Web Appendix for 'Comparison of Two Correlated ROC Curves at a Given Specificity or Sensitivity Level' by Leonidas E. Bantis and Ziding Feng

Web-Appendix A (Simulation results)

Table 1: Comparison of sizes of Z and Z^* tests. Simulation results of 1000 replications for the scenario in which data representing diseased and healthy individuals are generated by the same bivariate normal distributions (valid null hypothesis since $ROC_1(t) = ROC_2(t)$). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7$, and 0.8. Normality is also assumed for estimation. Sample sizes explored are $(100, 100)$, $(200, 200)$ and $(100, 200)$. Correlation is set equal to $\rho = 0.2, 0.4, 0.6$ for the data of both the healthy and diseased individuals.

			Size results							
			Z test				Z^* test			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.051	0.054	0.056	0.046	0.048	0.053	0.054	0.048
	0.2	0.7	0.051	0.055	0.056	0.040	0.049	0.055	0.055	0.051
		0.8	0.050	0.058	0.044	0.018	0.049	0.060	0.057	0.051
		0.6	0.057	0.055	0.055	0.049	0.051	0.053	0.053	0.049
$100, 100$	0.4	0.7	0.049	0.052	0.048	0.032	0.047	0.052	0.049	0.046
		0.8	0.048	0.058	0.047	0.011	0.048	0.059	0.055	0.050
		0.6	0.052	0.052	0.057	0.041	0.051	0.052	0.055	0.045
	0.6	0.7	0.049	0.050	0.048	0.027	0.045	0.050	0.050	0.045
		0.8	0.047	0.050	0.036	0.010	0.046	0.054	0.050	0.045
		0.6	0.056	0.060	0.062	0.059	0.054	0.060	0.062	0.060
	0.2	0.7	0.057	0.057	0.060	0.051	0.053	0.057	0.059	0.061
		0.8	0.051	0.059	0.054	0.043	0.049	0.059	0.063	0.063
		0.6	0.052	0.066	0.065	0.056	0.051	0.065	0.065	0.058
$200, 200$	0.4	0.7	0.057	0.058	0.060	0.046	0.056	0.058	0.060	0.056
		0.8	0.057	0.060	0.054	0.035	0.055	0.060	0.061	0.061
		0.6	0.050	0.064	0.059	0.045	0.050	0.064	0.058	0.047
	0.6	0.7	0.055	0.059	0.055	0.039	0.055	0.059	0.055	0.052
		0.8	0.053	0.050	0.050	0.032	0.053	0.049	0.060	0.058
		0.6	0.049	0.049	0.045	0.052	0.046	0.048	0.043	0.051
	0.2	0.7	0.056	0.049	0.047	0.042	0.055	0.049	0.048	0.048
		0.8	0.057	0.040	0.040	0.024	0.056	0.043	0.046	0.050
		0.6	0.056	0.040	0.044	0.045	0.051	0.040	0.044	0.045
$100, 200$	0.4	0.7	0.056	0.042	0.041	0.040	0.054	0.044	0.041	0.046
		0.8	0.056	0.041	0.038	0.024	0.058	0.041	0.042	0.047
		0.6	0.054	0.035	0.043	0.042	0.051	0.035	0.044	0.042
	0.6	0.7	0.054	0.044	0.040	0.041	0.053	0.045	0.039	0.043
		0.8	0.052	0.042	0.032	0.023	0.056	0.043	0.046	0.050

Table 2: Comparison of the power of Z and Z^* tests. Simulation results of 1000 replications for the normal setting (non-valid null hypothesis) under different scenarios. Normality is also assumed for estimation. Sample sizes explored are $(100, 100)$, $(200, 200)$, $(300, 300)$, and $(100, 300)$. Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for the data of both the healthy and diseased individuals. The true difference $ROC_2(t) - ROC_1(t)$ to be detected is denoted by d.

Power results									
		Z test				Z^* test			
n_A, n_B	ρ	$t = 0.540$	$t = 0.648$	$t = 0.753$	$t = 0.863$	$t = 0.540$	$t = 0.648$	$t = 0.753$	$t = 0.863$
		$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$
Assuming Normality									
	0.2	0.786	0.704	0.523	0.203	0.812	0.756	0.702	0.624
$50, 50$	0.4	0.872	0.796	0.585	0.216	0.897	0.863	0.800	0.6920
	0.6	0.970	0.913	0.694	0.245	0.977	0.964	0.925	0.8390
	0.2	0.975	0.963	0.918	0.713	0.978	0.971	0.951	0.900
$100, 100$	0.4	0.993	0.988	0.962	0.770	0.993	0.991	0.980	0.949
	0.6	1.000	0.997	0.989	0.862	1.000	0.999	0.995	0.986
	0.2	1.000	0.999	0.999	0.980	1.000	0.999	0.999	0.991
$200, 200$	0.4	1.000	1.000	1.000	0.994	1.000	1.000	1.000	0.999
	0.6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Box-Cox									
	0.2	0.769	0.655	0.486	0.181	0.755	0.692	0.602	0.505
$50, 50$	0.4	0.861	0.745	0.543	0.192	0.866	0.788	0.690	0.564
	0.6	0.933	0.859	0.626	0.223	0.939	0.899	0.825	0.696
	0.2	0.972	0.953	0.891	0.656	0.970	0.961	0.909	0.825
$100, 100$	0.4	0.991	0.979	0.930	0.696	0.988	0.977	0.957	0.879
	0.6	1.000	0.996	0.979	0.782	0.998	0.993	0.984	0.946
	0.2	1.000	1.000	0.993	0.960	1.000	1.000	0.994	0.978
$200, 200$	0.4	1.000	1.000	1.000	0.982	0.999	0.998	0.998	0.991
	0.6	1.000	1.000	1.000	0.999	1.000	1.000	1.000	0.999

Table 3: Comparison of sizes of Z and Z^* tests. Simulation results of 1000 replications where data representing the diseased and healthy individuals are generated by the same bivariate normal distributions (valid null hypothesis). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7, and 0.8$. The Box-Cox approach is used for estimation. Sample sizes explored are $(100, 100)$, $(200, 200)$ and $(100, 200)$. Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for the data of both the healthy and diseased individuals.

			Size results							
			Z test				Z^* test			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.054	0.055	0.058	0.037	0.047	0.054	0.057	0.039
	0.2	0.7	0.054	0.056	0.049	0.035	0.047	0.053	0.051	0.040
		0.8	0.052	0.057	0.049	0.025	0.047	0.049	0.045	0.045
		0.6	0.053	0.053	0.054	0.039	0.049	0.051	0.049	0.041
$100, 100$	0.4	0.7	0.048	0.055	0.047	0.035	0.044	0.052	0.047	0.040
		0.8	0.048	0.051	0.045	0.015	0.046	0.050	0.045	0.045
		0.6	0.060	0.057	0.054	0.038	0.054	0.051	0.051	0.044
	0.6	0.7	0.051	0.056	0.047	0.022	0.046	0.053	0.051	0.035
		0.8	0.045	0.050	0.031	0.012	0.045	0.049	0.046	0.039
		0.6	0.054	0.061	0.061	0.056	0.053	0.059	0.058	0.055
	0.2	0.7	0.054	0.059	0.056	0.054	0.051	0.058	0.058	0.060
		0.8	0.050	0.061	0.062	0.050	0.050	0.062	0.061	0.058
		0.6	0.050	0.064	0.063	0.056	0.047	0.064	0.062	0.054
$200, 200$	0.4	0.7	0.055	0.057	0.060	0.050	0.049	0.056	0.057	0.053
		0.8	0.060	0.060	0.058	0.041	0.053	0.062	0.057	0.052
		0.6	0.051	0.063	0.060	0.047	0.043	0.064	0.062	0.049
	0.6	0.7	0.052	0.059	0.055	0.042	0.045	0.057	0.056	0.053
		0.8	0.057	0.055	0.053	0.033	0.054	0.057	0.053	0.052
		0.6	0.053	0.047	0.047	0.047	0.052	0.047	0.050	0.052
	0.2	0.7	0.060	0.050	0.049	0.047	0.053	0.049	0.045	0.046
		0.8	0.056	0.045	0.044	0.027	0.056	0.045	0.046	0.044
		0.6	0.056	0.042	0.048	0.051	0.045	0.041	0.045	0.051
$100, 200$	0.4	0.7	0.059	0.043	0.040	0.047	0.055	0.044	0.044	0.054
		0.8	0.053	0.046	0.043	0.028	0.053	0.044	0.048	0.044
		0.6	0.055	0.038	0.042	0.047	0.043	0.037	0.045	0.050
	0.6	0.7	0.054	0.046	0.047	0.045	0.045	0.044	0.045	0.051
		0.8	0.053	0.043	0.047	0.023	0.052	0.045	0.041	0.040

Table 4: Exploration of Z^* test size. Simulation results of 1000 replications where data representing the diseased and healthy individuals are generated by bivariate lognormal and bivariate gamma distributions (valid null hypothesis). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7, and 0.8$. The Box-Cox approach is used for estimation. Sample sizes explored are $(100, 100)$, $(200, 200)$ and $(100, 200)$. Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for the data of both the healthy and diseased individuals.

