```
<< LinearAlgebra`MatrixManipulation`;
<< NumericalMath`TrigFit`;
<< Graphics`Graphics`;
<< Graphics`Arrow`;
Off[General::"spell"];
Off[General::"spell1"];
```

(* Map Yeast Alpha Factor Cell Cycle Time Course Expression Data *)

(* Read Data *)

```
stream = "Desktop/Networks/Data/Alpha_Factor.txt.nb";
externalmatrix = ReadList[stream, Word, RecordLists -> True, NullWords -> True];
{externalgenes, externalarrays} = Dimensions[externalmatrix] - {2, 3}
Clear[stream];
```

```
{4636, 18}
```

```
externalgenenames = TakeRows[
   TakeColumns[externalmatrix, {1, 3}],
   {3, externalgenes + 2}];
externalarraynames = TakeColumns[
   TakeRows[externalmatrix, {1, 2}],
   {4, externalarrays + 3}];
externalmatrix = TakeColumns[
   TakeRows[externalmatrix, {3, externalgenes + 2}],
   {4, externalarrays + 3}];
externalmatrix = ToExpression[externalmatrix];

sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[externalarraynames[[2, a]]
      ]]],
    {a, 1, externalarrays}]];
size = Sort[sizes, OrderedQ[{#2, #1}] &][[1]];
Do[
  Do[externalarraynames[[2, a]] = StringJoin[ToString[externalarraynames[[2, a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, externalarrays}];
```

(* Display Sorted External Arrays *)

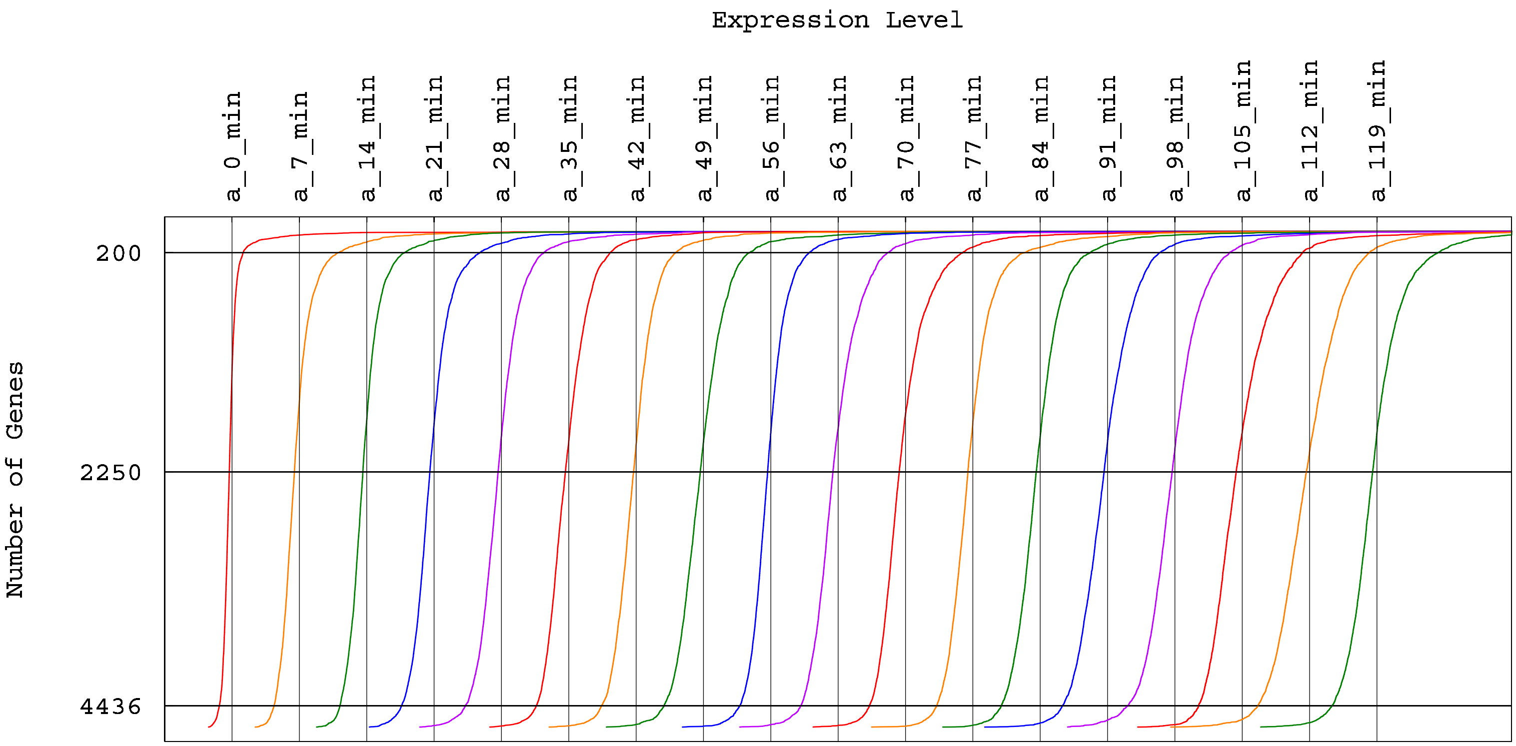
```
arraypatterns = Transpose[externalmatrix];
```

(* Center External Arrays *)

```
average = Table[1, {a, 1, externalgenes}];
average = N[average / Sqrt[Dot[average, average]]];
arraypatterns = arraypatterns - N[Outer[Times, Dot[arraypatterns, average], average]];
```

```
(* Normalize External Arrays *)

Do[
 arraypatterns[[a]] =
  arraypatterns[[a]] / Sqrt[Dot[arraypatterns[[a]], arraypatterns[[a]]]],
 {a, 1, externalarrays}]

(* Sort External Arrays *)

Do[
 arraypatterns[[a]] = Sort[arraypatterns[[a]], OrderedQ[{{#2}, {#1}}] &],
 {a, 1, externalarrays}]

(* Create Sorted External Arrays Graph Display *)

p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "Expression Level";
labely = ColumnForm[
   {" ", "Number of Genes", " ", " ", " ", " ", " ", " ", " ", " "},
   Center];
framex = Table[{0.02 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
framey = {{-200, "200"}, {-2250, "2250"}, {-externalgenes + 200, "4436"}};

Do[{
   coordinates = Table[
     If[arraypatterns[[n, a]] + 0.02 * n < -0.02, -0.02,
      If[arraypatterns[[n, a]] + 0.02 * n > 0.4, 0.4,
       arraypatterns[[n, a]] + 0.02 * n]],
     {a, 1, externalgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, externalgenes}],
   line = Line[coordinates],
   g = Show[
     Graphics[{color[[Mod[n, 5] + 1]], line}],
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {{{0.02 * n, RGBColor[0, 0, 0]}}, {{-200, RGBColor[0, 0, 0]},
        {-2250, RGBColor[0, 0, 0]}, {-externalgenes + 200, RGBColor[0, 0, 0]}}},
     PlotRange -> {{0, 0.4}, {135, -externalgenes + 1 - 135}},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1600}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 700}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → 1 / 1.2 / GoldenRatio,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
```

```
(* Display Sorted External Arrays *)

Show[Table[p[[a]], {a, 1, externalarrays}],
  DisplayFunction → $DisplayFunction];
```



```
(* Estimate Significance of Association of External Data with the Cell Cycle *)

(* Use Microarray Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {2}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table1 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Use Traditional Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {3}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table2 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Display Significance of Association of External Arrays with the Cell Cycle *)

headerx = {{
    ColumnForm[{" ", " ", " "}, Left],
    ColumnForm[{" ", " ", "Classification"}, Left],
    ColumnForm[{" ", "External", "Array"}, Left],
    ColumnForm[{"Most Likely", "Parallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Parallel", "Association"}, Left],
    ColumnForm[{"Most Likely", "Antiparallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Antiparallel", "Association"}, Left]},
   {" ", " ", " ", " ", " ", " ", " "}};
spacerx = {{" ", " ", " ", " ", " "}};
headery = Table[" ", {a, 1, 2 * externalarrays + 1}, {b, 1, 2}];
headery[[1]] = {"(a)", "Microarray"};
headery[[externalarrays + 2]] = {"(b)", "Traditional"};
association =
  AppendColumns[headerx,
   AppendRows[headery,
    AppendColumns[table1, spacerx, table2]]];
TableForm[association, TableSpacing → {1, 1}]
```

