.01	1.00

Table B.24: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
I$'$	1	0.05	-0.00 (0.04)	0.04	0.04	0.14	0.03 (0.05)	0.21	0.11	0.51
	2	0.10	0.04 (0.04)	0.04	0.04	0.24	0.06 (0.04)	0.27	0.11	0.51
	3	0.15	0.13 (0.04)	0.13	0.12	0.54	0.13 (0.05)	0.43	0.15	0.57
	4	0.20	0.14 (0.04)	0.97	0.15	0.54	0.16 (0.04)	0.98	0.18	0.76
	5	0.25	0.19 (0.04)	0.98	0.19	0.65	0.21 (0.05)	1.00	0.19	0.76
	6	0.00	0.02 (0.03)	0.02	0.01	1.00	0.02 (0.04)	0.04	0.03	1.00
	7	0.00	-0.02 (0.04)	0.00	0.00	1.00	-0.01 (0.04)	0.01	0.00	1.00
	8	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.06	0.02	1.00
	9	0.00	-0.03 (0.03)	0.03	0.01	1.00	-0.01 (0.03)	0.07	0.02	1.00
	10	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.02 (0.03)	0.06	0.02	1.00
II$'$	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.04)	0.07	0.04	1.00
	2	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.04)	0.08	0.04	1.00
	3	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	4	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.04)	0.08	0.04	1.00
	5	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.07	0.04	1.00
	6	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.01	1.00
	7	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.04	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.05	0.02	1.00
	10	0.00	0.01 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.05	0.01	1.00

Table B.25: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.04 (0.03)	0.02	0.03	0.16	0.05 (0.04)	0.19	0.08	0.59
	2	0.00	0.01 (0.04)	0.02	0.00	1.00	0.01 (0.04)	0.10	0.06	1.00
	3	0.15	0.17 (0.04)	0.94	0.15	0.73	0.16 (0.04)	0.98	0.15	0.73
	4	0.00	0.03 (0.04)	0.01	0.00	1.00	0.04 (0.05)	0.07	0.04	1.00
I$'$	5	0.25	0.26 (0.04)	1.00	0.16	0.81	0.26 (0.04)	1.00	0.17	0.78
	6	0.10	0.08 (0.03)	0.39	0.09	0.57	0.09 (0.03)	0.72	0.11	0.68
	7	0.20	0.19 (0.03)	1.00	0.11	0.89	0.19 (0.03)	1.00	0.12	0.89
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.05	0.01	1.00
	9	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.06	0.02	1.00
	10	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.06	0.02	1.00
	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.04)	0.08	0.04	1.00
	3	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
II$'$	5	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	6	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.03)	0.05	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.04	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.04	0.01	1.00

Table B.26: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I$'$ and II$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.04 (0.04)	0.03	0.04	0.22	0.06 (0.04)	0.21	0.10	0.70
	2	0.00	-0.00 (0.04)	0.03	0.01	1.00	0.01 (0.04)	0.17	0.09	1.00
	3	0.15	0.17 (0.04)	0.94	0.16	0.76	0.17 (0.04)	0.97	0.17	0.73
	4	0.00	0.02 (0.05)	0.01	0.00	1.00	0.02 (0.06)	0.07	0.06	1.00
I$'$	5	0.25	0.25 (0.04)	1.00	0.18	0.92	0.26 (0.05)	1.00	0.18	0.95
	6	0.10	0.08 (0.03)	0.43	0.09	0.65	0.09 (0.03)	0.75	0.11	0.76
	7	0.20	0.19 (0.03)	1.00	0.11	0.89	0.20 (0.03)	1.00	0.12	0.95
	8	0.00	-0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.06	0.02	1.00
	9	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.07	0.02	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.03)	0.06	0.02	1.00
	1	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	2	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.04)	0.08	0.04	1.00
	3	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
	4	0.00	-0.00 (0.03)	0.02	0.01	1.00	0.00 (0.04)	0.08	0.04	1.00
II$'$	5	0.00	-0.00 (0.03)	0.02	0.00	1.00	-0.00 (0.04)	0.08	0.04	1.00
	6	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.04	0.02	1.00
	7	0.00	0.00 (0.03)	0.02	0.01	1.00	0.01 (0.03)	0.05	0.02	1.00
	8	0.00	0.00 (0.03)	0.02	0.01	1.00	0.00 (0.03)	0.05	0.02	1.00
	9	0.00	-0.00 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.04	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.04	0.01	1.00

Table B.27: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.05 (0.04)	0.03	0.05	0.32	0.07 (0.04)	0.45	0.10	0.68
	2	0.10	0.08 (0.03)	0.21	0.09	0.50	0.10 (0.03)	0.86	0.13	0.82
	3	0.15	0.16 (0.04)	0.70	0.15	0.66	0.16 (0.04)	0.98	0.18	0.87
	4	0.20	0.16 (0.03)	1.00	0.16	0.84	0.19 (0.04)	1.00	0.15	1.00
	5	0.25	0.22 (0.04)	1.00	0.16	0.71	0.24 (0.04)	1.00	0.17	0.87
	6	0.00	0.01 (0.04)	0.02	0.01	1.00	0.02 (0.04)	0.05	0.03	1.00
	7	0.00	-0.00 (0.04)	0.00	0.00	1.00	0.00 (0.04)	0.01	0.00	1.00
	8	0.00	-0.00 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.08	0.03	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.08	0.03	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.07	0.03	1.00
IV$'$	1	0.05	0.07 (0.03)	0.16	0.07	0.56	0.07 (0.03)	0.24	0.08	0.60
	2	0.10	0.11 (0.03)	0.91	0.12	0.85	0.11 (0.03)	0.94	0.12	0.88
	3	0.15	0.17 (0.04)	0.98	0.14	0.65	0.16 (0.04)	0.98	0.15	0.75
	4	0.20	0.19 (0.03)	1.00	0.11	0.94	0.19 (0.03)	1.00	0.12	0.94
	5	0.25	0.24 (0.04)	1.00	0.14	0.92	0.24 (0.04)	1.00	0.14	0.90
	6	0.00	0.02 (0.04)	0.02	0.00	1.00	0.02 (0.04)	0.06	0.03	1.00
	7	0.00	0.01 (0.04)	0.00	0.00	1.00	0.01 (0.04)	0.02	0.01	1.00
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.09	0.04	1.00
	9	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.08	0.04	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.08	0.04	1.00