			Size results (Box-Cox)							
			Bivariate Gammas				Bivariate Lognormals			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.042	0.051	0.047	0.054	0.054	0.055	0.058	0.043
	0.2	0.7	0.045	0.042	0.049	0.049	0.060	0.057	0.055	0.043
		0.8	0.045	0.049	0.039	0.043	0.060	0.055	0.043	0.035
		0.6	0.030	0.039	0.046	0.045	0.057	0.056	0.058	0.050
$100, 100$	0.4	0.7	0.041	0.042	0.049	0.044	0.055	0.052	0.047	0.039
		0.8	0.039	0.050	0.048	0.046	0.054	0.054	0.048	0.035
		0.6	0.025	0.050	0.051	0.042	0.059	0.054	0.059	0.046
	0.6	0.7	0.036	0.051	0.058	0.048	0.054	0.053	0.051	0.038
		0.8	0.047	0.049	0.041	0.036	0.052	0.051	0.039	0.033
		0.6	0.048	0.059	0.057	0.054	0.069	0.079	0.082	0.068
	0.2	0.7	0.047	0.051	0.049	0.047	0.062	0.071	0.071	0.058
		0.8	0.044	0.049	0.048	0.039	0.066	0.065	0.067	0.055
		0.6	0.038	0.050	0.055	0.049	0.055	0.068	0.071	0.060
$200, 200$	0.4	0.7	0.045	0.040	0.033	0.033	0.058	0.062	0.060	0.051
		0.8	0.044	0.054	0.047	0.043	0.062	0.062	0.059	0.053
		0.6	0.036	0.045	0.051	0.051	0.054	0.065	0.062	0.046
	0.6	0.7	0.033	0.042	0.044	0.041	0.057	0.057	0.058	0.044
		0.8	0.036	0.040	0.044	0.043	0.057	0.053	0.057	0.051
		0.6	0.039	0.040	0.052	0.053	0.065	0.056	0.050	0.051
	0.2	0.7	0.039	0.049	0.048	0.045	0.061	0.054	0.052	0.051
		0.8	0.044	0.049	0.048	0.045	0.068	0.053	0.048	0.050
		0.6	0.034	0.041	0.037	0.054	0.058	0.054	0.047	0.050
$100, 200$	0.4	0.7	0.041	0.049	0.045	0.040	0.055	0.052	0.044	0.052
		0.8	0.045	0.042	0.038	0.032	0.055	0.043	0.047	0.047
		0.6	0.038	0.044	0.051	0.053	0.049	0.041	0.044	0.044
	0.6	0.7	0.029	0.041	0.042	0.045	0.052	0.051	0.042	0.042
		0.8	0.037	0.041	0.038	0.042	0.051	0.041	0.046	0.040

Table 5: Power simulation results of 1000 replications for the non-normal settings (non-valid null hypothesis) under different scenarios. The kernel-based method is considered for estimation. Sample sizes explored are $(100, 100)$, $(200, 200)$, $(300, 300)$, and $(100, 300)$. Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for data of both the healthy and diseased individuals. The true difference $ROC_2(t) - ROC_1(t)$ to be detected is denoted by d.

Power results									
		Bivariate Gammas				Bivariate Lognormals			
n_A, n_B	ρ	$t = 0.540$	$t = 0.648$	$t = 0.753$	$t = 0.863$	$t = 0.3790$	$t = 0.5350$	$t = 0.6890$	$t = 0.847$
		$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$
Box-Cox approach									
	0.2	0.503	0.462	0.347	0.217	0.733	0.642	0.503	0.337
50, 50	0.4	0.600	0.544	0.419	0.273	0.842	0.755	0.607	0.397
	0.6	0.758	0.687	0.532	0.333	0.953	0.893	0.756	0.526
	0.2	0.806	0.739	0.610	0.430	0.942	0.905	0.820	0.637
100, 100	0.4	0.892	0.833	0.688	0.493	0.986	0.967	0.894	0.726
	0.6	0.973	0.949	0.842	0.621	1.000	0.994	0.975	0.855
	0.2	0.977	0.965	0.885	0.718	0.996	0.992	0.969	0.890
200, 200	0.4	0.999	0.991	0.944	0.801	1.000	1.000	0.993	0.949
	0.6	1.000	0.999	0.990	0.897	1.000	1.000	1.000	0.993
Kernel-based approach									
	0.2	0.4180	0.3930	0.3000	0.1790	0.6250	0.5200	0.3340	0.1620
50, 50	0.4	0.5190	0.4620	0.3690	0.2270	0.7200	0.5810	0.4000	0.1930
	0.6	0.6310	0.5710	0.4390	0.2770	0.8480	0.7250	0.5310	0.2610
	0.2	0.7240	0.6850	0.5620	0.3690	0.9010	0.7960	0.5910	0.3050
100, 100	0.4	0.8060	0.7900	0.6440	0.4210	0.9500	0.8790	0.6950	0.3670
	0.6	0.9270	0.8970	0.7720	0.5140	0.9910	0.9570	0.8320	0.5160
	0.2	0.9340	0.9320	0.8490	0.6340	0.9950	0.9720	0.8730	0.5470
200, 200	0.4	0.9790	0.9770	0.9170	0.7300	0.9990	0.9910	0.9260	0.6400
	0.6	0.9970	0.9930	0.9700	0.8260	1.0000	1.0000	0.9770	0.7920

Table 6: Exploration of Z^* test size. Simulation results of 1000 replications where data representing diseased and healthy individuals are generated by bivariate lognormal and bivariate gamma distributions (valid null hypothesis since $ROC_1(t) = ROC_2(t)$). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7$, and 0.8. The kernel-based approach is used for estimation. Sample sizes explored are (100, 100), (200, 200) and (100, 200). Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for the data of both the healthy and diseased individuals.

			Size results (Kernel-based Z^* statistic)							
			Bivariate Gammas				Bivariate Lognormals			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.034	0.036	0.040	0.049	0.044	0.045	0.053	0.057
	0.2	0.7	0.033	0.039	0.032	0.046	0.042	0.052	0.057	0.057
		0.8	0.047	0.049	0.051	0.051	0.039	0.049	0.057	0.060
		0.6	0.035	0.031	0.037	0.045	0.048	0.055	0.054	0.053
100, 100	0.4	0.7	0.039	0.035	0.041	0.039	0.048	0.058	0.049	0.054
		0.8	0.040	0.038	0.043	0.048	0.048	0.052	0.046	0.051
		0.6	0.036	0.0440	0.0430	0.0310	0.0480	0.062	0.055	0.043
	0.6	0.7	0.042	0.0450	0.0460	0.0570	0.0530	0.061	0.048	0.045
		0.8	0.043	0.0440	0.0410	0.0440	0.0490	0.047	0.047	0.043
		0.6	0.054	0.052	0.050	0.042	0.051	0.055	0.055	0.063
	0.2	0.7	0.049	0.046	0.045	0.051	0.051	0.056	0.057	0.071
		0.8	0.044	0.045	0.043	0.050	0.053	0.050	0.072	0.067
		0.6	0.046	0.045	0.046	0.044	0.048	0.052	0.058	0.060
200, 200	0.4	0.7	0.043	0.037	0.039	0.040	0.050	0.054	0.058	0.070
		0.8	0.035	0.053	0.038	0.043	0.046	0.054	0.070	0.068
		0.6	0.045	0.044	0.041	0.047	0.045	0.053	0.068	0.059
	0.6	0.7	0.043	0.042	0.043	0.051	0.045	0.051	0.064	0.067
		0.8	0.033	0.039	0.048	0.060	0.046	0.054	0.068	0.065
		0.6	0.051	0.049	0.055	0.058	0.050	0.045	0.046	0.056
	0.2	0.7	0.049	0.049	0.044	0.052	0.056	0.043	0.052	0.058
		0.8	0.051	0.045	0.050	0.054	0.053	0.048	0.057	0.062
		0.6	0.046	0.043	0.042	0.049	0.047	0.040	0.048	0.054
100, 200	0.4	0.7	0.044	0.042	0.040	0.050	0.055	0.046	0.051	0.057
		0.8	0.045	0.040	0.049	0.038	0.052	0.042	0.054	0.061
		0.6	0.039	0.042	0.053	0.053	0.056	0.044	0.052	0.052
	0.6	0.7	0.044	0.038	0.043	0.049	0.058	0.051	0.049	0.050
		0.8	0.041	0.035	0.037	0.041	0.058	0.051	0.051	0.055

Table 7: Size simulation results with respect to the AUC of 1000 replications where data representing the diseased and healthy individuals are generated by bivariate lognormal and bivariate gamma distributions (valid null hypothesis $ROC_1(t) = ROC_2(t)$). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7$, and 0.8. The Box-Cox, kernel-based and DeLong's methods are compared. Sample sizes explored are (100, 100), (200, 200) and (100,200). Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for the data of both the healthy and diseased individuals.

AUC size results								
			Bivariate Gammas			Bivariate Lognormals		
n_A, n_B	ρ	AUC	$Box-Cox$	$Kernel$	$DeLong$	$Box-Cox$	$Kernel$	$DeLong$
		0.6	0.051	0.040	0.042	0.056	0.051	0.055
	0.2	0.7	0.049	0.048	0.043	0.057	0.053	0.052
		0.8	0.059	0.055	0.044	0.054	0.054	0.055
		0.6	0.040	0.037	0.044	0.058	0.059	0.058
100, 100	0.4	0.7	0.042	0.042	0.043	0.055	0.055	0.056
		0.8	0.041	0.043	0.043	0.055	0.055	0.051
		0.6	0.052	0.045	0.048	0.052	0.062	0.061
	0.6	0.7	0.047	0.045	0.044	0.057	0.059	0.058
		0.8	0.047	0.050	0.045	0.053	0.055	0.045
		0.6	0.057	0.059	0.054	0.058	0.063	0.059
	0.2	0.7	0.057	0.050	0.053	0.059	0.053	0.057
		0.8	0.046	0.041	0.044	0.063	0.056	0.049
		0.6	0.053	0.050	0.046	0.062	0.059	0.057
200, 200	0.4	0.7	0.040	0.038	0.038	0.060	0.049	0.050
		0.8	0.048	0.048	0.049	0.061	0.053	0.053
		0.6	0.047	0.045	0.040	0.065	0.056	0.062
	0.6	0.7	0.044	0.044	0.042	0.059	0.051	0.056
		0.8	0.041	0.036	0.038	0.057	0.056	0.050
		0.6	0.045	0.042	0.039	0.045	0.051	0.052
	0.2	0.7	0.049	0.047	0.045	0.051	0.049	0.044
		0.8	0.051	0.052	0.052	0.046	0.048	0.047
		0.6	0.044	0.042	0.043	0.047	0.044	0.047
100, 200	0.4	0.7	0.043	0.040	0.045	0.051	0.045	0.043
		0.8	0.043	0.047	0.054	0.049	0.051	0.046
		0.6	0.039	0.044	0.046	0.041	0.043	0.043
	0.6	0.7	0.031	0.030	0.031	0.043	0.044	0.045
		0.8	0.039	0.039	0.045	0.048	0.054	0.046

Table 8: Power simulation results with respect to the AUC of 1000 replications where data representing diseased and healthy individuals are generated by bivariate lognormal and bivariate gamma distributions (non-valid null hypothesis). The Box-Cox, kernel-based and DeLong's methods are compared. Sample sizes explored are (50, 50), (100, 100), and (200,200). Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for data of both the healthy and diseased individuals.