	Classification	External Array	Most Likely Parallel Association	P-Value of Parallel Association	Most Likely Antiparallel Association	P-Value of Antiparallel Association
(a)	Microarray	a_0_min	G2/M	3.3×10^{-6}	G1	$5.\times10^{-27}$
		a_7_min	M/G1	5.7×10^{-4}	S	1.3×10^{-6}
		a_14_min	G1	4.3×10^{-26}	G2/M	3.3×10^{-6}
		a_21_min	G1	$4.\times10^{-57}$	G2/M	1.1×10^{-18}
		a_28_min	G1	4.6×10^{-19}	G2/M	6.2×10^{-16}
		a_35_min	S	2.1×10^{-10}	M/G1	2.1×10^{-20}
		a_42_min	S/G2	1.2×10^{-11}	M/G1	4.8×10^{-25}
		a_49_min	G2/M	6.2×10^{-16}	M/G1	4.8×10^{-30}
		a_56_min	G2/M	3.1×10^{-31}	G1	6.9×10^{-51}
		a_63_min	G2/M	6.2×10^{-16}	G1	6.8×10^{-16}
		a_70_min	M/G1	1.6×10^{-21}	S/G2	4.1×10^{-9}
		a_77_min	G1	2.5×10^{-62}	S/G2	1.4×10^{-7}
		a_84_min	G1	5.8×10^{-34}	G2/M	1.4×10^{-21}
		a_91_min	G1	1.8×10^{-8}	G2/M	8.5×10^{-9}
		a_98_min	S/G2	3.3×10^{-3}	M/G1	$2.\times10^{-3}$
		a_105_min	G2/M	4.8×10^{-4}	M/G1	3.3×10^{-18}
		a_112_min	G2/M	3.1×10^{-10}	M/G1	3.3×10^{-18}
		a_119_min	G2/M	3.3×10^{-14}	G1	9.7×10^{-21}
(b)	Traditional	a_0_min	M/G1	4.4×10^{-6}	G1	$7.\times10^{-14}$
		a_7_min	M/G1	1.3×10^{-2}	S	3.4×10^{-6}
		a_14_min	G1	4.2×10^{-7}	S	3.4×10^{-6}
		a_21_min	G1	$7.\times10^{-14}$	M/G1	1.7×10^{-7}
		a_28_min	G1	$2.\times10^{-11}$	M/G1	4.5×10^{-9}
		a_35_min	S	3.4×10^{-6}	M/G1	4.4×10^{-6}
		a_42_min	S	$1.\times10^{-2}$	M/G1	1.1×10^{-12}
		a_49_min	G2/M	7.6×10^{-3}	M/G1	1.1×10^{-12}
		a_56_min	G2/M	2.7×10^{-8}	G1	3.5×10^{-15}
		a_63_min	G2/M	5.8×10^{-4}	G1	4.2×10^{-8}
		a_70_min	M/G1	1.7×10^{-7}	S	3.4×10^{-6}
		a_77_min	G1	2.3×10^{-22}	S/G2	5.5×10^{-3}
		a_84_min	G1	1.6×10^{-16}	G2/M	1.1×10^{-6}
		a_91_min	S	3.4×10^{-6}	G2/M	6.6×10^{-2}
		a_98_min	S	3.4×10^{-6}	M/G1	1.3×10^{-2}
		a_105_min	G2/M	7.6×10^{-3}	M/G1	8.8×10^{-5}
		a_112_min	S/G2	5.5×10^{-2}	G1	3.8×10^{-6}
		a_119_min	G2/M	$3.\times10^{-5}$	G1	4.2×10^{-8}

```
(* GSVD Sort External Arrays *)

genes = genes1;
genenames = TakeColumns[genenames1, 1];
arrays = arrays1;
arraynames = arraynames1;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{3625}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialarraylets = AppendRows[counter, gsvdphases, genenames, arraylets1];
partialarraylets = Sort[partialarraylets, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialarraylets, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialarraylets, {3, 3}],
   {1, partialgenes}];
partialarraylets = TakeRows[
   TakeColumns[partialarraylets, {4, arrays + 3}],
   {1, partialgenes}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Project Data Onto GSVD Basis and Calculate Contributions of Arraylets to External Arrays *)

partialarraylets = Transpose[partialarraylets];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialarraylets =
  partialarraylets - N[Outer[Times, Dot[partialarraylets, average], average]];
partialarraylets = Transpose[partialarraylets];

arraycontributions = Dot[PseudoInverse[partialarraylets], partialexternalmatrix];
average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
arraycontributions =
  arraycontributions - N[Outer[Times, Dot[arraycontributions, average], average]];
```

```
(* Project Arrays from 6 D Arraylets Subspace Onto 2 D Subspace  *)

externalcoordinates = Table[{
    (-Sqrt[3] * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        2 * Sqrt[3] * (arraycontributions[[4, a]] - arraycontributions[[16, a]]) +
        (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]],
    (3 * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        Sqrt[3] * (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]]},
   {a, 1, externalarrays}];

gsvdarrays = externalarrays;
gsvdcoordinates = externalcoordinates;
```

```
(* Classify GSVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.0000468343}

{-0.000239433}

endph1 = 856;
beginph2 = 857;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.00152245}

{0.999794}

endph2 = 1657;
beginph3 = 1658;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.0003}

{0.999906}

endph3 = 2226;
beginph4 = 2227;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.0009}

{0.99987}

endph4 = 3064;
beginph5 = 3065;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00019}

{1.99935}

(* 3625 yeast genes, 856 in M/G1, 801 in G1, 569 in S, 838 in S/G2, 561 in G2/M. *)
```

```
(* Display GSVD Reconstructed Sorted External Data *)

(* GSVD Reconstruct Sorted External Data *)

subspace = Table[0, {a, 1, arrays}];
Do[subspace[[a]] = 1, {a, 3, 5}];
Do[subspace[[a]] = 1, {a, 14, 16}];
matrix = Dot[partialarraylets, Dot[DiagonalMatrix[subspace], arraycontributions]];

(* Center GSVD Reconstructed Sorted External Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];

(* Create GSVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(c) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 700}, {0, 0}, {0, 1}];
g3 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create GSVD Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.01],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

a_0_min	$0.137614 \text{Cos}[\frac{\pi x}{1812}] - 0.0435982 \text{Sin}[\frac{\pi x}{1812}]$
a_7_min	$0.159545 \text{Cos}[\frac{\pi x}{1812}] + 0.0346513 \text{Sin}[\frac{\pi x}{1812}]$
a_14_min	$0.0676952 \text{Sin}[\frac{\pi x}{1812}]$
a_21_min	$-0.090067 \text{Cos}[\frac{\pi x}{1812}] + 0.0749473 \text{Sin}[\frac{\pi x}{1812}]$
a_28_min	$-0.0980538 \text{Cos}[\frac{\pi x}{1812}]$
a_35_min	$-0.079547 \text{Cos}[\frac{\pi x}{1812}] - 0.0742371 \text{Sin}[\frac{\pi x}{1812}]$
a_42_min	$-0.122127 \text{Sin}[\frac{\pi x}{1812}]$
a_49_min	$0.0247122 \text{Cos}[\frac{\pi x}{1812}] - 0.0901914 \text{Sin}[\frac{\pi x}{1812}]$
a_56_min	$0.0517049 \text{Cos}[\frac{\pi x}{1812}] - 0.031871 \text{Sin}[\frac{\pi x}{1812}]$
a_63_min	$0.0872474 \text{Cos}[\frac{\pi x}{1812}] + 0.0312449 \text{Sin}[\frac{\pi x}{1812}]$
a_70_min	$0.0814359 \text{Sin}[\frac{\pi x}{1812}]$
a_77_min	$-0.0305353 \text{Cos}[\frac{\pi x}{1812}] + 0.122854 \text{Sin}[\frac{\pi x}{1812}]$
a_84_min	$-0.023398 \text{Cos}[\frac{\pi x}{1812}] + 0.0164896 \text{Sin}[\frac{\pi x}{1812}]$
a_91_min	$-0.105753 \text{Cos}[\frac{\pi x}{1812}] + 0.050148 \text{Sin}[\frac{\pi x}{1812}]$
a_98_min	$-0.0485223 \text{Cos}[\frac{\pi x}{1812}]$
a_105_min	$0.0124528 \text{Sin}[\frac{\pi x}{1812}]$
a_112_min	$-0.0423649 \text{Cos}[\frac{\pi x}{1812}] - 0.118191 \text{Sin}[\frac{\pi x}{1812}]$
a_119_min	$0.0551446 \text{Cos}[\frac{\pi x}{1812}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(d) Expression Level";
framex = Table[{0.5 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.5 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.5 * n < 0, 0,
      If[matrix[[n, a]] + 0.5 * n > 9.5, 9.5,
       matrix[[n, a]] + 0.5 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.5 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 9.5}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1350}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 675}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 2.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g4 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];

(* Create GSVD Basis External Array Correlations Bar Chart Display *)

partialarraylets = Transpose[partialarraylets];
partialarraylets = Drop[partialarraylets, {17, 18}];
partialarraylets = Drop[partialarraylets, {6, 13}];
partialarraylets = Drop[partialarraylets, {1, 2}];
partialarraylets = Transpose[partialarraylets];

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
u = Transpose[SingularValues[partialarraylets][[1]]];
gsvdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[gsvdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]
```

```
0.843396
```

```
limit = 1;
```

```
gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(b) GSVD Basis Correlation";
labely = " ";
g = BarChart[
   Table[gsvdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 4}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 3}, {0, 0}, {0, 1}];
q4 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* SVD Sort External Arrays *)

genes = genes2;
genenames = TakeColumns[genenames2, 1];
arrays = arrays2;
arraynames = arraynames2;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{3954}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialeigenarrays = AppendRows[counter, svdphases, genenames, eigenarrays];
partialeigenarrays = Sort[partialeigenarrays, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialeigenarrays, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialeigenarrays, {3, 3}],
   {1, partialgenes}];
partialeigenarrays = TakeRows[
   TakeColumns[partialeigenarrays, {4, arrays + 3}],
   {1, partialgenes}];
partialeigenarrays = TakeColumns[partialeigenarrays, {1, 6}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Center Gene Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];

(* Center Array Data *)

partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
(* Create SVD Basis External Array Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialeigenarrays][[1]]];
svdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[svdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.588331

limit = 0.6;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(a) SVD Basis Correlation";
labely = "Arrays";
g = BarChart[
   Table[svdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 4}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 3}, {0, 0}, {0, 1}];
q3 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Project Data Onto SVD Basis *)

externalarraycorrelations = Dot[PseudoInverse[partialeigenarrays], partialexternalmatrix];
partialexternalmatrix = Dot[partialeigenarrays, externalarraycorrelations];

(* Calculate Contributions of Eigenarrays to External Arrays *)

partialexternalmatrix = Transpose[partialexternalmatrix];
externalcoordinates = Table[
   {externalarraycorrelations[[1, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]],
    externalarraycorrelations[[2, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]]},
   {a, 1, externalarrays}];
partialexternalmatrix = Transpose[partialexternalmatrix];

svdarrays = externalarrays;
svdcoordinates = externalcoordinates;
```

```
(* Classify SVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.000221307}

{-0.000315169}

endph1 = 867;
beginph2 = 868;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.00276875}

{0.999788}

endph2 = 1649;
beginph3 = 1650;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.00101}

{0.999626}

endph3 = 2245;
beginph4 = 2246;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.00001}

{0.999872}

endph4 = 3225;
beginph5 = 3226;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00032}

{1.99995}

(* 3954 yeast genes, 867 in M/G1, 782 in G1, 596 in S, 980 in S/G2, 729 in G2/M. *)
```

```
(* Display SVD Reconstructed and Sorted External Data *)

(* SVD Reconstruct Sorted External Data *)

matrix = partialexternalmatrix;

(* Create SVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(a) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

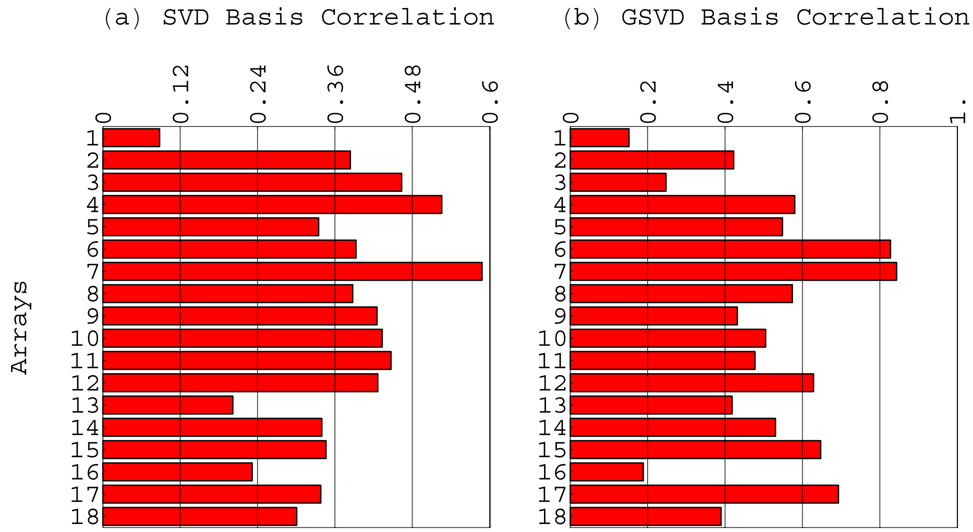
```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 700}, {0, 0}, {0, 1}];
g1 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.02],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

