Supplementary Material for:
Learning partial differential equations via data discovery and sparse optimization

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1 Supplementary Data

Terms	noise = 10.0%	noise = 10.0%
1	0	0
u	0	0
u^2	0	0
u_x	959.34	n.a.
u_x^2	0	0
$u\,u_x$	0	5.9965
u_{xx}	0	0
u_{xx}^2	0	0
$u\,u_{xx}$	0	0
$u_x\,u_{xx}$	0	0
u_{xxx}	0	0.9990
u_{xxx}^2	0	0
$u\,u_{xxx}$	0	0
$u_x\,u_{xxx}$	0	0
$u_{xx}\,u_{xxx}$	0	0

Table 1: Example 3.8: Travelling Waves, Solitons and Ambiguity. The data are simulated by equation (3.2) over $t \in [0, 0.1]$ using 512 grid points. In the first column, the method selects the one-way wave equation as the learned governing equation. This is indeed the sparsest governing equation (because it only uses one term for the spatial component of the model). In the second column, u_x is removed as a potential term and the method is applied to the remaining terms. This results in the correct identification of the intended governing equation.

Terms	noise = 50.24%	noise = 111.82%
1	0	0
u	0	-1.7136
u^2	0	0
u^3	0	0
u_x	0	0
u_x^2	0	0
u_x^3	0	0
$u\,u_x$	0.9222	0.9434
$u^2\,u_x$	0	0
$u\,u_x^2$	0	0
u_{xx}	0.0110	0.0034
u_{xx}^2	0	0
u_{xx}^3	0	0
$u\,u_{xx}$	0	0
$u^2\,u_{xx}$	0	0
$u\,u_{xx}^2$	0	0
$u_x\,u_{xx}$	0	0.0002
$u_x^2\,u_{xx}$	0	0
$u_x\,u_{xx}^2$	0	0
$u\,u_x\,u_{xx}$	0	0

Table 2: Example 3.9: Additive Noise on the Data. This example uses data from example 3.1 with various noise levels (applied directly to the data). Spectral smoothing is applied with $\beta = 5 \times 10^{-5}$. When the noise level between the smoothed velocity and the velocity calculated from the noisy data is about 50%, the learned model is accurate (first column). When the noise level between the smoothed velocity and the velocity calculated from the noisy data is large, the learned model loses accuracy and introduces false terms to compensate for the noise.

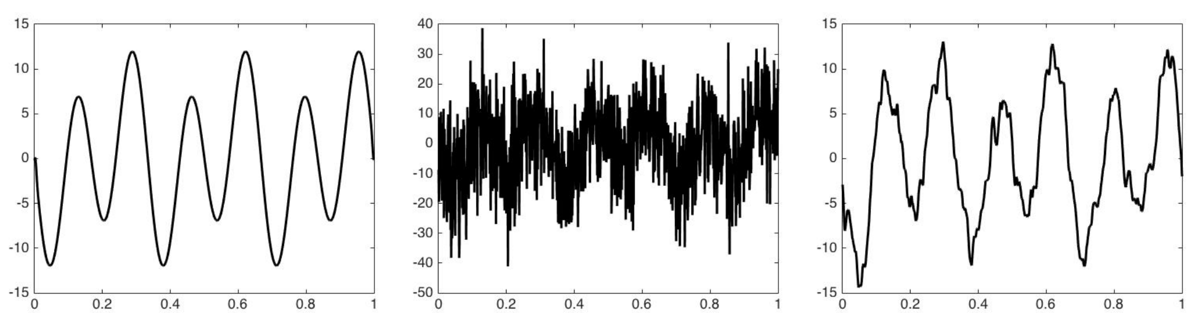

Figure 1: Example 3.9: Additive Noise on the Data. The plots display the various velocity estimates when the noise level is 50%. The first plot is the true velocity from the data generated by Burgers' equation. The second plot is the numerical approximation of the velocity when the data itself is noisy. The third plot is the numerical approximation to the velocity with spectral filtering applied to the data.