Table B.28: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.04 (0.04)	0.04	0.04	0.29	0.07 (0.04)	0.66	0.13	0.79
	2	0.10	0.07 (0.04)	0.12	0.08	0.42	0.10 (0.04)	0.81	0.14	0.82
	3	0.15	0.15 (0.05)	0.29	0.15	0.61	0.16 (0.04)	0.96	0.19	0.89
	4	0.20	0.16 (0.04)	0.99	0.16	0.82	0.19 (0.04)	1.00	0.16	0.97
III$'$	5	0.25	0.20 (0.04)	0.99	0.18	0.68	0.24 (0.04)	1.00	0.18	0.84
	6	0.00	0.02 (0.04)	0.02	0.00	1.00	0.02 (0.04)	0.05	0.03	1.00
	7	0.00	-0.01 (0.04)	0.00	0.00	1.00	-0.00 (0.04)	0.02	0.01	1.00
	8	0.00	-0.01 (0.03)	0.02	0.01	1.00	0.01 (0.03)	0.08	0.03	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.07	0.03	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.06	0.02	1.00
	1	0.05	0.07 (0.03)	0.16	0.07	0.56	0.07 (0.03)	0.24	0.08	0.60
	2	0.10	0.11 (0.03)	0.91	0.12	0.85	0.11 (0.03)	0.94	0.12	0.88
	3	0.15	0.17 (0.04)	0.98	0.14	0.65	0.16 (0.04)	0.98	0.15	0.75
	4	0.20	0.19 (0.03)	1.00	0.11	0.94	0.19 (0.03)	1.00	0.12	0.94
IV$'$	5	0.25	0.24 (0.04)	1.00	0.14	0.92	0.24 (0.04)	1.00	0.14	0.90
	6	0.00	0.02 (0.04)	0.02	0.00	1.00	0.02 (0.04)	0.06	0.03	1.00
	7	0.00	0.01 (0.04)	0.00	0.00	1.00	0.01 (0.04)	0.02	0.01	1.00
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.09	0.04	1.00
	9	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.08	0.04	1.00
	10	0.00	-0.01 (0.03)	0.02	0.00	1.00	0.00 (0.03)	0.08	0.04	1.00

Table B.29: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.06 (0.03)	0.07	0.05	0.39	0.06 (0.03)	0.27	0.09	0.71
	2	0.00	0.02 (0.03)	0.02	0.01	1.00	0.02 (0.04)	0.12	0.06	1.00
	3	0.15	0.18 (0.04)	0.99	0.15	0.63	0.17 (0.04)	0.99	0.16	0.87
	4	0.00	0.02 (0.04)	0.00	0.01	1.00	0.01 (0.04)	0.06	0.04	1.00
III$'$	5	0.25	0.25 (0.04)	1.00	0.15	0.79	0.25 (0.04)	1.00	0.16	0.92
	6	0.10	0.10 (0.03)	0.83	0.10	0.74	0.10 (0.03)	0.96	0.11	0.74
	7	0.20	0.19 (0.03)	1.00	0.11	0.92	0.20 (0.03)	1.00	0.11	0.92
	8	0.00	0.02 (0.03)	0.02	0.01	1.00	0.02 (0.03)	0.09	0.04	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.00 (0.03)	0.07	0.03	1.00
	10	0.00	-0.00 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.06	0.02	1.00
	1	0.05	0.08 (0.03)	0.16	0.07	0.50	0.08 (0.03)	0.28	0.09	0.62
	2	0.00	0.04 (0.04)	0.01	0.00	1.00	0.03 (0.04)	0.04	0.02	1.00
	3	0.15	0.17 (0.04)	0.95	0.15	0.62	0.17 (0.04)	0.99	0.15	0.69
	4	0.00	-0.00 (0.04)	0.00	0.00	1.00	0.00 (0.04)	0.02	0.00	1.00
IV$'$	5	0.25	0.25 (0.04)	1.00	0.14	0.96	0.25 (0.04)	1.00	0.14	1.00
	6	0.10	0.10 (0.03)	0.70	0.11	0.77	0.10 (0.03)	0.86	0.13	0.79
	7	0.20	0.19 (0.03)	1.00	0.13	0.90	0.20 (0.03)	1.00	0.13	0.90
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.04)	0.14	0.07	1.00
	9	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.01 (0.04)	0.14	0.07	1.00
	10	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.13	0.06	1.00

Table B.30: Results from simulation studies with 100 replications using factor-analytic models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios III$'$ and IV$'$ where the correlation between x_1 and x_2 equals 0.6. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