a_0_min	$-0.0298304 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.0555269 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_7_min	$0.0852254 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.0987516 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_14_min	$-0.0347534 - 0.0324876 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.113899 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_21_min	$-0.0332091 - 0.0670885 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.130139 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_28_min	$-0.0958274 \operatorname{Cos}[\frac{2\pi x}{3953}]$
a_35_min	$-0.0208353 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_42_min	$-0.0403921 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.104686 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_49_min	$0.043178 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.0755126 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_56_min	$0.0628262 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.0545885 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_63_min	$0.124974 \operatorname{Cos}[\frac{2\pi x}{3953}]$
a_70_min	$0.056966 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.0545245 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_77_min	$-0.0206309 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.0959781 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_84_min	$-0.0429149 \operatorname{Cos}[\frac{2\pi x}{3953}]$
a_91_min	$-0.0507345 \operatorname{Cos}[\frac{2\pi x}{3953}] + 0.0223237 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_98_min	$-0.0647489 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.0463648 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_105_min	$0.0301683 \operatorname{Cos}[\frac{2\pi x}{3953}]$
a_112_min	$0.0256747 - 0.115957 \operatorname{Sin}[\frac{2\pi x}{3953}]$
a_119_min	$0.0217212 + 0.0668363 \operatorname{Cos}[\frac{2\pi x}{3953}] - 0.05363 \operatorname{Sin}[\frac{2\pi x}{3953}]$

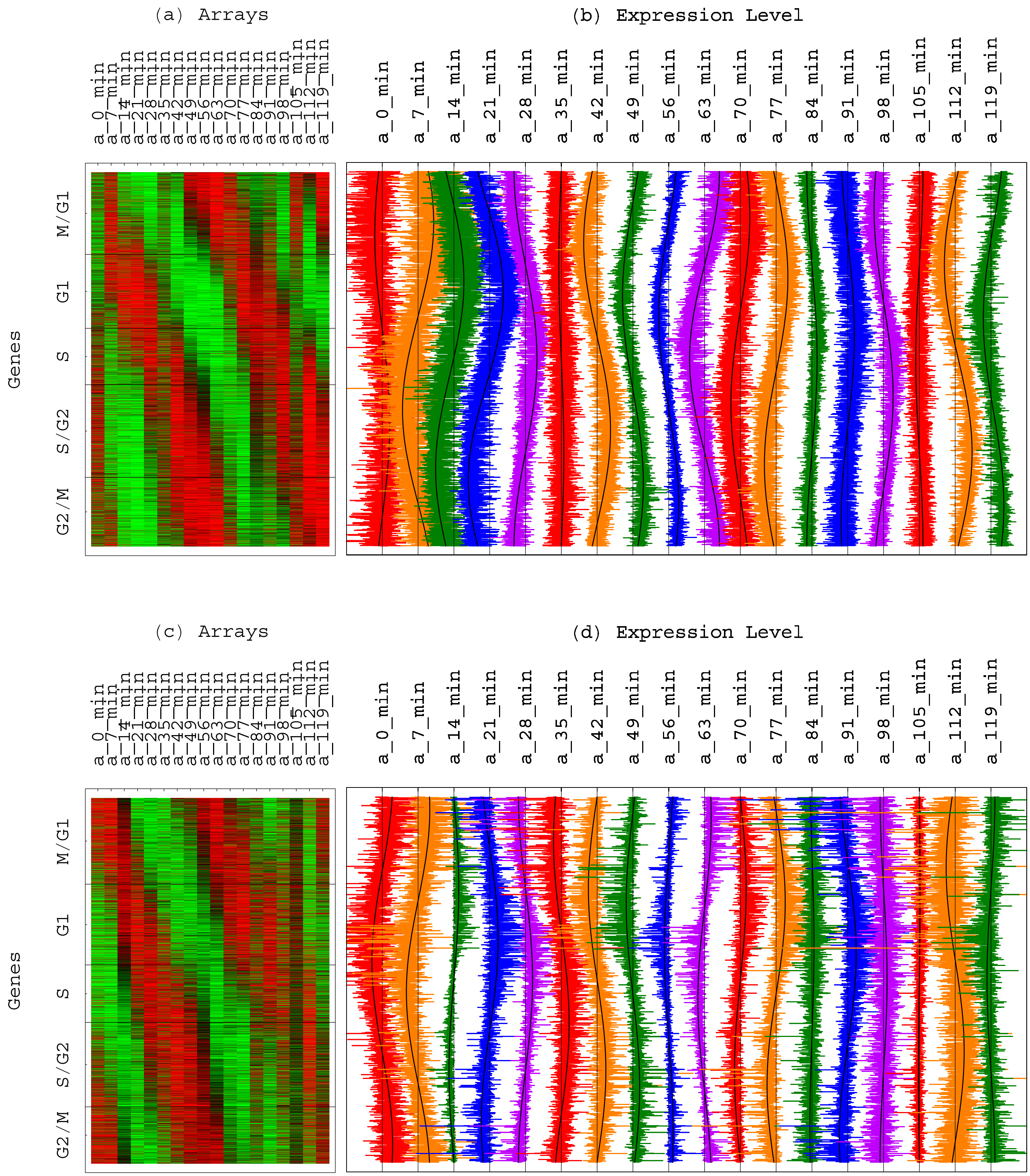
```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(b) Expression Level";
framex = Table[{0.3 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.3 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.3 * n < 0, 0,
      If[matrix[[n, a]] + 0.3 * n > 5.7, 5.7,
       matrix[[n, a]] + 0.3 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.3 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 5.7}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1350}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 675}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 2.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g2 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];
```

```
(* Display Correlations of External Arrays with SVD and GSVD Bases *)

Show[GraphicsArray[{q3, q4}],
  GraphicsSpacing -> -0.125];
```