AUC power results							
		Bivariate Gammas			Bivariate Lognormals		
n_A, n_B	ρ	$Box-Cox$	$Kernel$	$DeLong$	$Box-Cox$	$Kernel$	$DeLong$
	0.2	0.511	0.469	0.480	0.762	0.663	0.721
50, 50	0.4	0.610	0.569	0.575	0.857	0.754	0.830
	0.6	0.768	0.737	0.736	0.949	0.894	0.952
	0.2	0.809	0.776	0.771	0.953	0.909	0.949
100, 100	0.4	0.896	0.882	0.875	0.988	0.964	0.987
	0.6	0.972	0.969	0.969	0.994	0.994	0.998
	0.2	0.979	0.968	0.972	0.999	0.998	0.999
200, 200	0.4	0.998	0.995	0.993	1.000	1.000	1.000
	0.6	1.000	1.000	1.000	1.000	1.000	1.000

Web-Appendix B (technical details)

1 Appendix

1.1 Derivation of the partial derivatives for the delta method

$$
\begin{aligned}
\frac{\partial ROC_1(t)}{\partial \mu_{1A}} &= -\frac{\sqrt{2}}{2\sqrt{\pi}\sigma_{1B}} exp\left(-0.5\left(\frac{\mu_{1A}-\mu_{1B}-\Phi^{-1}(t)\sigma_{1A}}{\sigma_{1B}}\right)^2\right)\\
\frac{\partial ROC_1(t)}{\partial \mu_{1B}} &= \frac{\sqrt{2}}{2\sqrt{\pi}\sigma_{1B}} exp\left(-0.5\left(\frac{\mu_{1A}-\mu_{1B}-\Phi^{-1}(t)\sigma_{1A}}{\sigma_{1B}}\right)^2\right)\\
\frac{\partial ROC_1(t)}{\partial \sigma_{1A}} &= \frac{\Phi^{-1}(t)}{\sqrt{2\pi}\sigma_{1B}} exp\left(-0.5\left(\frac{\mu_{1A}-\mu_{1B}-\Phi^{-1}(t)\sigma_{1A}}{\sigma_{1B}}\right)^2\right)\\
\frac{\partial ROC_1(t)}{\partial \sigma_{1B}} &= \frac{\sqrt{2}}{2\sqrt{\pi}} exp\left(-0.5\left(\frac{\mu_{1A}-\mu_{1B}-\Phi^{-1}(t)\sigma_{1A}}{\sigma_{1B}}\right)^2\right)\left(\frac{\mu_{1A}-\mu_{1B}-\sigma_{1A}\Phi^{-1}(t)}{\sigma_{1B}^2}\right)
\end{aligned}
$$

and the expressions are similar for $\frac{\partial ROC_2(t)}{\partial \mu_{2A}}$, $\frac{\partial ROC_2(t)}{\partial \mu_{2B}}$, $\frac{\partial ROC_2(t)}{\partial \sigma_{2A}}$ and $\frac{\partial ROC_2(t)}{\partial \sigma_{2B}}$.

$$
\begin{aligned}
\frac{\partial \Phi^{-1}(R\hat{O}C_1(t))}{\partial \mu_{1A}} &= -\frac{1}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}(R\hat{O}C_1(t))}{\partial \mu_{1B}} &= \frac{1}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}(R\hat{O}C_1(t))}{\partial \sigma_{1A}} &= \frac{\Phi^{-1}(t)}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}(R\hat{O}C_1(t))}{\partial \sigma_{1B}} &= \frac{\mu_{1A}-\mu_{1B}-\Phi^{-1}(t)\sigma_{1A}}{\sigma_{1B}^2}.
\end{aligned}
\tag{1}
$$

Similarly for the expressions that correspond to $\Phi^{-1}(R\hat{O}C_2(t))$.

1.2 Derivation of the partial derivatives for the delta method in the presence of covariates

$$
\begin{aligned}
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \beta_0^{(1A)}} &= -\frac{1}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \beta_j^{(1A)}} &= -\frac{z_j^{(1A)}}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \beta_0^{(1B)}} &= \frac{1}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \beta_j^{(1A)}} &= \frac{z_j^{(1B)}}{\sigma_{1B}}\\
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \sigma_{1A}} &= -\frac{\sigma_{1B}\Phi^{-1}(t)}{\sigma_{1A}^2}\\
\frac{\partial \Phi^{-1}\left(ROC_{1,Z}(t)\right)}{\partial \sigma_{1B}} &= -\frac{(\beta_0^{(1B)}+\sum_{j=1}^{p}\beta_j^{(1B)}Z_{ji}^{(1B)})-(\beta_0^{(1A)}+\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)})}{\sigma_{1B}^2}+\frac{\Phi^{-1}(t)}{\sigma_{1A}^2}.
\end{aligned}
$$

where $j = 1, \ldots, p$. The expressions for $ROC_2(t)$ are similar.

1.3 Partial derivatives for the delta method (AUC)

$$\frac{\partial\Phi^{-1}(AUC_1)}{\partial\mu_{1A}} = \frac{-1}{\sigma_{1B}\sqrt{1+(\sigma_{1A}/\sigma_{1B})^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_1)}{\partial\mu_{1B}} = \frac{1}{\sigma_{1B}\sqrt{1+(\sigma_{1A}/\sigma_{1B})^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_1)}{\partial\sigma_{1A}} = \frac{\sigma_{1A}(\mu_{1A}-\mu_{1B})}{\sigma_{1B}^3(1+(\sigma_{1A}/\sigma_{1B})^2)^{3/2}}$$

$$\frac{\partial\Phi^{-1}(AUC_1)}{\partial\sigma_{1A}} = \frac{(\mu_{1A}-\mu_{1B})}{\sigma_{1B}^2(1+(\sigma_{1A}/\sigma_{1B})^2)^{3/2}}$$

1.4 Partial derivatives for the delta method in the presence of covariates (AUC)

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\beta_0^{(1A)}} = \frac{-1}{\sigma_{1B}\sqrt{1+\left(\frac{\sigma_{1A}}{\sigma_{1B}}\right)^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\beta_j^{(1A)}} = \frac{-z_j^{(1A)}}{\sigma_{1B}\sqrt{1+\left(\frac{\sigma_{1A}}{\sigma_{1B}}\right)^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\beta_0^{(1B)}} = \frac{1}{\sigma_{1B}\sqrt{1+\left(\frac{\sigma_{1A}}{\sigma_{1B}}\right)^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\beta_j^{(1B)}} = \frac{z_j^{(1B)}}{\sigma_{1B}\sqrt{1+\left(\frac{\sigma_{1A}}{\sigma_{1B}}\right)^2}}$$

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\sigma_{1A}} = -\frac{\sigma_{1A}}{\sigma_{1B}^3\left(\frac{\sigma_{1A}^2}{\sigma_{1B}^2}+1\right)^{3/2}}$$

$$\frac{\partial\Phi^{-1}(AUC_{1,Z})}{\partial\sigma_{1A}} = \frac{\sigma_{1A}^2}{\sigma_{1B}^4\left(\frac{\sigma_{1A}^2}{\sigma_{1B}^2}+1\right)^{3/2}} - \frac{1}{\sigma_{1B}^2\left(\frac{\sigma_{1A}^2}{\sigma_{1B}^2}+1\right)^{1/2}}$$

The expressions are similar for $\Phi^{-1}(AUC_{2,Z})$.

1.5 Derivation of the information matrix for the binormal-bivariate setting

Fisher's information matrix in this case is of the form:

$$
I=-\begin{pmatrix}
\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^2} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}\partial \mu_{2A}} & 0 & 0 & 0 & 0 & 0\\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^2} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}\partial \sigma_{2A}} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}\partial cov_A} & 0\\
0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^2} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}\partial \mu_{2B}} & 0 & 0 & 0\\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^2} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}\partial \sigma_{2B}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}\partial cov_B}\\
\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}\partial \mu_{1A}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^2} & 0 & 0 & 0 & 0 & 0\\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}\partial \sigma_{1A}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^2} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}\partial cov_A} & 0\\
0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}\partial \mu_{1B}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^2} & 0 & 0 & 0\\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}\partial \sigma_{1B}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^2} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}\partial cov_B}\\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A\partial \sigma_{1A}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A\partial \sigma_{2A}} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^2} & 0\\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B\partial \sigma_{1B}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B\partial \sigma_{2B}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^2}
\end{pmatrix}.
$$