Scenario	j	True	Factor-analytic with IB prior				Factor-analytic with MRF prior			
		$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
III$'$	1	0.05	0.06 (0.03)	0.09	0.06	0.53	0.07 (0.04)	0.33	0.11	0.76
	2	0.00	0.02 (0.04)	0.02	0.01	1.00	0.02 (0.04)	0.24	0.08	1.00
	3	0.15	0.19 (0.04)	0.98	0.16	0.74	0.17 (0.04)	0.98	0.18	0.89
	4	0.00	0.02 (0.04)	0.00	0.01	1.00	0.01 (0.04)	0.11	0.07	1.00
	5	0.25	0.24 (0.04)	1.00	0.16	0.89	0.26 (0.04)	1.00	0.17	0.97
	6	0.10	0.10 (0.03)	0.80	0.11	0.71	0.10 (0.03)	0.95	0.12	0.74
	7	0.20	0.19 (0.03)	1.00	0.11	0.89	0.20 (0.03)	1.00	0.11	0.92
	8	0.00	0.01 (0.03)	0.02	0.01	1.00	0.02 (0.03)	0.06	0.03	1.00
	9	0.00	-0.02 (0.03)	0.02	0.01	1.00	-0.01 (0.03)	0.06	0.02	1.00
	10	0.00	0.00 (0.03)	0.02	0.00	1.00	0.01 (0.03)	0.04	0.01	1.00
IV$'$	1	0.05	0.08 (0.03)	0.16	0.07	0.50	0.08 (0.03)	0.28	0.09	0.62
	2	0.00	0.04 (0.04)	0.01	0.00	1.00	0.03 (0.04)	0.04	0.02	1.00
	3	0.15	0.17 (0.04)	0.95	0.15	0.62	0.17 (0.04)	0.99	0.15	0.69
	4	0.00	-0.00 (0.04)	0.00	0.00	1.00	0.00 (0.04)	0.02	0.00	1.00
	5	0.25	0.25 (0.04)	1.00	0.14	0.96	0.25 (0.04)	1.00	0.14	1.00
	6	0.10	0.10 (0.03)	0.70	0.11	0.77	0.10 (0.03)	0.86	0.13	0.79
	7	0.20	0.19 (0.03)	1.00	0.13	0.90	0.20 (0.03)	1.00	0.13	0.90
	8	0.00	-0.01 (0.03)	0.02	0.00	1.00	-0.01 (0.04)	0.14	0.07	1.00
	9	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.01 (0.04)	0.14	0.07	1.00
	10	0.00	-0.02 (0.03)	0.02	0.00	1.00	-0.00 (0.03)	0.13	0.06	1.00

B.4 Sensitivity Analyses

We have shown improved operative characteristics of our proposed MRF prior when compared to the independent Bernoulli prior. In this section, we assess how sensitive the posterior inference is to the changes in the prior distribution of our proposed MRF prior, depending on the value of hyperparameter η. As noted in Section 2.3, we set η to η^{g-1}, using the our posterior simulation approach. We regard η^{g-1} as the greatest value that η can be while avoiding the phase transition, and thus it provides the optimal performance of our proposed MRF prior. Table B.31 and Table B.32 present the results from three different models corresponding to the prior choices with η=η^{g-1}, η^{g-3}, and η^{g-5} (the average chosen values of η are 1.26, 1.08, 0.88, respectively). As we expected, the proposed MRF prior tends to achieve higher power to detect the weak effects as the value of η increases and approaches to η^{g-1}, which is determined by our posterior simulation approach.

B.5 Simulation studies with larger n and p

We produce a simulation comparison for increasing n and increasing p. Specifically, we conduct two additional simulation studies, (n, p)=(500, 2) and (100, 5), and present the results in Table B.33-B.35. In contrast to the results from the simulation studies with n=100 (see Table 1 and Table B.1), Table B.33 shows that both the conventional and our proposed framework successfully identify the association with the covariates for the outcomes with effect sizes greater than 0.10 when n=500. However, the estimated inclusion probabilities for the outcomes with smallest effect size (0.05) are much smaller with the IB prior ($\gamma_{1,1}$=0.08, $\gamma_{1,2}$=0.63) while they are substantially increased and large enough for relevant variables to be selected using the MRF prior ($\gamma_{1,1}$=0.71, $\gamma_{1,2}$=0.88). In addition, from Table B.34-B.35, we see that the our proposed MRF prior provides more power to detect real effects by yielding higher inclusion probabilities when there is a true association comparing to the conventional independent prior for data with larger p (=5).

Table B.31: Results from simulation studies with 100 replications using unstructured models with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I where the correlation between x_1 and x_2 equals 0.3. The value of η is set to η^{g-1}, η^{g-3}, and η^{g-5}. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,1}$ (conditioning on $\gamma_{j,1}$=1), the medians of the posterior means of $\gamma_{j,1}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,1}$ and the coverage probability (CP) of the HPD intervals.

j	True	$\eta = \eta^{g-1}$ (proposed)				$\eta = \eta^{g-3}$				$\eta = \eta^{g-5}$			
	$\beta_{j,1}$	$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$		$\beta_{j,1}\|\gamma_{j,1}$=1	$\gamma_{j,1}$	$\beta_{j,1}$	
		PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
1	0.05	0.03 (0.04)	0.28	0.11	0.51	0.02 (0.04)	0.24	0.10	0.48	0.00 (0.04)	0.15	0.09	0.37
2	0.10	0.08 (0.04)	0.36	0.12	0.61	0.07 (0.04)	0.32	0.11	0.56	0.06 (0.04)	0.20	0.10	0.48
3	0.15	0.13 (0.04)	0.71	0.14	0.61	0.13 (0.04)	0.52	0.13	0.60	0.12 (0.04)	0.41	0.12	0.54
4	0.20	0.16 (0.04)	0.99	0.17	0.70	0.15 (0.04)	0.99	0.16	0.71	0.15 (0.04)	0.98	0.16	0.60
5	0.25	0.21 (0.04)	1.00	0.18	0.71	0.20 (0.04)	1.00	0.17	0.71	0.20 (0.04)	1.00	0.17	0.69
6	0.00	0.02 (0.05)	0.06	0.03	1.00	0.01 (0.05)	0.04	0.02	1.00	0.01 (0.05)	0.04	0.02	1.00
7	0.00	-0.01 (0.05)	0.02	0.02	1.00	0.00 (0.05)	0.01	0.01	1.00	-0.00 (0.05)	0.01	0.01	1.00
8	0.00	-0.01 (0.05)	0.06	0.05	1.00	-0.01 (0.05)	0.06	0.04	1.00	-0.01 (0.05)	0.04	0.03	1.00
9	0.00	-0.01 (0.05)	0.06	0.04	1.00	-0.02 (0.05)	0.05	0.03	1.00	-0.02 (0.05)	0.04	0.03	1.00
10	0.00	-0.02 (0.05)	0.07	0.05	1.00	-0.02 (0.05)	0.06	0.04	1.00	-0.03 (0.05)	0.04	0.04	1.00