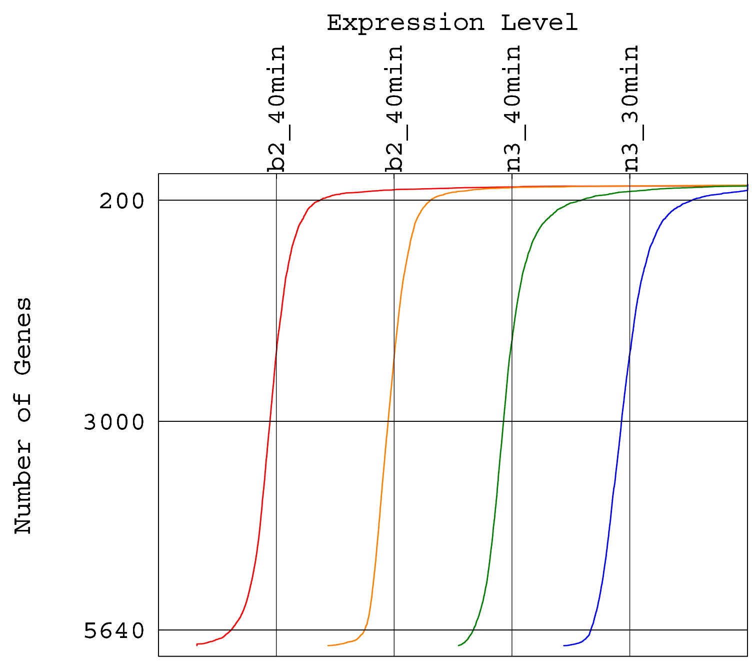

```
(* Display SVD and GSVD Reconstructed and Sorted External Data *)

q1 = Show[{
     Graphics[{Rectangle[{0, 0}, {58, 75}, g1]}],
     Graphics[{Rectangle[{56, 0}, {169, 75}, g2]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
q2 = Show[{
     Graphics[{Rectangle[{0, 0}, {58, 75}, g3]}],
     Graphics[{Rectangle[{56, 0}, {169, 75}, g4]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
Show[GraphicsArray[{{q1}, {q2}}],
  GraphicsSpacing → -0.11];
```


(a) Arrays
(b) Expression Level
(c) Arrays
(d) Expression Level
Genes
M/G1
G1
S
S/G2
G2/M
a_0_min
a_7_min
a_14_min
a_21_min
a_28_min
a_35_min
a_42_min
a_49_min
a_56_min
a_63_min
a_70_min
a_77_min
a_84_min
a_91_min
a_98_min
a_105_min
a_112_min
a_119_min

```
(* Map Yeast Cell Cycle Regulators Overexpression Data *)

(* Read Data *)

stream = "Desktop/Networks/Data/Regulator_Overexpress.txt.nb";
externalmatrix = ReadList[stream, Word, RecordLists -> True, NullWords -> True];
{externalgenes, externalarrays} = Dimensions[externalmatrix] - {2, 3}
Clear[stream];

{5840, 4}

externalgenenames = TakeRows[
   TakeColumns[externalmatrix, {1, 3}],
   {3, externalgenes + 2}];
externalarraynames = TakeColumns[
   TakeRows[externalmatrix, {1, 2}],
   {4, externalarrays + 3}];
externalmatrix = TakeColumns[
   TakeRows[externalmatrix, {3, externalgenes + 2}],
   {4, externalarrays + 3}];
externalmatrix = ToExpression[externalmatrix];

sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[externalarraynames[[2, a]]
      ]]],
    {a, 1, externalarrays}]];
size = Sort[sizes, OrderedQ[{#2, #1}] &][[1]];
Do[
  Do[externalarraynames[[2, a]] = StringJoin[ToString[externalarraynames[[2, a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, externalarrays}];

(* Display Sorted External Arrays *)

arraypatterns = Transpose[externalmatrix];

(* Center External Arrays *)

average = Table[1, {a, 1, externalgenes}];
average = N[average / Sqrt[Dot[average, average]]];
arraypatterns = arraypatterns - N[Outer[Times, Dot[arraypatterns, average], average]];

(* Normalize External Arrays *)

Do[
 arraypatterns[[a]] =
  arraypatterns[[a]] / Sqrt[Dot[arraypatterns[[a]], arraypatterns[[a]]]],
 {a, 1, externalarrays}]

(* Sort External Arrays *)

Do[
 arraypatterns[[a]] = Sort[arraypatterns[[a]], OrderedQ[{{#2}, {#1}}] &],
 {a, 1, externalarrays}]
```

```
(* Create Sorted External Arrays Graph Display *)

p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "Expression Level";
labely = ColumnForm[
   {" ", "Number of Genes", " ", " ", " ", " ", " ", " ", " ", " "},
   Center];
framex = Table[{0.02 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
framey = {{-200, "200"}, {-3000, "3000"}, {-externalgenes + 200, "5640"}};

Do[{
   coordinates = Table[
     If[arraypatterns[[n, a]] + 0.02 * n < -0.02, -0.02,
      If[arraypatterns[[n, a]] + 0.02 * n > 0.1, 0.1,
       arraypatterns[[n, a]] + 0.02 * n]],
     {a, 1, externalgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, externalgenes}],
   line = Line[coordinates],
   g = Show[
     Graphics[{color[[Mod[n, 5] + 1]], line}],
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {{{0.02 * n, RGBColor[0, 0, 0]}}, {{-200, RGBColor[0, 0, 0]},
        {-3000, RGBColor[0, 0, 0]}, {-externalgenes + 200, RGBColor[0, 0, 0]}}},
     PlotRange -> {{0, 0.1}, {135, -externalgenes + 1 - 135}},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1600}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 700}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     PlotRange → All,
     AspectRatio → 2 / 1.2 / GoldenRatio,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
```

```
(* Display Sorted External Arrays *)

Show[Table[p[[a]], {a, 1, externalarrays}],
  DisplayFunction → $DisplayFunction];
```



```
(* Estimate Significance of Association of External Data with the Cell Cycle *)

(* Use Microarray Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {2}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table1 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Use Traditional Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {3}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table2 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Display Significance of Association of External Arrays with the Cell Cycle *)

headerx = {{
    ColumnForm[{" ", " ", " "}, Left],
    ColumnForm[{" ", " ", "Classification"}, Left],
    ColumnForm[{" ", "External", "Array"}, Left],
    ColumnForm[{"Most Likely", "Parallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Parallel", "Association"}, Left],
    ColumnForm[{"Most Likely", "Antiparallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Antiparallel", "Association"}, Left]},
   {" ", " ", " ", " ", " ", " ", " "}};
spacerx = {{" ", " ", " ", " ", " "}};
headery = Table[" ", {a, 1, 2 * externalarrays + 1}, {b, 1, 2}];
headery[[1]] = {"(a)", "Microarray"};
headery[[externalarrays + 2]] = {"(b)", "Traditional"};
association =
  AppendColumns[headerx,
   AppendRows[headery,
    AppendColumns[table1, spacerx, table2]]];
TableForm[association, TableSpacing → {1, 1}]
```