We present all the derivations required:

$$
\begin{aligned}
\frac{\partial^2 \log L}{\partial \mu_{1A}^2} &= \frac{n_A}{{\sigma_{1A}}^2\left(\frac{{cov_A}^2}{{\sigma_{1A}}^2\,{\sigma_{2A}}^2}-1\right)}\\
\frac{\partial^2 logL}{\partial \sigma_{1A}^2} &= \frac{1}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^3}\sum_{i=1}^{n_A}\left(A_{1i}-A_{2i}-A_{3i}-A_{4i}+A_{5i}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
A_{1i} &= {\sigma_{1A}}^4\,{\sigma_{2A}}^6+{cov_A}^3\,\left(2\,\mu_{1A}\,\mu_{2A}\,{\sigma_{2A}}^2+2\,W_{1Ai}\,W_{2Ai}\,{\sigma_{2A}}^2-2\,W_{1Ai}\,\mu_{2A}\,{\sigma_{2A}}^2-2\,W_{2Ai}\,\mu_{1A}\,{\sigma_{2A}}^2\right)\\
A_{2i} &= {\sigma_{1A}}^2\,\left(3\,{W_{1Ai}}^2\,{\sigma_{2A}}^6-6\,W_{1Ai}\,\mu_{1A}\,{\sigma_{2A}}^6+3\,{\mu_{1A}}^2\,{\sigma_{2A}}^6\right)\\
A_{3i} &= {cov_A}^2\,\left({\sigma_{1A}}^2\,\left(3\,{W_{2Ai}}^2\,{\sigma_{2A}}^2-6\,W_{2Ai}\,\mu_{2A}\,{\sigma_{2A}}^2+3\,{\mu_{2A}}^2\,{\sigma_{2A}}^2\right)+{W_{1Ai}}^2\,{\sigma_{2A}}^4+{\mu_{1A}}^2\,{\sigma_{2A}}^4-2\,W_{1Ai}\,\mu_{1A}\,{\sigma_{2A}}^4\right)\\
A_{4i} &= {cov_A}^4\,\left({W_{2Ai}}^2-2\,W_{2Ai}\,\mu_{2A}+{\mu_{2A}}^2+{\sigma_{2A}}^2\right)\\
A_{5i} &= cov_A\,{\sigma_{1A}}^2\,\left(6\,\mu_{1A}\,\mu_{2A}\,{\sigma_{2A}}^4+6\,W_{1Ai}\,W_{2Ai}\,{\sigma_{2A}}^4-6\,W_{1Ai}\,\mu_{2A}\,{\sigma_{2A}}^4-6\,W_{2Ai}\,\mu_{1A}\,{\sigma_{2A}}^4\right)
\end{aligned}
$$

$$
\frac{\partial^2 logL}{\partial cov_A^2} = -\frac{1}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^3}\sum_{i=1}^{n_A}\left(B_{1i}-B_{2i}+B_{3i}-B_{4i}\right)
$$

$$
\begin{aligned}
B_{1i} &= {cov_A}^2\,\left(\left(3\,{W_{2Ai}}^2-6\,W_{2Ai}\,\mu_{2A}+3\,{\mu_{2A}}^2\right)\,{\sigma_{1A}}^2+\left(3\,{W_{1Ai}}^2-6\,W_{1Ai}\,\mu_{1A}+3\,{\mu_{1A}}^2\right)\,{\sigma_{2A}}^2\right)\\
B_{2i} &= {cov_A}^3\,\left(2\,W_{1Ai}\,W_{2Ai}-2\,W_{1Ai}\,\mu_{2A}-2\,W_{2Ai}\,\mu_{1A}+2\,\mu_{1A}\,\mu_{2A}\right)\\
B_{3i} &= {cov_A}^4+{\sigma_{2A}}^4\,\left({\sigma_{1A}}^2\,\left({W_{1Ai}}^2-2\,W_{1Ai}\,\mu_{1A}+{\mu_{1A}}^2\right)-{\sigma_{1A}}^4\right)+{\sigma_{1A}}^4\,{\sigma_{2A}}^2\,\left({W_{2Ai}}^2-2\,W_{2Ai}\,\mu_{2A}+{\mu_{2A}}^2\right)\\
B_{4i} &= cov_A\,{\sigma_{1A}}^2\,{\sigma_{2A}}^2\,\left(6\,W_{1Ai}\,W_{2Ai}-6\,W_{1Ai}\,\mu_{2A}-6\,W_{2Ai}\,\mu_{1A}+6\,\mu_{1A}\,\mu_{2A}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 logL}{\partial \mu_{1A}\partial \sigma_{1A}} &= \frac{2\,\sigma_{1A}\,{\sigma_{2A}}^2\,\sum_{i=1}^{n_A}\left(W_{2Ai}\,cov_A-cov_A\,\mu_{2A}-W_{1Ai}\,{\sigma_{2A}}^2+\mu_{1A}\,{\sigma_{2A}}^2\right)}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^2}=0\\
\frac{\partial^2 logL}{\partial \mu_{1A}\partial \mu_{2A}} &= \frac{n_A cov_A}{{\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2}\\
\frac{\partial^2 logL}{\partial \mu_{1A}\partial \sigma_{2A}} &= -\frac{\sigma_{2A}\,\sum_{i=1}^{n_A}\left(2\,{cov_A}^2\,\left(W_{1Ai}-\mu_{1A}\right)-2\,cov_A\,{\sigma_{1A}}^2\,\left(W_{2Ai}-\mu_{2A}\right)\right)}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^2}=0\\
\frac{\partial^2 logL}{\partial \mu_{1A}\partial cov_A} &= -\sum_{i=1}^{n_A}\frac{{cov_A}^2\,\left(W_{2Ai}-\mu_{2A}\right)-{\sigma_{2A}}^2\,\left(cov_A\,\left(2\,W_{1Ai}-2\,\mu_{1A}\right)-{\sigma_{1A}}^2\,\left(W_{2Ai}-\mu_{2A}\right)\right)}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^2}=0\\
\frac{\partial^2 logL}{\partial \sigma_{1A}\partial \mu_{2A}} &= -\frac{\sigma_{1A}\,\sum_{i=1}^{n_A}\left(2\,{cov_A}^2\,\left(W_{2Ai}-\mu_{2A}\right)-2\,cov_A\,{\sigma_{2A}}^2\,\left(W_{1Ai}-\mu_{1A}\right)\right)}{\left({\sigma_{1A}}^2\,{\sigma_{2A}}^2-{cov_A}^2\right)^2}=0
\end{aligned}
$$

$$\frac{\partial^2 logL}{\partial\sigma_{1A}\partial\sigma_{2A}} = \sum_{i=1}^{n_A}\left(\sigma_{1A}{}^3\,\left(2\,cov_A\,\left(C_{1i}\right)+2\,cov_A{}^2\right)-cov_A{}^2\,\sigma_{1A}\,\left(4\,W_{1A}{}^2-8\,W_{1A}\,\mu_{1A}+4\,\mu_{1A}{}^2\right)\right)\,\sigma_{2A}{}^3$$

$$+ \sum_{i=1}^{n_A}\left(\sigma_{1A}\,\left(2\,cov_A{}^3\,\left(C_{1i}\right)-2\,cov_A{}^4\right)-cov_A{}^2\,\sigma_{1A}{}^3\,\left(4\,W_{2A}{}^2-8\,W_{2A}\,\mu_{2A}+4\,\mu_{2A}{}^2\right)\right)\,\sigma_{2A}$$

where

$$C_{1i} = 2\,W_{1A}\,W_{2A}-2\,W_{1A}\,\mu_{2A}-2\,W_{2A}\,\mu_{1A}+2\,\mu_{1A}\,\mu_{2A}$$

$$\frac{\partial^2 logL}{\partial\sigma_{1A}\partial cov_A} = \sum_{i=1}^{n_A}\frac{\sigma_{1A}\,\left(cov_A{}^3\,D_{1i}+D_{2i}-cov_A{}^2\,\sigma_{2A}{}^2\,D_{3i}\right)-\sigma_{1A}{}^3\,\left(\sigma_{2A}{}^4\,D_{4i}+cov_A\,D_{5i}\right)}{\left(\sigma_{1A}{}^2\,\sigma_{2A}{}^2-cov_A{}^2\right)^3}$$

$$\begin{aligned}
D_{1i} &= 2\,W_{2Ai}{}^2-4\,W_{2Ai}\,\mu_{2A}+2\,\mu_{2A}{}^2+2\,\sigma_{2A}{}^2\\
D_{2i} &= cov_A\,\sigma_{2A}{}^4\,\left(4\,W_{1Ai}{}^2-8\,W_{1Ai}\,\mu_{1A}+4\,\mu_{1A}{}^2\right)\\
D_{3i} &= 6\,W_{1Ai}\,W_{2Ai}-6\,W_{1Ai}\,\mu_{2A}-6\,W_{2Ai}\,\mu_{1A}+6\,\mu_{1A}\,\mu_{2A}\\
D_{4i} &= 2\,W_{1Ai}\,W_{2Ai}-2\,W_{1Ai}\,\mu_{2A}-2\,W_{2Ai}\,\mu_{1A}+2\,\mu_{1A}\,\mu_{2A}\\
D_{5i} &= 2\,\sigma_{2A}{}^4-\sigma_{2A}{}^2\,\left(2\,W_{2Ai}{}^2-4\,W_{2Ai}\,\mu_{2A}+2\,\mu_{2A}{}^2\right)
\end{aligned}$$

Note that all remaining nonzero derivatives are completely analogous to the above results and can be derived by substituting the parameters of the corresponding group. We also derive:

$$\frac{\partial^2 logL}{\partial\mu_{1A}\partial\mu_{1B}}=0,\frac{\partial^2 logL}{\partial\mu_{1A}\partial\sigma_{1B}}=0,\frac{\partial^2 logL}{\partial\mu_{1A}\partial\mu_{2B}}=0,\frac{\partial^2 logL}{\partial\mu_{1A}\partial\sigma_{2B}}=0,\frac{\partial^2 logL}{\partial\mu_{1A}\partial cov_B}=0$$

$$\frac{\partial^2 logL}{\partial\sigma_{1A}\partial\mu_{1B}}=0,\frac{\partial^2 logL}{\partial\sigma_{1A}\partial\sigma_{1B}}=0,\frac{\partial^2 logL}{\partial\sigma_{1A}\partial\mu_{2B}}=0,\frac{\partial^2 logL}{\partial\sigma_{1A}\partial\sigma_{2B}}=0,\frac{\partial^2 logL}{\partial\sigma_{1A}\partial cov_B}=0$$

$$\frac{\partial^2 logL}{\partial\mu_{1B}\partial\mu_{2A}}=0,\frac{\partial^2 logL}{\partial\mu_{1B}\partial\sigma_{2A}}=0,\frac{\partial^2 logL}{\partial\mu_{1B}\partial cov_A}=0,$$

$$\frac{\partial^2 logL}{\partial\sigma_{1B}\partial\mu_{2A}}=0,\frac{\partial^2 logL}{\partial\sigma_{1B}\partial\sigma_{2A}}=0,\frac{\partial^2 logL}{\partial\sigma_{1B}\partial cov_A}=0,$$

$$\frac{\partial^2 logL}{\partial\mu_{2A}\partial\mu_{2B}}=0,\frac{\partial^2 logL}{\partial\mu_{2A}\partial\sigma_{2B}}=0,\frac{\partial^2 logL}{\partial\mu_{2A}\partial cov_B}=0,$$

$$\frac{\partial^2 logL}{\partial\sigma_{2A}\partial\mu_{2B}}=0,\frac{\partial^2 logL}{\partial\sigma_{2A}\partial\sigma_{2B}}=0,\frac{\partial^2 logL}{\partial\sigma_{2A}\partial cov_B}=0,$$

$$\frac{\partial^2 logL}{\partial\mu_{2B}\partial cov_A}=0,\frac{\partial^2 logL}{\partial\sigma_{2B}\partial cov_A}=0,\frac{\partial^2 logL}{\partial cov_A\partial cov_B}=0.$$