Table B.32: Results from simulation studies with 100 replications using unstructured models with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I where the correlation between x_1 and x_2 equals 0.3. The value of η is set to η^{g-1}, η^{g-3}, and η^{g-5}. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,2}$ (conditioning on $\gamma_{j,2}$=1), the medians of the posterior means of $\gamma_{j,2}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,2}$ and the coverage probability (CP) of the HPD intervals.

j	True	$\eta = \eta^{g-1}$ (proposed)				$\eta = \eta^{g-3}$				$\eta = \eta^{g-5}$			
	$\beta_{j,2}$	$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$		$\beta_{j,2}\|\gamma_{j,2}$=1	$\gamma_{j,2}$	$\beta_{j,2}$	
		PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
1	0.05	0.05 (0.04)	0.20	0.08	0.60	0.04 (0.03)	0.14	0.07	0.54	0.04 (0.03)	0.09	0.06	0.44
2	0.00	-0.00 (0.04)	0.13	0.06	1.00	-0.01 (0.04)	0.08	0.04	1.00	-0.01 (0.04)	0.06	0.03	1.00
3	0.15	0.15 (0.03)	0.99	0.14	0.85	0.15 (0.03)	0.99	0.13	0.78	0.15 (0.03)	0.99	0.13	0.77
4	0.00	-0.00 (0.04)	0.05	0.04	1.00	-0.00 (0.04)	0.03	0.03	1.00	-0.01 (0.04)	0.02	0.02	1.00
5	0.25	0.24 (0.04)	1.00	0.15	0.90	0.25 (0.04)	1.00	0.14	0.87	0.25 (0.03)	1.00	0.14	0.86
6	0.10	0.09 (0.05)	0.19	0.11	0.57	0.10 (0.05)	0.19	0.12	0.56	0.10 (0.05)	0.17	0.11	0.54
7	0.20	0.19 (0.05)	0.97	0.20	0.88	0.19 (0.05)	0.97	0.20	0.88	0.19 (0.05)	0.97	0.20	0.87
8	0.00	-0.01 (0.05)	0.06	0.04	1.00	-0.02 (0.05)	0.05	0.04	1.00	-0.02 (0.05)	0.04	0.03	1.00
9	0.00	-0.02 (0.05)	0.06	0.04	1.00	-0.02 (0.05)	0.05	0.03	1.00	-0.02 (0.05)	0.04	0.02	1.00
10	0.00	-0.01 (0.05)	0.06	0.04	1.00	-0.01 (0.05)	0.04	0.03	1.00	-0.02 (0.05)	0.04	0.02	1.00

Table B.33: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I with $\underline{n=500}$ where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,k}$ (conditioning on $\gamma_{j,k}$=1), the medians of the posterior means of $\gamma_{j,k}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,k}$ and the coverage probability (CP) of the HPD intervals.

k	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,k}$	$\beta_{j,k}\|\gamma_{j,k}$=1	$\gamma_{j,k}$	$\beta_{j,k}$		$\beta_{j,k}\|\gamma_{j,k}$=1	$\gamma_{j,k}$	$\beta_{j,k}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	0.06 (0.02)	0.08	0.05	0.51	0.06 (0.02)	0.71	0.07	0.83
	2	0.10	0.08 (0.02)	1.00	0.08	0.76	0.10 (0.02)	1.00	0.08	0.87
	3	0.15	0.13 (0.02)	1.00	0.08	0.74	0.14 (0.02)	1.00	0.08	0.88
	4	0.20	0.18 (0.02)	1.00	0.08	0.73	0.19 (0.02)	1.00	0.08	0.90
1	5	0.25	0.23 (0.02)	1.00	0.08	0.74	0.24 (0.02)	1.00	0.08	0.88
	6	0.00	0.00 (0.02)	0.00	0.00	1.00	0.00 (0.02)	0.01	0.01	1.00
	7	0.00	-0.02 (0.02)	0.00	0.00	1.00	-0.01 (0.02)	0.01	0.00	1.00
	8	0.00	-0.01 (0.02)	0.01	0.00	1.00	-0.00 (0.02)	0.07	0.02	1.00
	9	0.00	-0.01 (0.02)	0.01	0.00	1.00	-0.00 (0.02)	0.07	0.02	1.00
	10	0.00	-0.01 (0.02)	0.01	0.00	1.00	-0.01 (0.02)	0.08	0.02	1.00
	1	0.05	0.05 (0.02)	0.63	0.06	0.71	0.05 (0.02)	0.88	0.07	0.86
	2	0.00	-0.01 (0.02)	0.00	0.00	1.00	-0.01 (0.02)	0.07	0.02	1.00
	3	0.15	0.15 (0.02)	1.00	0.06	0.95	0.15 (0.02)	1.00	0.06	0.95
	4	0.00	-0.00 (0.02)	0.00	0.00	1.00	-0.00 (0.02)	0.06	0.02	1.00
2	5	0.25	0.24 (0.02)	1.00	0.06	0.91	0.25 (0.02)	1.00	0.06	0.91
	6	0.10	0.10 (0.02)	1.00	0.09	0.93	0.10 (0.02)	1.00	0.09	0.94
	7	0.20	0.20 (0.02)	1.00	0.08	0.94	0.20 (0.02)	1.00	0.08	0.94
	8	0.00	-0.00 (0.02)	0.01	0.00	1.00	-0.00 (0.02)	0.05	0.01	1.00
	9	0.00	-0.00 (0.02)	0.01	0.00	1.00	0.00 (0.02)	0.05	0.01	1.00
	10	0.00	-0.00 (0.02)	0.01	0.00	1.00	0.00 (0.02)	0.06	0.02	1.00