	Classification	External Array	Most Likely Parallel Association	P-Value of Parallel Association	Most Likely Antiparallel Association	P-Value of Antiparallel Association
(a)	Microarray	b2_40min	G2/M	2.1×10^{-67}	G1	7.3×10^{-26}
		b2_40min	G2/M	$1. \times 10^{-55}$	G1	1.2×10^{-31}
		n3_40min	G1	1.2×10^{-61}	M/G1	6.5×10^{-13}
		n3_30min	G1	5.1×10^{-48}	G2/M	4.5×10^{-18}
(b)	Traditional	b2_40min	G2/M	$7. \times 10^{-14}$	G1	9.2×10^{-9}
		b2_40min	G2/M	$7. \times 10^{-14}$	G1	$2. \times 10^{-14}$
		n3_40min	G1	5.2×10^{-27}	G2/M	$1. \times 10^{-4}$
		n3_30min	G1	1.1×10^{-15}	G2/M	$1. \times 10^{-4}$

```
(* GSVD Sort External Arrays *)

genes = genes1;
genenames = TakeColumns[genenames1, 1];
arrays = arrays1;
arraynames = arraynames1;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{4383}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialarraylets = AppendRows[counter, gsvdphases, genenames, arraylets1];
partialarraylets = Sort[partialarraylets, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialarraylets, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialarraylets, {3, 3}],
   {1, partialgenes}];
partialarraylets = TakeRows[
   TakeColumns[partialarraylets, {4, arrays + 3}],
   {1, partialgenes}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Project Data Onto GSVD Basis and Calculate Contributions of Arraylets to External Arrays *)

partialarraylets = Transpose[partialarraylets];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialarraylets =
  partialarraylets - N[Outer[Times, Dot[partialarraylets, average], average]];
partialarraylets = Transpose[partialarraylets];

arraycontributions = Dot[PseudoInverse[partialarraylets], partialexternalmatrix];
average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
arraycontributions =
  arraycontributions - N[Outer[Times, Dot[arraycontributions, average], average]];
```

```
(* Project Arrays from 6 D Arraylets Subspace Onto 2 D Subspace  *)

externalcoordinates = Table[{
    (-Sqrt[3] * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        2 * Sqrt[3] * (arraycontributions[[4, a]] - arraycontributions[[16, a]]) +
        (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]],
    (3 * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        Sqrt[3] * (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]]},
   {a, 1, externalarrays}];
```

```
(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

coordinates = gsvdcoordinates;
arrays = gsvdarrays;

points1 = {Point[coordinates[[2]]], Point[coordinates[[11]]]};
points2 = {Point[coordinates[[3]]], Point[coordinates[[4]]], Point[coordinates[[5]]],
   Point[coordinates[[12]]], Point[coordinates[[13]]], Point[coordinates[[14]]]};
points3 = {Point[coordinates[[6]]], Point[coordinates[[15]]]};
points4 = {Point[coordinates[[7]]]};
points5 = {Point[coordinates[[1]]], Point[coordinates[[8]]], Point[coordinates[[9]]],
   Point[coordinates[[10]]], Point[coordinates[[16]]], Point[coordinates[[17]]],
   Point[coordinates[[18]]]};
points6 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[2]]]};
points7 = {Point[externalcoordinates[[3]]], Point[externalcoordinates[[4]]]};
textcoordinates = Flatten[{coordinates, externalcoordinates}, 1];
Do[
  textcoordinates[[a, 1]] = If[
    textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.085,
    textcoordinates[[a, 1]] + 0.095],
  {a, 1, 9}];
Do[
  textcoordinates[[a, 1]] =
   If[textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.11,
    textcoordinates[[a, 1]] + 0.12],
  {a, 10, arrays + externalarrays}];
textcoordinates[[3]] = textcoordinates[[3]] + {0.18, -0.06};
textcoordinates[[5]] = textcoordinates[[5]] + {-0.22, 0};
textcoordinates[[6]] = textcoordinates[[6]] + {-0.06, 0.1};
textcoordinates[[7]] = textcoordinates[[7]] + {0.1, -0.095};
textcoordinates[[8]] = textcoordinates[[8]] + {0.16, -0.085};
textcoordinates[[9]] = textcoordinates[[9]] + {0.1, -0.1};
textcoordinates[[11]] = textcoordinates[[11]] + {0.23, 0};
textcoordinates[[15]] = textcoordinates[[15]] - {0.23, 0};
textcoordinates[[16]] = textcoordinates[[16]] - {0.23, 0.06};
textcoordinates[[19]] = textcoordinates[[19]] + {0, 0.02};
textcoordinates[[21]] = textcoordinates[[21]] - {0, 0.08};
textcoordinates[[22]] = textcoordinates[[22]] - {0, 0.04};
```

```
texts = Table[Text[a, textcoordinates[[a]]], {a, 1, arrays + externalarrays}];
p = Show[
   {Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[0, 0, 1], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points5}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points6}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points7}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.1, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(b)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/3 ", {0.925, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=-π/3", {0.925, -1.15}]}],
    Graphics[texts],
    Graphics[{RGBColor[0, 0, 0], Arrow[{1.12, -0.15}, externalcoordinates[[1]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{1.12, -0.15}, externalcoordinates[[2]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Text["CLB2", {1.12, -0.25}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.12, 0.125}, externalcoordinates[[3]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.12, 0.125}, externalcoordinates[[4]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Text["CLN3", {-1.12, 0.175}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], 1.25}, {1.25 / Tan[Pi / 3.], -1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], -1.25}, {1.25 / Tan[Pi / 3.], 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Circle[{0, 0}, 0.4,
       {ArcTan[coordinates[[1, 2]] / coordinates[[1, 1]]],
        ArcTan[coordinates[[3, 2]] / coordinates[[3, 1]]]}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[
       {0.4 * Cos[-0.05 + ArcTan[coordinates[[3, 2]] / coordinates[[3, 1]]]],
        0.4 * Sin[-0.05 + ArcTan[coordinates[[3, 2]] / coordinates[[3, 1]]]]},
       {0.4 * Cos[ArcTan[coordinates[[3, 2]] / coordinates[[3, 1]]]],
        0.4 * Sin[ArcTan[coordinates[[3, 2]] / coordinates[[3, 1]]]]},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
s2 = Show[p,
   AspectRatio → 1.0,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify GSVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.0000468343}

{-0.000239433}

endph1 = 1068;
beginph2 = 1069;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.00152245}

{0.999794}

endph2 = 2044;
beginph3 = 2045;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.0003}

{0.999906}

endph3 = 2729;
beginph4 = 2730;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.0009}

{0.99987}

endph4 = 3713;
beginph5 = 3714;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00019}

{1.99935}

(* 4383 yeast genes, 1068 in M/G1, 976 in G1, 685 in S, 984 in S/G2, 670 in G2/M. *)
```

```
(* Display GSVD Reconstructed Sorted External Data *)

(* GSVD Reconstruct Sorted External Data *)

subspace = Table[0, {a, 1, arrays}];
Do[subspace[[a]] = 1, {a, 3, 5}];
Do[subspace[[a]] = 1, {a, 14, 16}];
matrix = Dot[partialarraylets, Dot[DiagonalMatrix[subspace], arraycontributions]];

(* Center GSVD Reconstructed Sorted External Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];

(* Create GSVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(c) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 0.4, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 2200}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1100}, {0, 0}, {0, 1}];
g3 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create GSVD Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.05],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

b2_40min	$0.210075 \text{Cos}[\frac{\pi x}{2191}]$
b2_40min	$0.174958 \text{Cos}[\frac{\pi x}{2191}] - 0.0545018 \text{Sin}[\frac{\pi x}{2191}]$
n3_40min	$-0.277423 \text{Cos}[\frac{\pi x}{2191}] + 0.0549437 \text{Sin}[\frac{\pi x}{2191}]$
n3_30min	$-0.107609 \text{Cos}[\frac{\pi x}{2191}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(d) Expression Level";
framex = Table[{1 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 1 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 1 * n < 0, 0,
      If[matrix[[n, a]] + 1 * n > 5, 5,
       matrix[[n, a]] + 1 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{1 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 5}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 2200}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 1100}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 1.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g4 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];

(* Create GSVD Basis External Array Correlations Bar Chart Display *)

partialarraylets = Transpose[partialarraylets];
partialarraylets = Drop[partialarraylets, {17, 18}];
partialarraylets = Drop[partialarraylets, {6, 13}];
partialarraylets = Drop[partialarraylets, {1, 2}];
partialarraylets = Transpose[partialarraylets];

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
u = Transpose[SingularValues[partialarraylets][[1]]];
gsvdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[gsvdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]
```

```
0.43626
```

```
limit = 0.5;
```

```
gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(b) GSVD Basis Correlation";
labely = " ";
g = BarChart[
   Table[gsvdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.88}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.66}, {0, 0}, {0, 1}];
q4 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* SVD Sort External Arrays *)

genes = genes2;
genenames = TakeColumns[genenames2, 1];
arrays = arrays2;
arraynames = arraynames2;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{4404}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialeigenarrays = AppendRows[counter, svdphases, genenames, eigenarrays];
partialeigenarrays = Sort[partialeigenarrays, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialeigenarrays, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialeigenarrays, {3, 3}],
   {1, partialgenes}];
partialeigenarrays = TakeRows[
   TakeColumns[partialeigenarrays, {4, arrays + 3}],
   {1, partialgenes}];
partialeigenarrays = TakeColumns[partialeigenarrays, {1, 6}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Center Gene Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];

(* Center Array Data *)

partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
(* Create SVD Basis External Array Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialeigenarrays][[1]]];
svdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[svdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.590854

limit = 0.6;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(a) SVD Basis Correlation";
labely = "Arrays";
g = BarChart[
   Table[svdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.88}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.66}, {0, 0}, {0, 1}];
q3 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Project Data Onto SVD Basis *)

externalarraycorrelations = Dot[PseudoInverse[partialeigenarrays], partialexternalmatrix];
partialexternalmatrix = Dot[partialeigenarrays, externalarraycorrelations];

(* Calculate Contributions of Eigenarrays to External Arrays *)

partialexternalmatrix = Transpose[partialexternalmatrix];
externalcoordinates = Table[
   {externalarraycorrelations[[1, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]],
    externalarraycorrelations[[2, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]]},
   {a, 1, externalarrays}];
partialexternalmatrix = Transpose[partialexternalmatrix];

(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

coordinates = svdcoordinates;
arrays = svdarrays;

points1 = {Point[coordinates[[2]]], Point[coordinates[[11]]]};
points2 = {Point[coordinates[[3]]], Point[coordinates[[4]]], Point[coordinates[[5]]],
   Point[coordinates[[12]]], Point[coordinates[[13]]], Point[coordinates[[14]]]};
points3 = {Point[coordinates[[6]]], Point[coordinates[[15]]]};
points4 = {Point[coordinates[[7]]]};
points5 = {Point[coordinates[[1]]], Point[coordinates[[8]]], Point[coordinates[[9]]],
   Point[coordinates[[10]]], Point[coordinates[[16]]], Point[coordinates[[17]]],
   Point[coordinates[[18]]]};
points6 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[2]]]};
points7 = {Point[externalcoordinates[[4]]], Point[externalcoordinates[[3]]]};
textcoordinates = Flatten[{coordinates, externalcoordinates}, 1];
Do[
  textcoordinates[[a, 1]] = If[
    textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.085,
    textcoordinates[[a, 1]] + 0.095],
  {a, 1, 9}];
Do[
  textcoordinates[[a, 1]] =
   If[textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.11,
    textcoordinates[[a, 1]] + 0.12],
  {a, 10, arrays + externalarrays}];
textcoordinates[[1]] = textcoordinates[[1]] - {0.085, 0.1};
textcoordinates[[2]] = textcoordinates[[2]] + {0.18, 0.04};
textcoordinates[[3]] = textcoordinates[[3]] - {0, 0.04};
textcoordinates[[6]] = textcoordinates[[6]] - {0.18, 0};
textcoordinates[[7]] = textcoordinates[[7]] - {0.18, 0};
textcoordinates[[9]] = textcoordinates[[9]] - {0, 0.04};
textcoordinates[[12]] = textcoordinates[[12]] + {0, 0.04};
textcoordinates[[13]] = textcoordinates[[13]] - {0.23, 0};
textcoordinates[[14]] = textcoordinates[[14]] - {0.11, 0.13};
textcoordinates[[17]] = textcoordinates[[17]] + {0.12, 0.1};
textcoordinates[[19]] = textcoordinates[[19]] + {0.12, -0.12};
textcoordinates[[20]] = textcoordinates[[20]] + {0.14, 0.12};
textcoordinates[[21]] = textcoordinates[[21]] + {-0.12, 0.2};
textcoordinates[[22]] = textcoordinates[[22]] + {0.04, 0.02};
```

```
texts = Table[Text[a, textcoordinates[[a]]], {a, 1, arrays + externalarrays}];
p = Show[
   {Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[0, 0, 1], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points5}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points7}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points6}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.2, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(a)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/2", {0.2, 1.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[texts],
    Graphics[{RGBColor[0, 0, 0], Arrow[{1.1, -0.26}, {0.7, -0.08},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Text["CLB2", {1.1, -0.34}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.1, 0.37}, externalcoordinates[[3]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.1, 0.37}, externalcoordinates[[4]],
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Text["CLN3", {-1.1, 0.44}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5, {0. * Pi, 2 * Pi}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{0, -1.25}, {0, 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Circle[{0, 0}, 0.7,
       {-Pi + ArcTan[coordinates[[1, 2]] / coordinates[[1, 1]]],
        ArcTan[coordinates[[8, 2]] / coordinates[[8, 1]]]}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[
       {0.7 * Cos[-0.05 + ArcTan[coordinates[[8, 2]] / coordinates[[8, 1]]]],
        0.7 * Sin[-0.05 + ArcTan[coordinates[[8, 2]] / coordinates[[8, 1]]]]},
       {0.7 * Cos[ArcTan[coordinates[[8, 2]] / coordinates[[8, 1]]]],
        0.7 * Sin[ArcTan[coordinates[[8, 2]] / coordinates[[8, 1]]]]},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {{{0, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
s1 = Show[p,
   AspectRatio -> 1.,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify SVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.000221307}

{-0.000315169}

endph1 = 967;
beginph2 = 968;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.000187677}

{0.999788}

endph2 = 1827;
beginph3 = 1828;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.00101}

{0.999626}

endph3 = 2481;
beginph4 = 2482;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.00001}

{0.999872}

endph4 = 3556;
beginph5 = 3557;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00032}

{1.99995}

(* 4404 yeast genes, 967 in M/G1, 920 in G1, 654 in S, 1075 in S/G2, 848 in G2/M. *)
```