1.6 Box-Cox transformation

The negative Fisher information matrix in this setting is

$$
\begin{pmatrix}
\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^{(\lambda_1)2}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^{(\lambda_1)} \partial \mu_{2A}^{(\lambda_2)}} & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^{(\lambda_1)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^{(\lambda_1)} \partial \lambda_2} \\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)2}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)} \partial \sigma_{2A}^{(\lambda_2)}} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)} \partial cov_A^{(\lambda_{1,2})}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)} \partial \lambda_2} \\
0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^{(\lambda_1)2}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^{(\lambda_1)} \partial \mu_{2B}^{(\lambda_2)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^{(\lambda_1)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^{(\lambda_1)} \partial \lambda_2} \\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)2}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)} \partial \sigma_{2B}^{(\lambda_2)}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)} \partial cov_B^{(\lambda_{1,2})}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)} \partial \lambda_2} \\
\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^{(\lambda_2)} \partial \mu_{1A}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^{(\lambda_2)2}} & 0 & 0 & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^{(\lambda_2)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^{(\lambda_2)} \partial \lambda_2} \\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)} \partial \sigma_{1A}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)2}} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)} \partial cov_A^{(\lambda_{1,2})}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)} \partial \lambda_2} \\
0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^{(\lambda_2)} \partial \mu_{1B}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^{(\lambda_2)2}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^{(\lambda_2)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^{(\lambda_2)} \partial \lambda_2} \\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)} \partial \sigma_{1B}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)2}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)} \partial cov_B^{(\lambda_{1,2})}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)} \partial \lambda_2} \\
0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})} \partial \sigma_{1A}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})} \partial \sigma_{2A}^{(\lambda_2)}} & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})2}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})} \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})} \partial \lambda_2} \\
0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})} \partial \sigma_{1B}^{(\lambda_1)}} & 0 & 0 & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})} \partial \sigma_{2B}^{(\lambda_2)}} & 0 & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})2}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B \partial \lambda_2} \\
\frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \mu_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \sigma_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \mu_{1B}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \sigma_{1B}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \mu_{2A}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \sigma_{2A}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \mu_{2B}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \sigma_{2B}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 cov_A} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial cov_B^{(\lambda_{1,2})}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1^2} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1 \partial \lambda_2} \\
\frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{1B}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \sigma_{1B}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{1A}^{(\lambda_1)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \mu_{2B}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \sigma_{2B}^{(\lambda_2)}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial cov_A^{(\lambda_{1,2})}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 cov_B^{(\lambda_{1,2})}} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2 \partial \lambda_1} & \frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_2^2}
\end{pmatrix}.
$$

The derivatives included in the upper left 10×10 part of the above matrix are completely analogous to the ones derived in the previous subsection. Counting the matrix columns from left to right, note also that given the derivatives of the 11th column, the derivatives of the 12th column can be straightforwardly obtained by simple substitution of the corresponding parameters involved. Regarding the derivatives of the 11th column, we derive:

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1A}^{(\lambda_1)} \partial \lambda_1} = \sum_{i=1}^{n_A} \frac{\sigma_{2A}{}^{(\lambda_2)^2} \left(W_{1Ai}{}^{\lambda_1} \lambda_1 \log(W_{1Ai}) - W_{1Ai}{}^{\lambda_1} + 1\right)}{\lambda_1{}^2 \left(\sigma_{1A}^{(\lambda_1)^2} \sigma_{2A}{}^{(\lambda_2)^2} - cov_A^{(\lambda_{1,2})^2}\right)}$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1A}^{(\lambda_1)} \partial \lambda_1} = \sum_{i=1}^{n_A} \frac{2\, \sigma_{1A}^{(\lambda_1)} \sigma_{2A}{}^{(\lambda_2)^2} \, (K_{1i}) \, (K_{2i})}{\lambda_1{}^3 \lambda_2 \left(\sigma_{1A}^{(\lambda_1)^2} \sigma_{2A}{}^{(\lambda_2)^2} - cov_A^{(\lambda_{1,2})^2}\right)^2}$$

where,

$$K_{1i} = W_{1Ai}{}^{\lambda_1} \lambda_1 \log(W_{1Ai}) - W_{1Ai}{}^{\lambda_1} + 1$$

$$K_{2i} = cov_A^{(\lambda_{1,2})} \lambda_1 (1 - W_{2Ai}{}^{\lambda_2} \lambda_2) - \lambda_2 \, \sigma_{2A}{}^{(\lambda_2)^2} (1 + W_{1Ai}^{\lambda_1}) + (\lambda_1 \lambda_2)(cov_A^{(\lambda_{1,2})} \mu_{2A}^{(\lambda_2)} - \mu_{1A}^{(\lambda_1)} \sigma_{2A}{}^{(\lambda_2)^2})$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{1B}^{(\lambda_1)} \partial \lambda_1} = \sum_{i=1}^{n_B} \frac{\sigma_{2B}{}^{(\lambda_1)^2} \left(W_{1Bi}{}^{\lambda_1} \lambda_1 \log(W_{1Bi}) - W_{1Bi}{}^{\lambda_1} + 1\right)}{\lambda_1{}^2 \left(\sigma_{1B}{}^{(\lambda_1)^2} \sigma_{2B}{}^{(\lambda_1)^2} - cov_B^{(\lambda_{1,2})^2}\right)}$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{1B}^{(\lambda_1)} \partial \lambda_1} = \sum_{i=1}^{n_B} \frac{2\, \sigma_{1B}{}^{(\lambda_1)} \sigma_{2B}{}^{(\lambda_1)^2} \left(K'_{1i}\right) \left(K'_{2i}\right)}{\lambda_1{}^3 \lambda_2 \left(\sigma_{1B}{}^{(\lambda_1)^2} \sigma_{2B}{}^{(\lambda_1)^2} - cov_B^{(\lambda_{1,2})^2}\right)^2}$$

where,

$$K'_{1i} = W_{1Bi}{}^{\lambda_1} \lambda_1 \log(W_{1Bi}) - W_{1Bi}{}^{\lambda_1} + 1$$

$$K'_{2i} = cov_B^{(\lambda_{1,2})} \lambda_1 (1 - W_{2Bi}{}^{\lambda_2} \lambda_2) - \lambda_2 \, \sigma_{2B}{}^{(\lambda_1)^2} (1 + W_{1Bi}^{\lambda_1}) + (\lambda_1 \lambda_2)(cov_B^{(\lambda_{1,2})} \mu_{2B}^{(\lambda_2)} - \mu_{1B}^{(\lambda_1)} \sigma_{2B}{}^{(\lambda_1)^2})$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2A}^{(\lambda_2)} \partial \lambda_1} = \sum_{i=1}^{n_A} - \frac{cov_A^{(\lambda_{1,2})} \left(W_{1Ai}{}^{\lambda_1} \lambda_1 \log(W_{1Ai}) - W_{1Ai}{}^{\lambda_1} + 1\right)}{\lambda_1{}^2 \left(\sigma_{1A}^{(\lambda_1)^2} \sigma_{2A}{}^{(\lambda_2)^2} - cov_A^{(\lambda_{1,2})^2}\right)}$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2A}^{(\lambda_2)} \partial \lambda_1} = \sum_{i=1}^{n_A} - \frac{2\, cov_A^{(\lambda_{1,2})} \sigma_{2A}{}^{(\lambda_2)} \, (K_{1i}) \, (L_{1i})}{\lambda_1{}^3 \lambda_2 \left(\sigma_{1A}^{(\lambda_1)^2} \sigma_{2A}{}^{(\lambda_2)^2} - cov_A^{(\lambda_{1,2})^2}\right)^2}$$

where

$$L_{1i} = cov_A^{(\lambda_{1,2})} \lambda_2 (1 - W_{1Ai}{}^2) - \lambda_1 \sigma_{1A}^{(\lambda_1)^2} (1 + W_{2Ai})^{\lambda_2} + \lambda_1 \lambda_2 (cov_A^{(\lambda_{1,2})} \mu_{1A}^{(\lambda_1)} - \mu_{2A}^{(\lambda_2)} \sigma_{1A}^{(\lambda_1)^2})$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \mu_{2B}^{(\lambda_2)} \partial \lambda_1} = \sum_{i=1}^{n_B} - \frac{cov_B^{(\lambda_{1,2})} \left(W_{1Bi}{}^{\lambda_1} \, \lambda_1 \, \log(W_{1Bi}) - W_{1Bi}{}^{\lambda_1} + 1 \right)}{\lambda_1{}^2 \left(\sigma_{1B}{}^{(\lambda_1)}{}^2 \, \sigma_{2B}{}^{(\lambda_1)}{}^2 - cov_B^{(\lambda_{1,2})}{}^2 \right)}$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \sigma_{2B}^{(\lambda_2)} \partial \lambda_1} = \sum_{i=1}^{n_B} - \frac{2 \, cov_B^{(\lambda_{1,2})} \, \sigma_{2B}{}^{(\lambda_1)} \left(K_{1i}^{'} \right) \left(L_{1i}^{'} \right)}{\lambda_1{}^3 \, \lambda_2 \left(\sigma_{1B}{}^{(\lambda_1)}{}^2 \, \sigma_{2B}{}^{(\lambda_1)}{}^2 - cov_B^{(\lambda_{1,2})}{}^2 \right)^2}$$