Table B.34: Results from simulation studies with 100 replications using unstructured models with a conventional independent Bernoulli (IB) prior and with our proposed MRF prior for $\boldsymbol{\gamma}_k$ under Scenarios I with p=5 where the correlation between x_1 and x_2 equals 0.3. The medians of the posterior means (PM) and posterior standard deviations (SD) of $\beta_{j,k}$ (conditioning on $\gamma_{j,k}$=1), the medians of the posterior means of $\gamma_{j,k}$ (marginal posterior probabilities of inclusion) are provided. We also present the average width (WI) of the 95% highest posterior density (HPD) intervals for $\beta_{j,k}$ and the coverage probability (CP) of the HPD intervals. (Continued in Table B.35)

k	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,k}$	$\beta_{j,k}\|\gamma_{j,k}$=1	$\gamma_{j,k}$	$\beta_{j,k}$		$\beta_{j,k}\|\gamma_{j,k}$=1	$\gamma_{j,k}$	$\beta_{j,k}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	-0.02 (0.04)	0.02	0.04	0.11	0.04 (0.05)	0.31	0.12	0.55
	2	0.10	0.05 (0.04)	0.02	0.06	0.26	0.09 (0.05)	0.46	0.13	0.61
	3	0.15	0.12 (0.04)	0.18	0.11	0.45	0.15 (0.05)	0.85	0.17	0.70
	4	0.20	0.15 (0.04)	0.74	0.14	0.54	0.19 (0.05)	0.99	0.18	0.77
1	5	0.25	0.19 (0.04)	0.98	0.17	0.59	0.23 (0.05)	1.00	0.20	0.81
	6	0.00	-0.00 (0.05)	0.02	0.01	1.00	0.01 (0.05)	0.03	0.02	1.00
	7	0.00	-0.01 (0.06)	0.01	0.02	1.00	0.01 (0.06)	0.02	0.03	1.00
	8	0.00	-0.04 (0.05)	0.02	0.03	1.00	-0.00 (0.05)	0.05	0.03	1.00
	9	0.00	-0.05 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.05	0.04	1.00
	10	0.00	-0.05 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.04	0.03	1.00
	1	0.05	0.04 (0.03)	0.02	0.02	0.18	0.05 (0.04)	0.14	0.08	0.57
	2	0.00	-0.01 (0.04)	0.01	0.01	1.00	-0.00 (0.04)	0.08	0.06	0.99
	3	0.15	0.16 (0.04)	0.93	0.15	0.78	0.14 (0.04)	0.96	0.16	0.80
	4	0.00	-0.01 (0.04)	0.01	0.01	1.00	0.01 (0.04)	0.07	0.06	1.00
2	5	0.25	0.25 (0.04)	1.00	0.16	0.89	0.25 (0.04)	1.00	0.17	0.90
	6	0.10	0.10 (0.05)	0.08	0.10	0.45	0.12 (0.05)	0.18	0.14	0.65
	7	0.20	0.18 (0.05)	0.66	0.20	0.69	0.19 (0.05)	0.87	0.21	0.77
	8	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.04	0.02	1.00
	9	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.04	0.03	1.00
	10	0.00	-0.03 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.04	0.03	1.00

Table B.35: (Continued from Table B.34)

k	j	True	Unstructured with IB prior				Unstructured with MRF prior			
		$\beta_{j,k}$	$\beta_{j,k} \mid \gamma_{j,k}=1$	$\gamma_{j,k}$	$\beta_{j,k}$		$\beta_{j,k} \mid \gamma_{j,k}=1$	$\gamma_{j,k}$	$\beta_{j,k}$	
			PM (SD)	PM	WI	CP	PM (SD)	PM	WI	CP
	1	0.05	-0.02 (0.04)	0.02	0.04	0.12	0.04 (0.05)	0.25	0.11	0.53
	2	0.10	0.05 (0.04)	0.03	0.06	0.24	0.08 (0.04)	0.30	0.12	0.55
	3	0.15	0.12 (0.04)	0.11	0.11	0.43	0.14 (0.05)	0.67	0.15	0.60
	4	0.20	0.15 (0.04)	0.86	0.15	0.52	0.19 (0.05)	0.98	0.18	0.68
3	5	0.25	0.19 (0.04)	0.97	0.18	0.51	0.21 (0.05)	1.00	0.20	0.66
	6	0.00	0.02 (0.05)	0.02	0.01	1.00	0.03 (0.05)	0.04	0.03	1.00
	7	0.00	-0.00 (0.06)	0.01	0.00	1.00	0.01 (0.05)	0.02	0.02	1.00
	8	0.00	-0.03 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.04	0.03	1.00
	9	0.00	-0.05 (0.06)	0.02	0.02	1.00	-0.01 (0.05)	0.04	0.03	1.00
	10	0.00	-0.04 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.04	0.03	1.00
	1	0.05	0.05 (0.03)	0.02	0.03	0.26	0.07 (0.04)	0.26	0.11	0.66
	2	0.00	-0.01 (0.04)	0.01	0.02	1.00	0.02 (0.04)	0.10	0.06	1.00
	3	0.15	0.15 (0.04)	0.95	0.15	0.79	0.15 (0.04)	0.98	0.17	0.87
	4	0.00	-0.01 (0.04)	0.01	0.02	1.00	0.01 (0.05)	0.09	0.06	1.00
4	5	0.25	0.25 (0.04)	1.00	0.16	0.85	0.26 (0.04)	1.00	0.18	0.88
	6	0.10	0.09 (0.05)	0.06	0.08	0.38	0.11 (0.05)	0.18	0.13	0.60
	7	0.20	0.18 (0.05)	0.62	0.20	0.72	0.19 (0.05)	0.87	0.22	0.79
	8	0.00	-0.04 (0.05)	0.02	0.02	0.99	-0.01 (0.05)	0.05	0.03	1.00
	9	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.02 (0.05)	0.04	0.02	1.00
	10	0.00	-0.04 (0.05)	0.02	0.01	1.00	-0.02 (0.05)	0.04	0.02	1.00
	1	0.05	-0.02 (0.04)	0.04	0.05	0.17	0.05 (0.04)	0.34	0.12	0.49
	2	0.10	0.05 (0.04)	0.03	0.06	0.28	0.09 (0.05)	0.39	0.12	0.56
	3	0.15	0.12 (0.04)	0.22	0.13	0.48	0.15 (0.05)	0.81	0.16	0.60
	4	0.20	0.14 (0.04)	0.64	0.16	0.48	0.18 (0.05)	0.98	0.18	0.63
5	5	0.25	0.19 (0.04)	0.97	0.19	0.55	0.23 (0.05)	1.00	0.20	0.68
	6	0.00	0.00 (0.05)	0.02	0.00	1.00	0.02 (0.05)	0.03	0.02	1.00
	7	0.00	0.00 (0.06)	0.01	0.01	1.00	0.02 (0.05)	0.02	0.02	1.00
	8	0.00	-0.04 (0.05)	0.02	0.02	1.00	-0.01 (0.05)	0.04	0.03	1.00
	9	0.00	-0.05 (0.05)	0.02	0.03	1.00	-0.01 (0.05)	0.05	0.03	1.00
	10	0.00	-0.05 (0.05)	0.02	0.03	1.00	-0.02 (0.05)	0.04	0.03	1.00