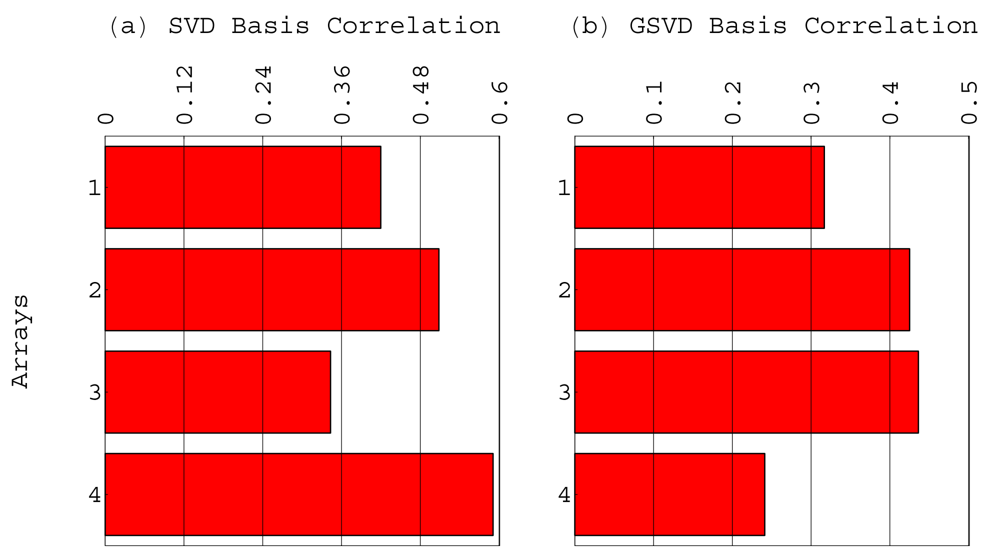
```
(* Display SVD Reconstructed and Sorted External Data *)

(* SVD Reconstruct Sorted External Data *)

matrix = partialexternalmatrix;

(* Create SVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(a) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 0.4, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 2200}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1100}, {0, 0}, {0, 1}];
g1 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.05],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

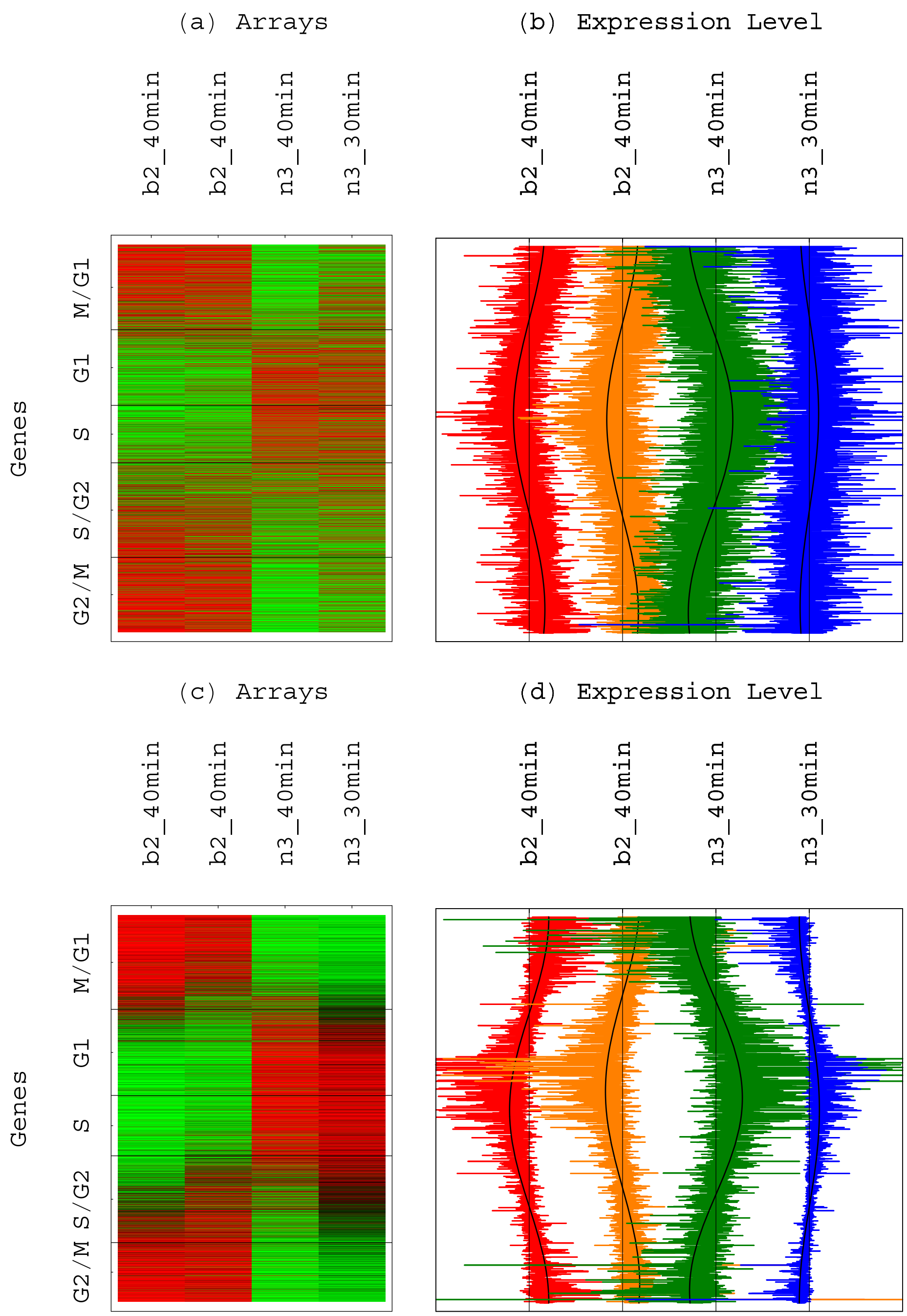
b2_40min	$0.218194 \operatorname{Cos}[\frac{2\pi x}{4403}] - 0.0879994 \operatorname{Sin}[\frac{2\pi x}{4403}]$
b2_40min	$0.224201 \operatorname{Cos}[\frac{2\pi x}{4403}] - 0.0760136 \operatorname{Sin}[\frac{2\pi x}{4403}]$
n3_40min	$-0.0810224 - 0.315992 \operatorname{Cos}[\frac{2\pi x}{4403}] + 0.106885 \operatorname{Sin}[\frac{2\pi x}{4403}]$
n3_30min	$-0.126403 \operatorname{Cos}[\frac{2\pi x}{4403}] + 0.0571277 \operatorname{Sin}[\frac{2\pi x}{4403}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(b) Expression Level";
framex = Table[{1.4 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 1.4 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 1.4 * n < 0, 0,
      If[matrix[[n, a]] + 1.4 * n > 7, 7,
       matrix[[n, a]] + 1.4 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{1.4 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 7}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 2200}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 1100}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 1.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g2 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];

(* Display Correlations of External Arrays with SVD and GSVD Bases *)

Show[GraphicsArray[{q3, q4}],
  GraphicsSpacing -> -0.125];
```

