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Supplementary Materials for

High-dimensional quantum cloning and applications to quantum hacking

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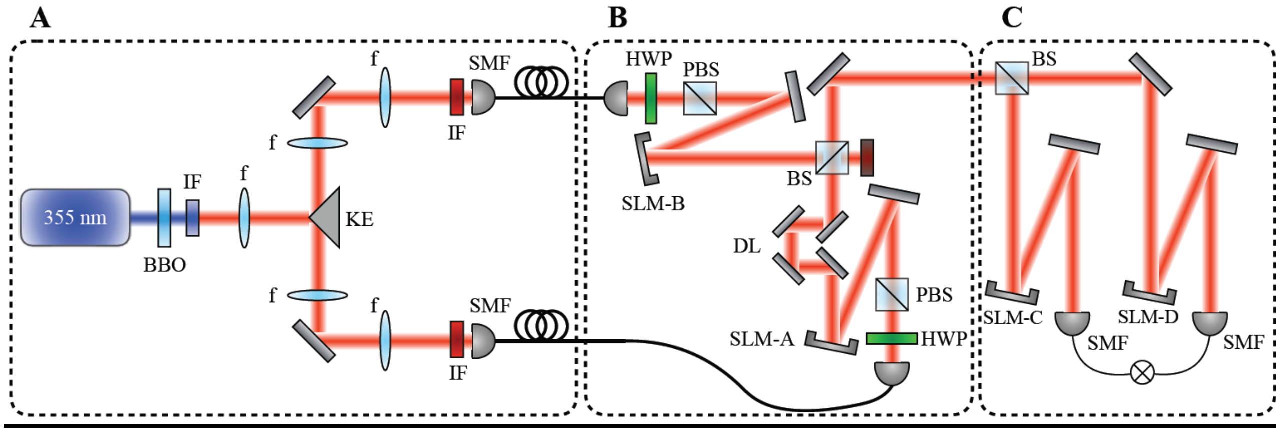

fig. S1. Detailed experimental setup.

Cloning fidelities for various mutually unbiased bases:

In quantum information, mutually unbiased bases (MUBs) are orthogonal bases such that two elements, $|\psi_i\rangle$ and $|\phi_j\rangle$, belonging to different MUBs obeys the following inner product relation

$$|\langle\psi_i|\phi_j\rangle|^2 = \frac{1}{d}$$

An important result in quantum information is the existence of a number of MUBs for a specific dimension. In the particular case of dimensions that are powers of prime numbers, the number of MUBs is given by $d+1$. Let us now give explicit expressions for each elements of every MUBs, $|\psi_n^{(\alpha)}\rangle$, where $n \in [1, d]$ and $\alpha \in [1, d+1]$. The first MUB is given by the logical basis, i.e. $\left|\psi_n^{(1)}\right\rangle = |n\rangle$. Further, the elements of each MUBs with $\alpha \geq 2$ are given by the following expressions

$$|\psi_n^{(\alpha)}\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} \exp\left[\frac{2\pi i}{d}\left((n-1)(d-j+1) - (\alpha-2)\left(\sum_{m=j-2}^{d+1} m\right)\right)\right] |i\rangle$$

In order to test the universality of our cloner, we perform cloning fidelity measurements for each element of every MUBs, see fig. S2.