C Application to NAS data

In order to supplement the results from the application in the main paper, we provide the empirical correlations between gene-specific methylation scores in the NAS data in Table C.1 and the posterior means and posterior standard deviations, 95% highest posterior density intervals for the regression coefficients corresponding to all of the 27 outcomes in Table C.2. We also provide trace plots in Figure C.1 for assessing convergence of the MCMC sampler.

Table C.1: NAS data: empirical correlations between gene-specific methylation scores in the asthma pathway.

	HLA	HLA	HLA	HLA	HLA	HLA	HLA	HLA	HLA	HLA	HLA	IL4	CD40	CD40	FCE	MS4	FCE	IL9	IL10	IL13	IL5	TNF	IL3	PRG2	RNA	EPX	CCL
	-DMA	-DMB	-DOA	-DOB	-DPA1	-DPB1	-DQA1	-DQA2	-DRA	-DRB1	-DRB5		LG		R1A	A2	R1G							SE3		11	
HLA-DMA	1.00																										
HLA-DMB	0.04	1.00																									
HLA-DOA	-0.00	-0.09	1.00																								
HLA-DOB	0.06	0.23	0.09	1.00																							
HLA-DPA1	0.05	0.03	0.11	0.06	1.00																						
HLA-DPB1	-0.08	0.23	0.05	0.10	-0.04	1.00																					
HLA-DQA1	-0.03	-0.09	0.13	0.03	0.11	-0.06	1.00																				
HLA-DQA2	-0.05	0.05	-0.00	-0.04	0.12	-0.01	0.13	1.00																			
HLA-DRA	0.08	0.22	0.11	0.12	0.18	0.02	0.16	0.18	1.00																		
HLA-DRB1	-0.16	-0.11	-0.05	0.01	-0.16	0.00	-0.01	-0.10	-0.10	1.00																	
HLA-DRB5	0.13	-0.03	0.24	0.33	-0.12	0.19	0.00	0.02	-0.07	-0.02	1.00																
IL4	-0.08	-0.02	0.21	-0.10	0.19	0.04	0.10	0.09	0.13	0.14	-0.12	1.00															
CD40LG	-0.09	-0.06	-0.00	-0.18	-0.03	-0.02	-0.20	-0.06	-0.07	0.03	-0.09	-0.20	1.00														
CD40	-0.16	0.21	-0.06	0.09	0.19	-0.10	-0.17	0.06	-0.07	0.03	-0.18	0.13	0.12	1.00													
FCER1A	-0.09	0.26	0.04	-0.01	0.08	0.03	0.01	-0.01	0.07	-0.31	0.13	-0.11	-0.10	0.15	1.00												
MS4A2	-0.08	0.11	0.03	0.21	-0.02	0.29	0.04	0.02	-0.09	-0.16	0.23	-0.02	-0.18	-0.10	0.09	1.00											
FCER1G	-0.05	-0.07	0.03	-0.03	0.01	0.07	-0.01	0.05	0.12	0.10	-0.01	0.23	-0.03	0.07	-0.14	-0.12	1.00										
IL9	-0.02	0.25	-0.09	0.31	0.07	0.51	0.10	0.03	0.18	-0.14	0.08	0.00	-0.14	-0.10	0.01	0.49	0.01	1.00									
IL10	0.03	-0.12	0.32	-0.05	-0.07	0.05	0.15	-0.04	-0.08	-0.03	0.13	-0.13	0.09	-0.13	0.02	0.18	-0.13	0.10	1.00								
IL13	0.19	-0.08	0.16	-0.05	0.07	-0.02	0.22	-0.11	0.12	-0.13	0.17	0.01	-0.07	-0.16	0.10	-0.00	-0.16	0.02	-0.13	1.00							
IL5	-0.09	-0.03	0.07	0.10	-0.07	0.18	0.02	0.19	-0.09	-0.01	0.16	-0.08	-0.05	0.12	0.06	0.36	-0.10	0.35	0.29	0.07	1.00						
TNF	-0.02	-0.31	0.04	-0.10	0.04	-0.07	-0.09	-0.01	-0.14	0.13	0.00	-0.01	0.08	0.11	-0.07	-0.13	0.32	-0.16	-0.00	0.02	0.14	1.00					
IL3	0.01	0.14	-0.17	0.05	-0.02	-0.04	0.26	0.04	0.07	-0.10	-0.14	-0.07	-0.12	-0.14	0.11	0.07	-0.03	0.13	0.00	0.04	-0.11	-0.02	1.00				
PRG2	-0.18	-0.39	0.14	-0.12	-0.01	-0.06	0.19	-0.06	0.01	0.08	0.08	0.38	-0.12	0.01	-0.24	-0.20	0.22	-0.14	-0.03	-0.05	-0.17	0.11	-0.16	1.00			
RNASE3	0.05	-0.16	0.09	0.06	0.07	0.03	0.31	0.37	0.20	-0.04	0.05	-0.00	-0.26	-0.15	-0.08	0.07	0.03	0.07	0.01	0.08	-0.00	0.07	0.30	-0.00	1.00		
EPX	-0.10	-0.22	-0.01	-0.15	-0.10	-0.10	-0.11	-0.22	-0.07	0.17	-0.11	0.27	0.12	0.02	-0.16	-0.08	0.39	-0.24	0.00	-0.08	-0.06	0.38	0.03	0.25	-0.16	1.00	
CCL11	-0.09	-0.08	0.23	0.31	0.11	0.10	0.05	0.16	0.01	-0.01	0.57	-0.13	-0.15	0.02	0.00	0.25	0.10	0.19	0.24	-0.05	0.37	0.00	-0.13	0.06	0.16	-0.14	1.00

Table C.2: Estimated covariate effects on the 27 outcomes in NAS data. Posterior mean (PM) and posterior standard deviation (SD) of $\beta_{j,k}$ (conditioning on $\gamma_{j,k}$=1), 95% highest posterior density (HPD) interval for $\beta_{j,k}$ are provided.