where

$$L_{1i}^{'} = cov_B^{(\lambda_{1,2})} \, \lambda_2 (1 - W_{1Bi}{}^2) - \lambda_1 \, \sigma_{1B}{}^{(\lambda_1)}{}^2 (1 + W_{2Bi})^{\lambda_2} + \lambda_1 \, \lambda_2 (cov_B^{(\lambda_{1,2})} \mu_{1B}^{(\lambda_1)} - \mu_{2B}^{(\lambda_2)} \sigma_{1B}{}^{(\lambda_1)}{}^2)$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})} \partial \lambda_1} = \sum_{i=1}^{n_A} - \frac{(K_{1i}) \; (M_{1i} + M_{2i})}{\lambda_1{}^3 \, \lambda_2 \left(\sigma_{1A}^{(\lambda_1)}{}^2 \, \sigma_{2A}{}^{(\lambda_2)}{}^2 - cov_A^{(\lambda_{1,2})}{}^2 \right)^2}$$

where

$$M_{1i} = cov_A^{(\lambda_{1,2})}{}^2 (1 - W_{2Ai}^{\lambda_2}) - 2 cov_A^{(\lambda_{1,2})} \lambda_2 \sigma_{2A}{}^{(\lambda_2)}{}^2 (1 - W_{1Ai}^{\lambda_1}) + \lambda_1 \sigma_{1A}^{(\lambda_1)}{}^2 \sigma_{2A}{}^{(\lambda_2)}{}^2 + 2 W_{1Ai}^{\lambda_1}$$

$$M_{2i} = \lambda_1 \lambda_2 \mu_{2A}^{(\lambda_2)} (cov_A^{(\lambda_{1,2})}{}^2 + \sigma_{1A}^{(\lambda_1)}{}^2 \sigma_{2A}{}^{(\lambda_2)}{}^2) - \lambda_1 \sigma_{2A}{}^{(\lambda_2)}{}^2 (W_{2Ai}^{\lambda_2} \sigma_{1A}^{(\lambda_1)}{}^2 + 2 cov_A^{(\lambda_{1,2})} \lambda_2 \mu_{1A}^{(\lambda_1)})$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})} \partial \lambda_1} = \sum_{i=1}^{n_B} - \frac{(K_{1i}) \; (M_{1i} + M_{2i})}{\lambda_1{}^3 \, \lambda_2 \left(\sigma_{1B}{}^{(\lambda_1)}{}^2 \, \sigma_{2B}{}^{(\lambda_1)}{}^2 - cov_B^{(\lambda_{1,2})}{}^2 \right)^2}$$

where

$$M_{1i}^{'} = cov_B^{(\lambda_{1,2})}{}^2 (1 - W_{2Bi}^{\lambda_2}) - 2 cov_B^{(\lambda_{1,2})} \lambda_2 \sigma_{2B}{}^{(\lambda_1)}{}^2 (1 - W_{1Bi}^{\lambda_1}) + \lambda_1 \sigma_{1B}{}^{(\lambda_1)}{}^2 \sigma_{2B}{}^{(\lambda_1)}{}^2 + 2 W_{1Bi}^{\lambda_1}$$

$$M_{2i}^{'} = \lambda_1 \lambda_2 \mu_{2B}^{(\lambda_2)} (cov_B^{(\lambda_{1,2})}{}^2 + \sigma_{1B}{}^{(\lambda_1)}{}^2 \sigma_{2B}{}^{(\lambda_1)}{}^2) - \lambda_1 \sigma_{2B}{}^{(\lambda_1)}{}^2 (W_{2Bi}^{\lambda_2} \sigma_{1B}{}^{(\lambda_1)}{}^2 + 2 cov_B^{(\lambda_{1,2})} \lambda_2 \mu_{1B}^{(\lambda_1)})$$

$$\frac{\partial^2 l(\boldsymbol{p})}{\partial \lambda_1^2} = \sum_{i=1}^{n_A} \frac{\frac{2 \, N_{1i}{}^2 - N_{2i}}{\sigma_{1A}^{(\lambda_1)}{}^2} + \frac{N_{3i}}{\sigma_{1A}^{(\lambda_1)}{}^2 \, \sigma_{2A}{}^{(\lambda_2)}{}^2}}{\frac{2 \, cov_A^{(\lambda_{1,2})}{}^2}{\sigma_{1A}^{(\lambda_1)}{}^2 \, \sigma_{2A}{}^{(\lambda_2)}{}^2} - 2} + \sum_{i=1}^{n_B} \frac{\frac{2 \, N_{1i}^{'}{}^2 - N_{2i}^{'}}{\sigma_{1B}{}^{(\lambda_1)}{}^2} + \frac{N_{3i}^{'}}{\sigma_{1B}{}^{(\lambda_1)}{}^2 \, \sigma_{2B}{}^{(\lambda_1)}{}^2}}{\frac{2 \, cov_B^{(\lambda_{1,2})}{}^2}{\sigma_{1B}{}^{(\lambda_1)}{}^2 \, \sigma_{2B}{}^{(\lambda_1)}{}^2} - 2}$$

where,

$$
\begin{aligned}
N_{1i} &= \frac{W_{1Ai}{}^{\lambda_1}-1}{\lambda_1{}^2}-\frac{W_{1Ai}{}^{\lambda_1}\,\log\left(W_{1Ai}\right)}{\lambda_1}\\
N_{2i} &= \left(2\,\mu_{1A}^{(\lambda_1)}-\frac{2\,\left(W_{1Ai}{}^{\lambda_1}-1\right)}{\lambda_1}\right)\left(\frac{2\,W_{1Ai}{}^{\lambda_1}-2}{\lambda_1{}^3}-\frac{2\,W_{1Ai}{}^{\lambda_1}\,\log\left(W_{1Ai}\right)}{\lambda_1{}^2}+\frac{W_{1Ai}{}^{\lambda_1}\log\left(W_{1Ai}\right)^2}{\lambda_1}\right)\\
N_{3i} &= 2\,cov_A^{(\lambda_{1,2})}\left(\mu_{2A}^{(\lambda_2)}-\frac{W_{2Ai}{}^{\lambda_2}-1}{\lambda_2}\right)\left(\frac{2\,W_{1Ai}{}^{\lambda_1}-2}{\lambda_1{}^3}-\frac{2\,W_{1Ai}{}^{\lambda_1}\,\log\left(W_{1Ai}\right)}{\lambda_1{}^2}+\frac{W_{1Ai}{}^{\lambda_1}\log\left(W_{1Ai}\right)^2}{\lambda_1}\right)\\
N_{1i}^{'} &= \frac{W_{1Bi}{}^{\lambda_1}-1}{\lambda_1{}^2}-\frac{W_{1Bi}{}^{\lambda_1}\,\log\left(W_{1Bi}\right)}{\lambda_1}\\
N_{2i}^{'} &= \left(2\,\mu_{1B}^{(\lambda_1)}-\frac{2\,\left(W_{1Bi}{}^{\lambda_1}-1\right)}{\lambda_1}\right)\left(\frac{2\,W_{1Bi}{}^{\lambda_1}-2}{\lambda_1{}^3}-\frac{2\,W_{1Bi}{}^{\lambda_1}\,\log\left(W_{1Bi}\right)}{\lambda_1{}^2}+\frac{W_{1Ai}{}^{\lambda_1}\log\left(W_{1Ai}\right)^2}{\lambda_1}\right)\\
N_{3i}^{'} &= 2\,cov_B^{(\lambda_{1,2})}\left(\mu_{2B}^{(\lambda_2)}-\frac{W_{2Bi}{}^{\lambda_2}-1}{\lambda_2}\right)\left(\frac{2\,W_{1Bi}{}^{\lambda_1}-2}{\lambda_1{}^3}-\frac{2\,W_{1Bi}{}^{\lambda_1}\,\log\left(W_{1Bi}\right)}{\lambda_1{}^2}+\frac{W_{1Bi}{}^{\lambda_1}\log\left(W_{1Bi}\right)^2}{\lambda_1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 l(\boldsymbol{p})}{\partial\lambda_1\partial\lambda_2} &= \sum_{i=1}^{n_A}\frac{2\,cov_A^{(\lambda_{1,2})}\left(W_{1Ai}{}^{\lambda_1}\,\lambda_1\,log\left(W_{1Ai}\right)-W_{1Ai}{}^{\lambda_1}+1\right)\left(W_{2Ai}{}^{\lambda_2}\,\lambda_2\,log\left(W_{2Ai}\right)-W_{2Ai}{}^{\lambda_2}+1\right)}{\lambda_1{}^2\,\lambda_2{}^2\left(2\,\sigma_{1A}^{(\lambda_1)}{}^2\,\sigma_{2A}{}^{(\lambda_2)}{}^2-2\,cov_A^{(\lambda_{1,2})}{}^2\right)}\\
&+ \sum_{i=1}^{n_B}\frac{2\,cov_B^{(\lambda_{1,2})}\left(W_{1Bi}{}^{\lambda_1}\,\lambda_1\,log\left(W_{1Bi}\right)-W_{1Bi}{}^{\lambda_1}+1\right)\left(W_{2Bi}{}^{\lambda_2}\,\lambda_2\,log\left(W_{2Bi}\right)-W_{2Bi}{}^{\lambda_2}+1\right)}{\lambda_1{}^2\,\lambda_2{}^2\left(2\,\sigma_{1B}{}^{(\lambda_1)}{}^2\,\sigma_{2B}{}^{(\lambda_1)}{}^2-2\,cov_B^{(\lambda_{1,2})}{}^2\right)}.
\end{aligned}
$$

Derivatives included in the last column of Fisher's information matrix: $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{1A}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1A}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{1B}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1B}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{2A}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1A}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{2B}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{2B}^{(\lambda_1)}\partial\lambda_2}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})}\partial\lambda_2}$,$\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})}\partial\lambda_2}$, are respectively completely analogous to $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{1A}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1A}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{1B}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1B}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{2A}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{1A}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\mu_{2B}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial\sigma_{2B}^{(\lambda_1)}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_A^{(\lambda_{1,2})}\partial\lambda_1}$, $\frac{\partial^2 l(\boldsymbol{p})}{\partial cov_B^{(\lambda_{1,2})}\partial\lambda_1}$ and can be obtained by simple substitution of W_{1A}, W_{1B}, λ_1, μ_{1A} and μ_{1B} with W_{2A}, W_{2B}, λ_2, μ_{2A}, μ_{2B}, respectively.