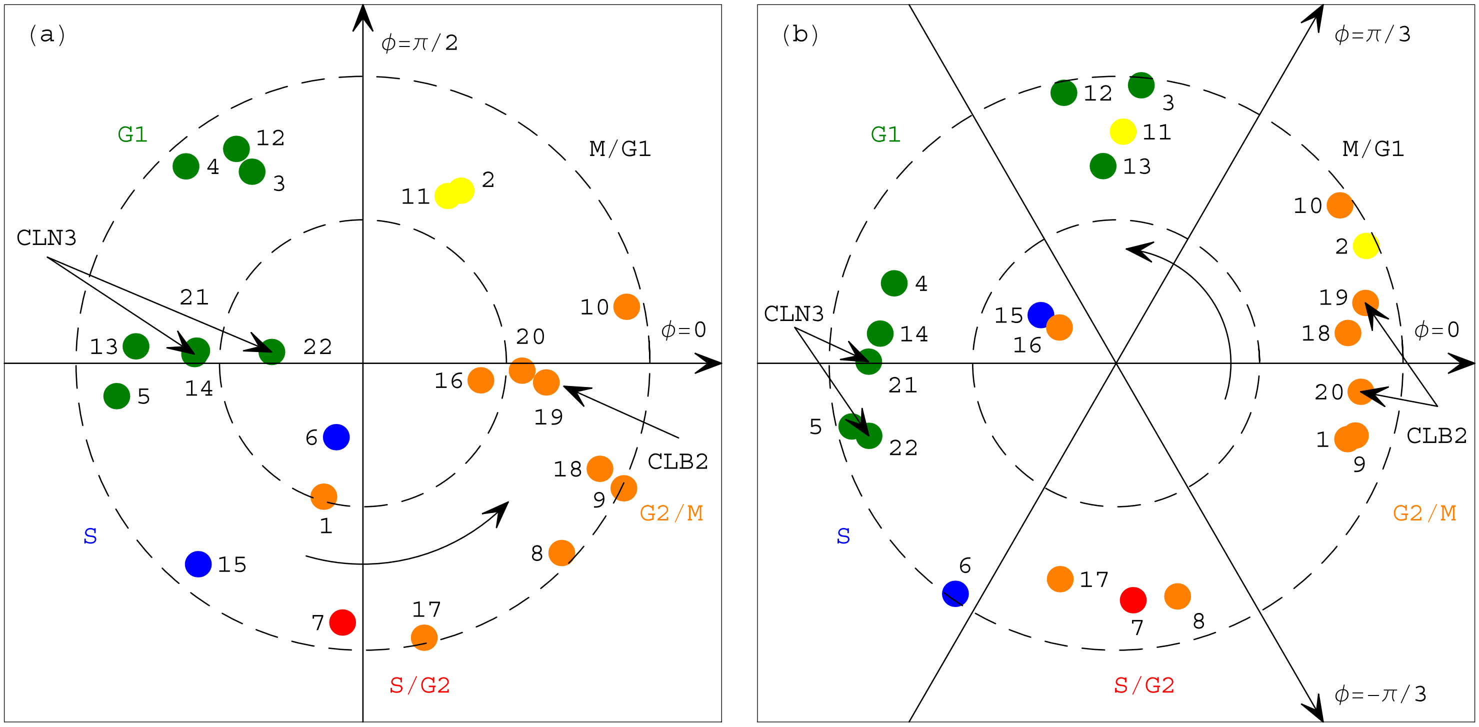


```
(* Display SVD and GSVD Sorted External Data *)

q1 = Show[{
    Graphics[{Rectangle[{0, 0}, {82, 75}, g1]}],
    Graphics[{Rectangle[{70, 0}, {155, 75}, g2]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
q2 = Show[{
    Graphics[{Rectangle[{0, 0}, {82, 75}, g3]}],
    Graphics[{Rectangle[{70, 0}, {155, 75}, g4]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
Show[GraphicsArray[{{q1}, {q2}}],
  GraphicsSpacing → -0.02];
```



```
(* Display Mapping of Alpha Factor Time Course and Regulators Overexpression Data *)
```