	Unstructured with IB prior				Unstructured with MRF prior			
	Black Carbon (k=1)		Sulfate (k=2)		Black Carbon (k=1)		Sulfate (k=2)	
	$\beta_{j,k}\|\gamma_{j,k}$=1	$\beta_{j,k}$	$\beta_{j,k}\|\gamma_{j,k}$=1	$\beta_{j,k}$	$\beta_{j,k}\|\gamma_{j,k}$=1	$\beta_{j,k}$	$\beta_{j,k}\|\gamma_{j,k}$=1	$\beta_{j,k}$
	PM (SD)	95% HPDI	PM (SD)	95% HPDI	PM (SD)	95% HPDI	PM (SD)	95% HPDI
CCL11	-0.17 (0.11)	(-0.10, 0.00)	0.32 (0.14)	(-0.00, 0.47)	-0.17 (0.13)	(-0.27, 0.00)	0.29 (0.11)	(0.00, 0.46)
EPX	0.18 (0.12)	(-0.00, 0.13)	0.03 (0.12)	(0.00, 0.00)	0.11 (0.12)	(-0.01, 0.23)	0.00 (0.13)	(-0.02, 0.02)
RNASE3	-0.23 (0.11)	(-0.28, 0.00)	-0.27 (0.11)	(-0.36, 0.00)	-0.18 (0.12)	(-0.27, 0.00)	-0.23 (0.11)	(-0.36, 0.00)
PRG2	0.04 (0.10)	(0.00, 0.00)	0.05 (0.10)	(0.00, 0.00)	0.02 (0.10)	(-0.04, 0.00)	0.04 (0.10)	(-0.03, 0.01)
IL3	0.06 (0.12)	(0.00, 0.00)	0.00 (0.14)	(0.00, 0.00)	0.07 (0.12)	(-0.00, 0.04)	-0.01 (0.13)	(-0.04, 0.05)
TNF	-0.11 (0.12)	(0.00, 0.00)	0.04 (0.13)	(0.00, 0.00)	-0.08 (0.12)	(-0.15, 0.01)	0.02 (0.12)	(-0.05, 0.01)
IL5	0.00 (0.12)	(0.00, 0.00)	0.27 (0.10)	(0.00, 0.39)	-0.02 (0.12)	(0.00, 0.00)	0.29 (0.11)	(0.00, 0.44)
IL13	0.08 (0.12)	(0.00, 0.00)	0.12 (0.11)	(0.00, 0.00)	0.08 (0.11)	(-0.00, 0.10)	0.10 (0.11)	(-0.00, 0.13)
IL10	0.01 (0.10)	(0.00, 0.00)	0.21 (0.10)	(0.00, 0.28)	-0.00 (0.11)	(0.00, 0.00)	0.23 (0.10)	(0.00, 0.34)
IL9	-0.22 (0.07)	(-0.32, 0.00)	0.12 (0.10)	(0.00, 0.00)	-0.23 (0.08)	(-0.35, 0.00)	0.14 (0.11)	(-0.00, 0.13)
FCER1G	-0.27 (0.09)	(-0.40, 0.00)	-0.22 (0.10)	(-0.24, 0.00)	-0.27 (0.09)	(-0.40, 0.00)	-0.23 (0.12)	(-0.30, 0.00)
MS4A2	-0.05 (0.11)	(0.00, 0.00)	-0.01 (0.11)	(0.00, 0.00)	-0.02 (0.11)	(-0.09, 0.04)	0.05 (0.11)	(-0.02, 0.12)
FCER1A	0.09 (0.10)	(0.00, 0.00)	-0.04 (0.09)	(0.00, 0.00)	0.11 (0.09)	(0.00, 0.13)	-0.06 (0.10)	(-0.04, 0.00)
CD40	0.01 (0.11)	(0.00, 0.00)	-0.03 (0.11)	(0.00, 0.00)	0.01 (0.09)	(-0.03, 0.00)	-0.00 (0.12)	(-0.04, 0.06)
CD40LG	0.11 (0.10)	(0.00, 0.00)	-0.14 (0.09)	(-0.08, 0.00)	0.12 (0.10)	(0.00, 0.15)	-0.16 (0.10)	(-0.24, 0.00)
IL4	-0.01 (0.09)	(0.00, 0.00)	-0.09 (0.10)	(0.00, 0.00)	-0.03 (0.09)	(-0.03, 0.02)	-0.11 (0.09)	(-0.13, 0.00)
HLA-DRB5	0.17 (0.07)	(0.00, 0.23)	0.24 (0.10)	(0.00, 0.30)	0.19 (0.08)	(0.00, 0.29)	0.20 (0.10)	(0.00, 0.29)
HLA-DRB1	-0.01 (0.10)	(0.00, 0.00)	0.14 (0.09)	(-0.00, 0.11)	0.02 (0.09)	(-0.01, 0.02)	0.09 (0.09)	(-0.00, 0.10)
HLA-DRA	-0.26 (0.09)	(-0.39, 0.00)	0.25 (0.10)	(0.00, 0.30)	-0.27 (0.09)	(-0.41, 0.00)	0.23 (0.10)	(0.00, 0.30)
HLA-DQA2	0.23 (0.09)	(0.00, 0.34)	0.13 (0.10)	(0.00, 0.00)	0.20 (0.09)	(0.00, 0.31)	0.13 (0.11)	(0.00, 0.14)
HLA-DQA1	0.10 (0.09)	(0.00, 0.00)	-0.15 (0.09)	(-0.13, 0.00)	0.07 (0.09)	(-0.00, 0.08)	-0.13 (0.10)	(-0.17, 0.00)
HLA-DPB1	0.14 (0.10)	(0.00, 0.11)	-0.19 (0.09)	(-0.27, 0.00)	0.14 (0.12)	(-0.00, 0.18)	-0.20 (0.09)	(-0.31, 0.00)
HLA-DPA1	-0.01 (0.10)	(0.00, 0.00)	-0.18 (0.09)	(-0.23, 0.00)	-0.04 (0.09)	(-0.00, 0.00)	-0.15 (0.10)	(-0.23, 0.00)
HLA-DOB	0.16 (0.09)	(0.00, 0.19)	-0.08 (0.08)	(0.00, 0.00)	0.16 (0.09)	(-0.00, 0.26)	-0.08 (0.11)	(-0.15, 0.00)
HLA-DOA	0.07 (0.08)	(0.00, 0.00)	-0.05 (0.10)	(0.00, 0.00)	0.11 (0.08)	(0.00, 0.15)	-0.03 (0.10)	(-0.09, 0.02)
HLA-DMB	0.07 (0.09)	(0.00, 0.00)	0.03 (0.08)	(0.00, 0.00)	0.02 (0.08)	(-0.02, 0.07)	0.02 (0.09)	(-0.01, 0.06)
HLA-DMA	-0.06 (0.08)	(0.00, 0.00)	0.16 (0.08)	(0.00, 0.19)	-0.03 (0.08)	(0.00, 0.00)	0.15 (0.09)	(0.00, 0.22)