1.7 Normality with covariates

Here, we restate and derive all derivatives of matrix M_A. All derivatives contained in matrix M_B are completely analogous to the ones that follow.

$$M_A = \begin{pmatrix} \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)^2}} & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \partial \beta_j^{(1A)}} & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \partial \beta_p^{(2A)}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \partial \sigma_{1A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \partial \sigma_{2A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \partial cov_A} \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)^2}} & \cdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)} \partial \sigma_{1A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)} \partial \sigma_{2A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)} \partial cov_A} \\ \vdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)} \partial \beta_{j'}^{(1A)}} & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_p^{(2A)} \partial \beta_0^{(1A)}} & \cdots & \cdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_p^{(2A)^2}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_p^{(2A)} \partial \sigma_{1A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_p^{(2A)} \partial \sigma_{2A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_p^{(2A)} \partial cov_A} \\ \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{1A} \partial \beta_0^{(1A)}} & \cdots & \cdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{1A} \beta_p^{(2A)}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{1A}^2} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{1A} \partial \sigma_{2A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{1A} \partial cov_A} \\ \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{2A} \partial \beta_0^{(1A)}} & \cdots & \cdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{2A} \beta_p^{(2A)}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{2A} \partial \sigma_{1A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{2A}^2} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \sigma_{2A} \partial cov_A} \\ \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A \partial \beta_0^{(1A)}} & \cdots & \cdots & \cdots & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A \beta_p^{(2A)}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A \partial \sigma_{1A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A \partial \sigma_{2A}} & \frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A^2} \end{pmatrix}_{((p+4)\times(p+4))}.$$

$$\begin{aligned}
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)^2}} &= -\frac{n_A}{\sigma_{1A}{}^2 - \frac{cov_A{}^2}{\sigma_{2A}{}^2}} \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \beta_j^{(1A)}} &= -\sum_{i=1}^{n_A} \frac{Z_{ji}^{(1A)}}{\sigma_{1A}{}^2 - \frac{cov_A{}^2}{\sigma_{2A}{}^2}} \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \beta_j^{(2A)}} &= \sum_{i=1}^{n_A} \frac{Z_{ji}^{(2A)}\, cov_A}{\mathrm{\sigma_{1A}}^2\, \mathrm{\sigma_{2A}}^2 - \mathrm{cov_A}^2} \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \sigma_{1A}} &= 0 \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} \sigma_{2A}} &= 0 \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_0^{(1A)} cov_A} &= 0 \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)^2}} &= -\sum_{i=1}^{n_A} \frac{Z_{ji}^{(1A)}}{\sigma_{1A}{}^2 - \frac{cov_A{}^2}{\sigma_{2A}{}^2}} \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(2A)^2}} &= -\sum_{i=1}^{n_A} \frac{Z_{ji}^{(2A)}}{\sigma_{2A}{}^2 - \frac{cov_A{}^2}{\sigma_{1A}{}^2}} \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(1A)} \beta_{j'}^{(1A)}} &= -\sum_{i=1}^{n_A} \frac{Z_j^{(1A)} Z_{j'}^{(1A)}}{\sigma_{1A}^2 - \frac{cov_A}{\sigma_{2A}^2}}, j \neq j' \\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial \beta_j^{(2A)} \beta_{j'}^{(2A)}} &= -\sum_{i=1}^{n_A} \frac{Z_j^{(2A)} Z_{j'}^{(2A)}}{\sigma_{2A}^2 - \frac{cov_A}{\sigma_{1A}^2}}, j \neq j'
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(1A)}\sigma_{1A}} &= \sigma_{1A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(1A)}cov_A\sigma_{2A}^2(W_{2Ai}-\beta_0^{(2A)}-\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sigma_{1A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(1A)}\sigma_{2A}^4(W_{1Ai}-\beta_0^{(1A)}-\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(2A)}\sigma_{2A}} &= \sigma_{2A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(2A)}cov_A\sigma_{1A}^2(W_{1Ai}-\beta_0^{(1A)}-\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sigma_{2A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(2A)}\sigma_{1A}^4(W_{2Ai}-\beta_0^{(2A)}-\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(1A)}\sigma_{2A}} &= \sigma_{2A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(1A)}cov_A\sigma_{1A}^2(W_{2Ai}-\beta_0^{(2A)}-\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sigma_{2A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(1A)}cov_A^2(W_{1Ai}-\beta_0^{(1A)}-\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(2A)}\sigma_{1A}} &= \sigma_{1A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(2A)}cov_A\sigma_{2A}^2(W_{1Ai}-\beta_0^{(1A)}-\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sigma_{1A}\sum_{i=1}^{n_A}\frac{2Z_{ki}^{(2A)}cov_A^2(W_{2Ai}-\beta_0^{(2A)}-\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(1A)}cov_A} &= -\sum_{i=1}^{n_A}\sigma_{2A}^2\frac{2Z_{ki}^{(1A)}cov_A(\beta_0^{(1A)}+\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)}-W_{1Ai})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sum_{i=1}^{n_A}\frac{Z_{ki}^{(1A)}\sigma_{1A}^2\sigma_{2A}^2(\beta_0^{(2A)}+\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)}-W_{2Ai})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sum_{i=1}^{n_A}\frac{Z_{ki}^{(1A)}cov_A^2(\beta_0^{(2A)}+\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)}-W_{2Ai})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\beta_k^{(2A)}cov_A} &= -\sum_{i=1}^{n_A}\sigma_{1A}^2\frac{2Z_{ki}^{(2A)}cov_A(\beta_0^{(2A)}+\sum_{j=1}^{p}\beta_j^{(2A)}Z_{ji}^{(2A)}-W_{2Ai})}{(\sigma_{2A}^2\sigma_{1A}^2-cov_A^2)^2}\\
&- \sum_{i=1}^{n_A}\frac{Z_{ki}^{(2A)}\sigma_{2A}^2\sigma_{1A}^2(\beta_0^{(1A)}+\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)}-W_{1Ai})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}\\
&- \sum_{i=1}^{n_A}\frac{Z_{ki}^{(2A)}cov_A^2(\beta_0^{(1A)}+\sum_{j=1}^{p}\beta_j^{(1A)}Z_{ji}^{(1A)}-W_{1Ai})}{(\sigma_{1A}^2\sigma_{2A}^2-cov_A^2)^2}
\end{aligned}
$$

and the remaining derivatives $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\sigma_{1A}^2}$, $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\sigma_{2A}^2}$, $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\sigma_{1A}\partial\sigma_{2A}}$, $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\sigma_{1A}\partial cov_A}$, $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial\sigma_{2A}\partial cov_A}$ and $\frac{\partial^2 l(\boldsymbol{p}_Z)}{\partial cov_A^2}$ are analogous to their counterparts presented in section (1.5).

1.8 Partial derivatives for the construction of the Z test in the setting where covariates are present

$$
\begin{aligned}
\frac{\partial ROC_{1,Z}(t)}{\partial \beta_0^{(1A)}} &= -\frac{\sqrt{2}}{2\sqrt{\pi}\,\sigma_{1\mathrm{B}}} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
\frac{\partial ROC_{1,Z}(t)}{\partial \beta_j^{(1A)}} &= -\frac{z_j^{(1A)}\sqrt{2}}{2\sqrt{\pi}\,\sigma_{1\mathrm{B}}} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
\frac{\partial ROC_{1,Z}(t)}{\partial \beta_0^{(1B)}} &= \frac{\sqrt{2}}{2\sqrt{\pi}\,\sigma_{1\mathrm{B}}} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
\frac{\partial ROC_{1,Z}(t)}{\partial \beta_j^{(1B)}} &= \frac{z_j^{(1B)}\sqrt{2}}{2\sqrt{\pi}\,\sigma_{1\mathrm{B}}} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
\frac{\partial ROC_{1,Z}(t)}{\partial \sigma_{1A}} &= \frac{-\sigma_{1B}\Phi^{-1}(t)}{\sqrt{2\pi}\,\sigma_{1\mathrm{A}}^2} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
\frac{\partial ROC_{1,Z}(t)}{\partial \sigma_{1B}} &= \frac{1}{3\pi\sqrt{2}} \exp\left(-\frac{1}{2}\left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1\mathrm{B}}} - \frac{\sigma_{1\mathrm{B}}\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)^2\right) \\
&\times \left(\frac{\beta_0^{(1A)} + \sum_{j=1}^{p} \beta_j^{(1A)} Z_j^{(1A)} - \left(\beta_0^{(1B)} + \sum_{j=1}^{p} \beta_j^{(1B)} Z_j^{(1B)}\right)}{\sigma_{1B}^2} + \frac{\Phi^{-1}(t)}{\sigma_{1\mathrm{A}}}\right)
\end{aligned}
$$

2 Figure: Box-Cox-based ROC curves for the prostate cancer data

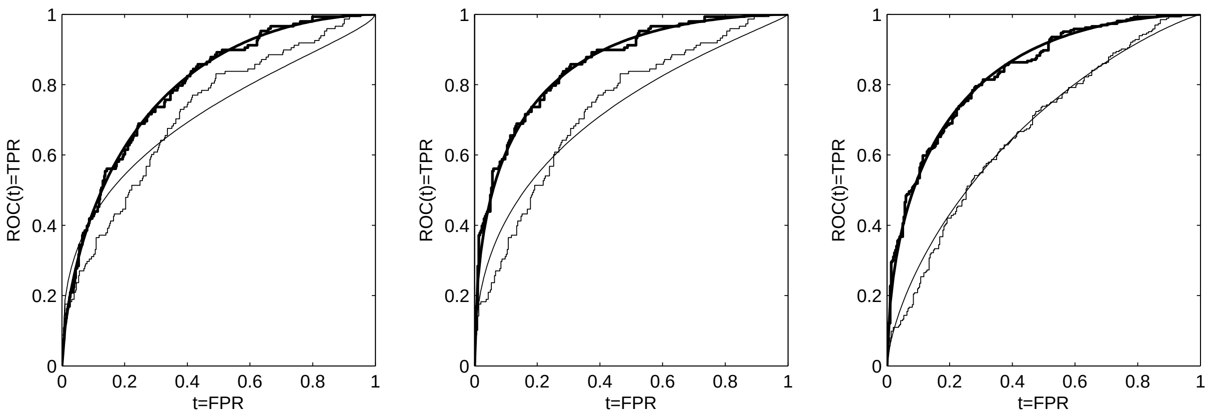

Figure 1: ROC estimates obtained from empirical-based and Box-Cox approaches for PSA alone (thin line) and the combined biomarker values (thick line). Left to right (i.e., scenarios (1) to (3)) for the empirical-based approach: ($AUC_{PSA} = 0.7185$, $AUC_C = 0.7933$), ($AUC_{PSA} = 0.7347$, $AUC_C = 0.8620$), ($AUC_{PSA} = 0.6682$, $AUC_C = 0.8342$). Left to right (i.e., scenarios (1) to (3)) for the Box-Cox approach: ($AUC_{PSA} = 0.7118$, $AUC_C = 0.7946$), ($AUC_{PSA} = 0.7242$, $AUC_C = 0.8597$), ($AUC_{PSA} = 0.6728$, $AUC_C = 0.8345$).