MUB	I	II	III	IV	V	VI	VII	VIII
$\|\psi_1\rangle$	0.62 ± 0.02	0.57 ± 0.01	0.594 ± 0.006	0.589 ± 0.007	0.602 ± 0.007	0.576 ± 0.005	0.590 ± 0.007	0.62 ± 0.01
$\|\psi_2\rangle$	0.61 ± 0.01	0.56 ± 0.01	0.617 ± 0.007	0.585 ± 0.007	0.598 ± 0.007	0.610 ± 0.006	0.592 ± 0.007	0.604 ± 0.008
$\|\psi_3\rangle$	0.610 ± 0.008	0.58 ± 0.01	0.617 ± 0.008	0.600 ± 0.007	0.584 ± 0.006	0.60751 ± 0.007	0.586 ± 0.006	0.578 ± 0.007
$\|\psi_4\rangle$	0.604 ± 0.006	0.57 ± 0.01	0.592 ± 0.008	0.585 ± 0.006	0.580 ± 0.006	0.580 ± 0.006	0.583 ± 0.006	0.576 ± 0.006
$\|\psi_5\rangle$	0.607 ± 0.008	0.559 ± 0.009	0.597 ± 0.008	0.591 ± 0.006	0.579 ± 0.006	0.593 ± 0.007	0.588 ± 0.006	0.584 ± 0.006
$\|\psi_6\rangle$	0.60 ± 0.01	0.58 ± 0.01	0.581 ± 0.007	0.577 ± 0.006	0.587 ± 0.006	0.591 ± 0.007	0.586 ± 0.006	0.599 ± 0.007
$\|\psi_7\rangle$	0.62 ± 0.02	0.555 ± 0.008	0.588 ± 0.007	0.585 ± 0.007	0.587 ± 0.006	0.583 ± 0.006	0.592 ± 0.007	0.628 ± 0.009

fig. S2. Experimental cloning fidelities for every element of each MUB in dimension 7. The elements $|\psi_i\rangle$ corresponds to the input state that is to be cloned. Experimental data are shown for all $d + 1$ MUBs (I)-(VIII). In the bottom, the transverse profile of each element are shown for every MUBs. The transverse profile is illustrated by plotting the transverse phase modulated by the intensity profile, allowing one to visualize both the phase and intensity pattern at the same time.

High-dimensional quantum state tomography:

In order to reconstruct the density matrix of an arbitrary state, we can perform quantum state tomography. For a given dimension d, where d is a power of a prime number, it is possible to reconstruct the density matrix of a given state by projecting it over each d elements of every $d + 1$ MUBs. In our case, this corresponds to a total of 56 projective measurements. Such projective measurements are expressed as $P_n^{(\alpha)}$, where n labels the states with $(n = 1, \dots, d)$ and α labels the MUB with $(\alpha = 1, \dots, d + 1)$. These measurements are formally connected to projection operators given by $\Pi_{\mathrm{n}}^{(\alpha)} = \left|\psi_n^{(\alpha)}\rangle\langle\psi_n^{(\alpha)}\right|$. Experimentally recorded projective measurements may now be expressed in terms of their associated projection operators, i.e. $P_n^{(\alpha)} = \mathrm{Tr}\left[\hat{\rho}\, \Pi_n^{(\alpha)}\right]$, where $\mathrm{Tr}[.]$ is the trace operator. Finally, the reconstructed density matrix may be expressed in terms of the projective measurements

$$\hat{\rho} = \sum_{\alpha=1}^{d+1} \sum_{n=1}^{d} P_n^{(\alpha)} \Pi_{\mathrm{n}}^{(\alpha)} - \hat{I}$$

where $\hat{I}$ is the d-dimensional identity matrix. For the case of a *Gaussian* state given earlier by, $|\psi_{\text{Gauss}}\rangle = \mathcal{N} \sum_{\ell=-3}^{\ell=3} \exp[-(\ell/2)^2] \, |\ell\rangle$, one may perform sequence of projective measurements to tomographically reconstruct its density matrix. Those projective measurements for the input and cloned states are shown in fig. S3. Furthermore, the cloning fidelity of the cloned state can be directly calculated by considering, $\mathcal{F} = \langle \psi_{\text{Gauss}} | \hat{\rho}_{\text{cl}} | \psi_{\text{Gauss}} \rangle$. From the set of measurements presented in fig. S3, one can readily see how a slight misalignment in one MUB can lead to larger misalignment in a different MUB. Moreover, all 56 measurements are in some sense *added* up to reconstruct the density matrix, hence leading to a degraded state fidelity, i.e. 0.80 ± 0.03 for the cloned state.