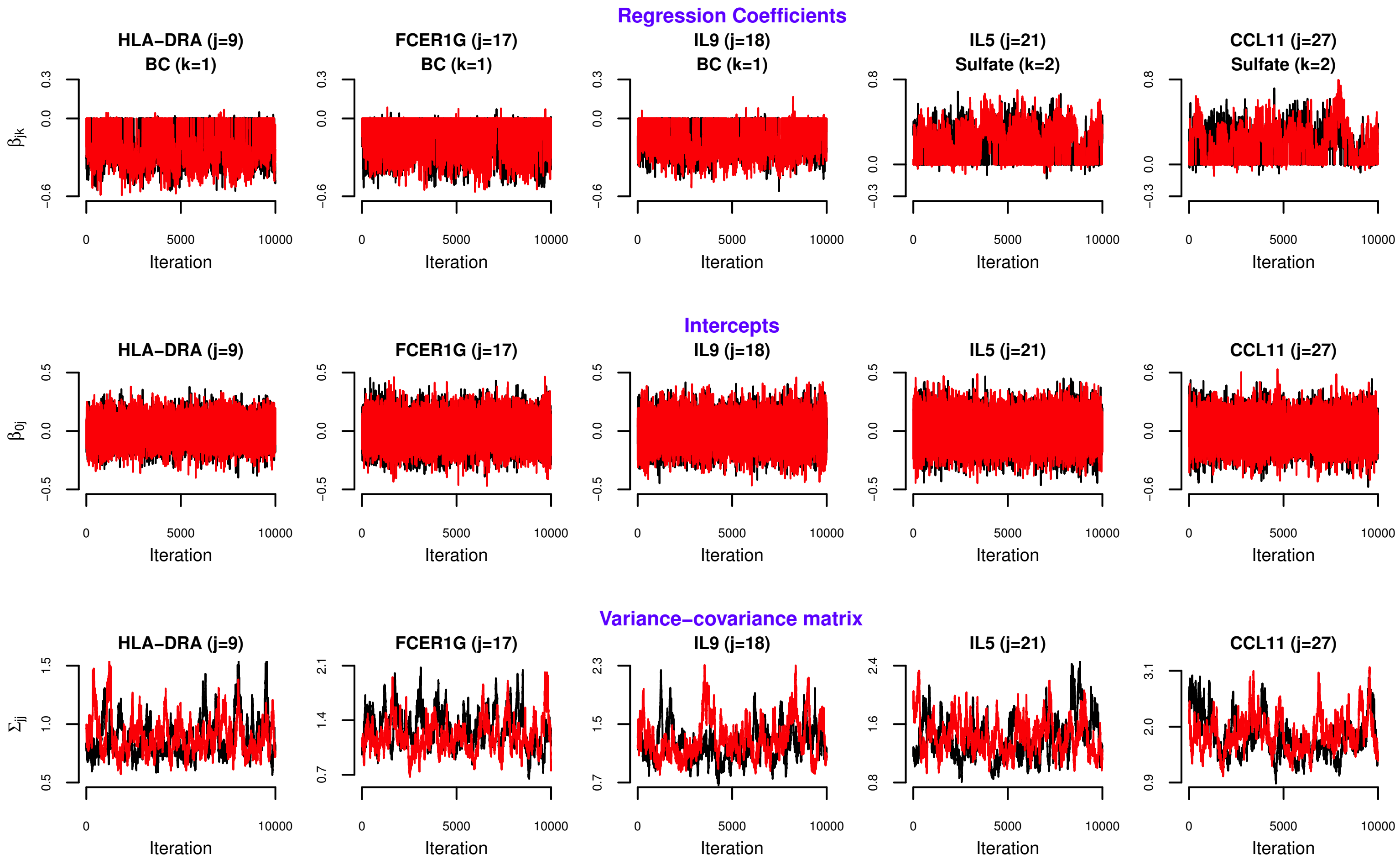

Figure C.1: Trace plots of model parameters for genes identified to have association with black carbon or sulfate in two MCMC chains (represented in red and black) from the fit of the unstructured model with MRF prior of Section 4.

References

Browne, W. J. (2006). MCMC algorithms for constrained variance matrices. *Computational Statistics and Data Analysis* **50,** 1655–1677.