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Web-Appendix C (Additional Simulation results)

Table 9: Comparison of sizes of BTI and BTII confidence intervals by Qin et al. (2006). Simulation results of 1000 replications for the scenario in which data representing diseased and healthy individuals are generated by the same bivariate normal distributions (valid null hypothesis since $ROC_1(t) = ROC_2(t)$). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7$, and 0.8. Sample sizes explored are $(100, 100)$, $(200, 200)$ and $(100, 200)$. Correlation is set equal to $\rho = 0.2, 0.4, 0.6$ for the data of both the healthy and diseased individuals.

			Size results							
			BTI				$BTII$			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.032	0.064	0.052	0.049	0.031	0.042	0.038	0.035
	0.2	0.7	0.046	0.045	0.042	0.030	0.029	0.038	0.027	0.018
		0.8	0.029	0.031	0.031	0.017	0.021	0.025	0.023	0.008
		0.6	0.048	0.059	0.040	0.040	0.034	0.047	0.029	0.024
$100, 100$	0.4	0.7	0.045	0.046	0.039	0.031	0.030	0.033	0.025	0.017
		0.8	0.030	0.035	0.025	0.021	0.020	0.022	0.020	0.008
		0.6	0.039	0.052	0.035	0.031	0.023	0.036	0.025	0.020
	0.6	0.7	0.035	0.044	0.041	0.027	0.026	0.027	0.017	0.012
		0.8	0.032	0.027	0.026	0.013	0.023	0.017	0.015	0.004
		0.6	0.060	0.047	0.057	0.041	0.048	0.039	0.052	0.036
	0.2	0.7	0.054	0.052	0.048	0.042	0.046	0.040	0.040	0.033
		0.8	0.051	0.049	0.043	0.031	0.043	0.040	0.033	0.020
		0.6	0.0470	0.043	0.044	0.032	0.037	0.043	0.039	0.025
$200, 200$	0.4	0.7	0.0470	0.046	0.047	0.034	0.039	0.043	0.036	0.030
		0.8	0.0470	0.054	0.035	0.025	0.029	0.044	0.027	0.018
		0.6	0.0360	0.046	0.052	0.028	0.026	0.034	0.035	0.027
	0.6	0.7	0.0450	0.043	0.041	0.035	0.032	0.037	0.030	0.025
		0.8	0.0390	0.046	0.036	0.023	0.037	0.031	0.026	0.012
		0.6	0.043	0.051	0.052	0.054	0.039	0.036	0.038	0.037
	0.2	0.7	0.043	0.046	0.038	0.036	0.031	0.036	0.035	0.023
		0.8	0.051	0.032	0.033	0.026	0.037	0.030	0.029	0.013
		0.6	0.040	0.034	0.044	0.043	0.033	0.025	0.034	0.028
$100, 200$	0.4	0.7	0.038	0.042	0.039	0.034	0.030	0.031	0.032	0.021
		0.8	0.048	0.030	0.032	0.027	0.034	0.021	0.023	0.015
		0.6	0.035	0.030	0.040	0.032	0.027	0.021	0.028	0.018
	0.6	0.7	0.043	0.039	0.039	0.025	0.028	0.020	0.022	0.013
		0.8	0.054	0.026	0.036	0.016	0.039	0.016	0.019	0.006

Table 10: Comparison of sizes of BTI and BTII confidence intervals by Qin et al. (2006). Simulation results of 1000 replications for the scenario in which data representing diseased and healthy individuals are generated by the same bivariate gamma distributions (valid null hypothesis since $ROC_1(t) = ROC_2(t)$). The data are generated such that $AUC_1 = AUC_2 = 0.6, 0.7$, and 0.8. Sample sizes explored are $(100, 100)$, $(200, 200)$ and $(100, 200)$. Correlation is set equal to $\rho = 0.2, 0.4, 0.6$ for the data of both the healthy and diseased individuals.

Size results										
			BTI				$BTII$			
n_A, n_B	ρ	$AUC_1 = AUC_2$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$	$t = 0.2$	$t = 0.4$	$t = 0.6$	$t = 0.8$
		0.6	0.038	0.040	0.038	0.042	0.027	0.032	0.028	0.032
	0.2	0.7	0.043	0.042	0.044	0.035	0.035	0.029	0.027	0.028
		0.8	0.042	0.045	0.035	0.022	0.030	0.033	0.025	0.010
		0.6	0.033	0.041	0.040	0.041	0.019	0.024	0.028	0.029
$100, 100$	0.4	0.7	0.039	0.040	0.041	0.037	0.021	0.022	0.026	0.022
		0.8	0.043	0.031	0.033	0.017	0.031	0.021	0.020	0.010
		0.6	0.033	0.049	0.036	0.036	0.022	0.034	0.022	0.023
	0.6	0.7	0.037	0.035	0.032	0.026	0.023	0.026	0.017	0.012
		0.8	0.029	0.036	0.030	0.008	0.017	0.023	0.012	0.002
		0.6	0.057	0.039	0.037	0.045	0.051	0.0310	0.036	0.035
	0.2	0.7	0.049	0.035	0.036	0.039	0.042	0.0350	0.034	0.029
		0.8	0.048	0.042	0.037	0.028	0.032	0.0410	0.032	0.017
		0.6	0.042	0.041	0.047	0.040	0.028	0.0310	0.035	0.028
$200, 200$	0.4	0.7	0.034	0.033	0.036	0.028	0.026	0.0280	0.020	0.022
		0.8	0.037	0.046	0.034	0.026	0.028	0.0400	0.027	0.014
		0.6	0.028	0.042	0.038	0.041	0.023	0.029	0.032	0.027
	0.6	0.7	0.037	0.039	0.035	0.046	0.023	0.028	0.023	0.029
		0.8	0.027	0.036	0.051	0.026	0.023	0.024	0.039	0.022
		0.6	0.043	0.049	0.048	0.052	0.034	0.036	0.043	0.041
	0.2	0.7	0.048	0.041	0.048	0.040	0.040	0.036	0.034	0.025
		0.8	0.049	0.037	0.036	0.021	0.035	0.030	0.030	0.014
		0.6	0.041	0.047	0.035	0.034	0.034	0.028	0.027	0.017
$100, 200$	0.4	0.7	0.038	0.045	0.039	0.037	0.024	0.033	0.026	0.026
		0.8	0.042	0.039	0.022	0.021	0.027	0.020	0.021	0.010
		0.6	0.034	0.040	0.046	0.039	0.024	0.029	0.037	0.026
	0.6	0.7	0.039	0.034	0.029	0.026	0.026	0.022	0.021	0.015
		0.8	0.037	0.033	0.036	0.016	0.022	0.023	0.015	0.007

Table 11: Power simulation results of 1000 replications for the normal setting (non-valid null hypothesis) under different scenarios. The BTI and BTII methods are considered. Sample sizes explored are $(50, 50)$, $(100, 100)$, and $(200, 200)$. Correlation is set equal to $\rho = 0.2, 0.4$, and 0.6 for data of both the healthy and diseased individuals. The true difference $ROC_2(t) - ROC_1(t)$ to be detected is denoted by d.

Power results									
Normals									
		BTI				BTII			
n_A, n_B	ρ	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$
	0.2	0.518	0.415	0.285	0.100	0.504	0.421	0.256	0.085
50, 50	0.4	0.574	0.454	0.294	0.097	0.568	0.436	0.269	0.087
	0.6	0.636	0.504	0.319	0.105	0.644	0.490	0.292	0.083
	0.2	0.826	0.754	0.622	0.306	0.848	0.774	0.632	0.302
100, 100	0.4	0.860	0.802	0.653	0.319	0.891	0.812	0.665	0.301
	0.6	0.923	0.833	0.695	0.338	0.944	0.870	0.725	0.329
	0.2	0.984	0.966	0.906	0.646	0.987	0.972	0.925	0.689
200, 200	0.4	0.995	0.972	0.930	0.667	0.997	0.984	0.936	0.703
	0.6	1.000	0.989	0.952	0.705	1.000	0.996	0.962	0.749
Gammass									
		BTI				BTII			
n_A, n_B	ρ	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$	$d = 0.20$	$d = 0.15$	$d = 0.10$	$d = 0.05$
	0.2	0.329	0.271	0.186	0.080	0.303	0.264	0.174	0.064
50, 50	0.4	0.341	0.294	0.204	0.083	0.324	0.265	0.182	0.076
	0.6	0.423	0.338	0.221	0.099	0.399	0.316	0.199	0.071
	0.2	0.562	0.518	0.376	0.194	0.560	0.509	0.372	0.176
100, 100	0.4	0.619	0.535	0.405	0.228	0.617	0.537	0.408	0.221
	0.6	0.743	0.623	0.445	0.217	0.754	0.624	0.445	0.194
	0.2	0.841	0.778	0.648	0.403	0.851	0.781	0.662	0.419
200, 200	0.4	0.892	0.851	0.747	0.477	0.893	0.861	0.755	0.480
	0.6	0.947	0.910	0.797	0.512	0.956	0.919	0.816	0.518