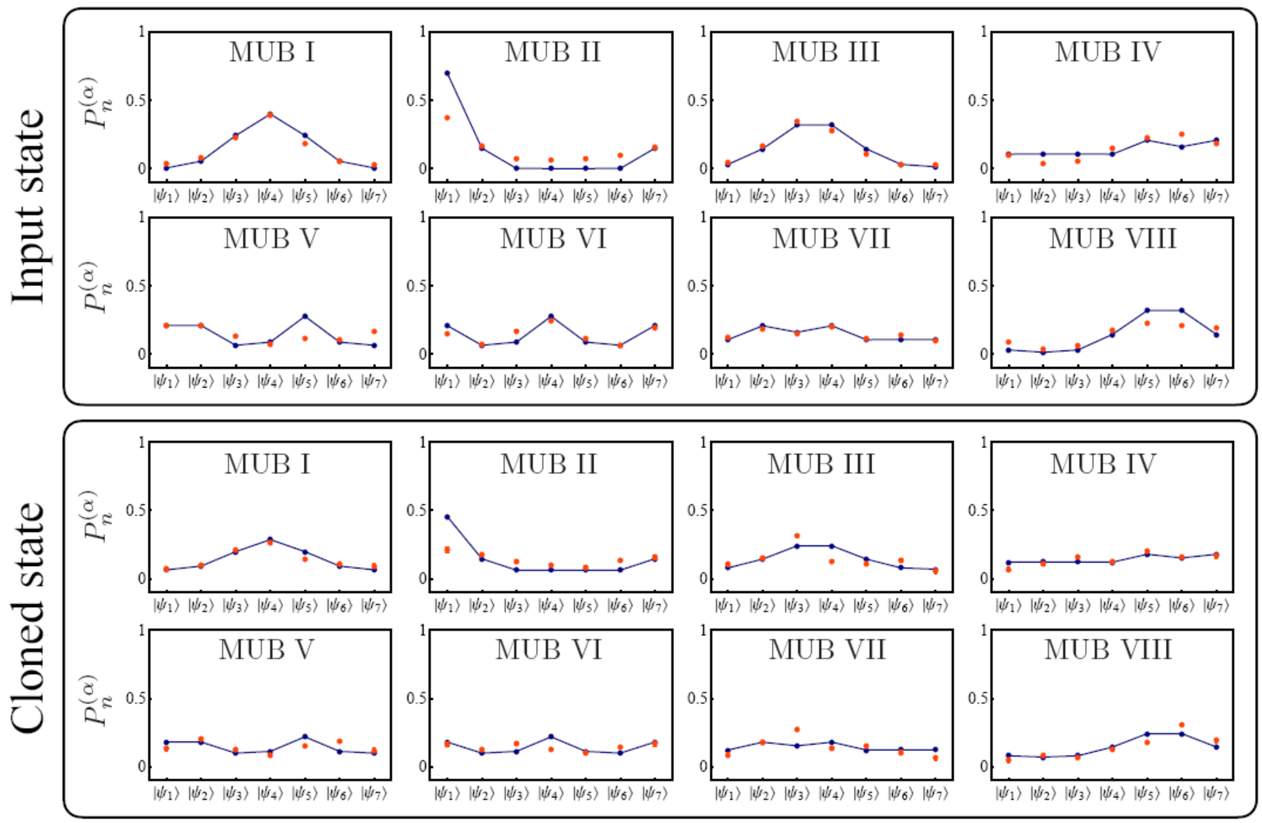

fig. S3. Projective measurements of the input and cloned Gaussian state. Projective measurements $P_n^{(\alpha)}$ are shown for $n = 1, \dots, d$ and $\alpha = 1, \dots, d+1$. The red points correspond to experimental results and the joined blue points correspond to theoretical values.