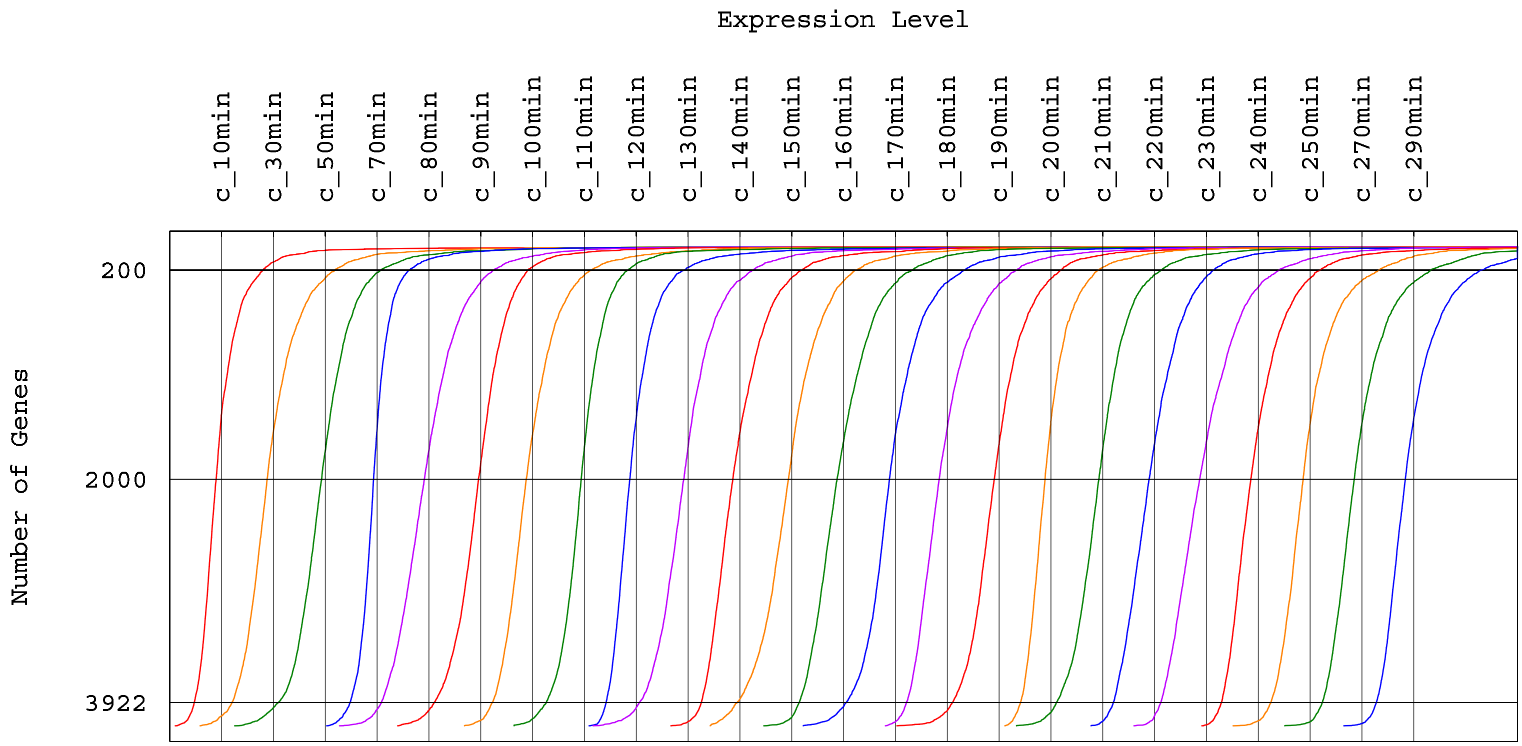
```
Show[GraphicsArray[{s1, s2}],
    GraphicsSpacing -> 0];
```



```
(* Map Yeast Cdc15 Cell Cycle Time Course Expression Data *)

(* Read Data *)

stream = "Desktop/Networks/Data/Cdc15.txt.nb";
externalmatrix = ReadList[stream, Word, RecordLists -> True, NullWords -> True];
{externalgenes, externalarrays} = Dimensions[externalmatrix] - {2, 3}
Clear[stream];

{4122, 24}

externalgenenames = TakeRows[
   TakeColumns[externalmatrix, {1, 3}],
   {3, externalgenes + 2}];
externalarraynames = TakeColumns[
   TakeRows[externalmatrix, {1, 2}],
   {4, externalarrays + 3}];
externalmatrix = TakeColumns[
   TakeRows[externalmatrix, {3, externalgenes + 2}],
   {4, externalarrays + 3}];
externalmatrix = ToExpression[externalmatrix];

sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[externalarraynames[[2, a]]
      ]]],
    {a, 1, externalarrays}]];
size = Sort[sizes, OrderedQ[{#2, #1}] &][[1]];
Do[
  Do[externalarraynames[[2, a]] = StringJoin[ToString[externalarraynames[[2, a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, externalarrays}];

(* Display Sorted External Arrays *)

arraypatterns = Transpose[externalmatrix];

(* Center External Arrays *)

average = Table[1, {a, 1, externalgenes}];
average = N[average / Sqrt[Dot[average, average]]];
arraypatterns = arraypatterns - N[Outer[Times, Dot[arraypatterns, average], average]];

(* Normalize External Arrays *)

Do[
 arraypatterns[[a]] =
  arraypatterns[[a]] / Sqrt[Dot[arraypatterns[[a]], arraypatterns[[a]]]],
 {a, 1, externalarrays}]

(* Sort External Arrays *)

Do[
 arraypatterns[[a]] = Sort[arraypatterns[[a]], OrderedQ[{{#2}, {#1}}] &],
 {a, 1, externalarrays}]
```

```
(* Create Sorted External Arrays Graph Display *)

p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "Expression Level";
labely = ColumnForm[
   {" ", "Number of Genes", " ", " ", " ", " ", " ", " ", " ", " "},
   Center];
framex = Table[{0.02 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
framey = {{-200, "200"}, {-2000, "2000"}, {-externalgenes + 200, "3922"}};

Do[{
   coordinates = Table[
     If[arraypatterns[[n, a]] + 0.02 * n < -0.02, -0.02,
      If[arraypatterns[[n, a]] + 0.02 * n > 0.52, 0.52,
       arraypatterns[[n, a]] + 0.02 * n]],
     {a, 1, externalgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, externalgenes}],
   line = Line[coordinates],
   g = Show[
     Graphics[{color[[Mod[n, 5] + 1]], line}],
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {{{0.02 * n, RGBColor[0, 0, 0]}}, {{-200, RGBColor[0, 0, 0]},
        {-2000, RGBColor[0, 0, 0]}, {-externalgenes + 200, RGBColor[0, 0, 0]}}},
     PlotRange -> {{0, 0.52}, {135, -externalgenes + 1 - 135}},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1600}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 700}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → 1 / 1.2 / GoldenRatio,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
```

```
(* Display Sorted External Arrays *)

Show[Table[p[[a]], {a, 1, externalarrays}],
  DisplayFunction → $DisplayFunction];
```



```
(* Estimate Significance of Association of External Data with the Cell Cycle *)

(* Use Microarray Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {2}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table1 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Use Traditional Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {3}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table2 = AppendRows[
   Table[{externalarraynames[[2, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Display Significance of Association of External Arrays with the Cell Cycle *)

headerx = {{
    ColumnForm[{" ", " ", " "}, Left],
    ColumnForm[{" ", " ", "Classification"}, Left],
    ColumnForm[{" ", "External", "Array"}, Left],
    ColumnForm[{"Most Likely", "Parallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Parallel", "Association"}, Left],
    ColumnForm[{"Most Likely", "Antiparallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Antiparallel", "Association"}, Left]},
   {" ", " ", " ", " ", " ", " ", " "}};
spacerx = {{" ", " ", " ", " ", " "}};
headery = Table[" ", {a, 1, 2 * externalarrays + 1}, {b, 1, 2}];
headery[[1]] = {"(a)", "Microarray"};
headery[[externalarrays + 2]] = {"(b)", "Traditional"};
association =
  AppendColumns[headerx,
   AppendRows[headery,
    AppendColumns[table1, spacerx, table2]]];
TableForm[association, TableSpacing → {1, 1}]
```

	Classification	External Array	Most Likely Parallel Association	P-Value of Parallel Association	Most Likely Antiparallel Association	P-Value of Antiparallel Association
(a)	Microarray	c_10min	M/G1	1.1×10^{-3}	G2/M	5.2×10^{-8}
		c_30min	G1	1.4×10^{-12}	G2/M	4.1×10^{-21}
		c_50min	G1	2.3×10^{-29}	G2/M	1.5×10^{-30}
		c_70min	S	2.5×10^{-8}	M/G1	2.8×10^{-19}
		c_80min	S/G2	2.2×10^{-9}	M/G1	9.6×10^{-15}
		c_90min	G2/M	3.8×10^{-20}	G1	$3.\times10^{-20}$
		c_100min	G2/M	3.8×10^{-20}	G1	2.3×10^{-29}
		c_110min	G2/M	$2.\times10^{-29}$	G1	1.9×10^{-27}
		c_120min	G2/M	9.3×10^{-15}	G1	6.8×10^{-12}
		c_130min	M/G1	4.2×10^{-18}	S	4.5×10^{-3}
		c_140min	G1	$3.\times10^{-20}$	S/G2	8.6×10^{-8}
		c_150min	G1	1.3×10^{-18}	G2/M	4.7×10^{-5}
		c_160min	G1	6.8×10^{-12}	G2/M	4.7×10^{-5}
		c_170min	G1	1.2×10^{-4}	M/G1	2.7×10^{-4}
		c_180min	S	1.1×10^{-3}	M/G1	5.9×10^{-5}
		c_190min	G2/M	4.7×10^{-4}	G1	$1.\times10^{-14}$
		c_200min	G2/M	4.6×10^{-10}	G1	1.5×10^{-5}
		c_210min	G2/M	1.5×10^{-11}	G1	$3.\times10^{-20}$
		c_220min	G2/M	2.3×10^{-17}	G1	1.5×10^{-5}
		c_230min	G2/M	1.4×10^{-5}	G1	$5.\times10^{-6}$
		c_240min	G2/M	1.1×10^{-8}	S	4.9×10^{-2}
		c_250min	M/G1	1.2×10^{-11}	S/G2	1.2×10^{-1}
		c_270min	M/G1	1.2×10^{-5}	G2/M	3.5×10^{-3}
		c_290min	M/G1	5.9×10^{-5}	G2/M	4.7×10^{-4}
(b)	Traditional	c_10min	G2/M	1.4×10^{-2}	S	2.1×10^{-2}
		c_30min	G1	2.9×10^{-5}	G2/M	1.4×10^{-3}
		c_50min	G1	4.9×10^{-14}	G2/M	1.6×10^{-7}
		c_70min	S	2.6×10^{-5}	M/G1	1.6×10^{-5}
		c_80min	S	$1.\times10^{-3}$	M/G1	7.6×10^{-7}
		c_90min	G2/M	9.3×10^{-5}	G1	3.2×10^{-9}
		c_100min	G2/M	9.3×10^{-5}	G1	3.2×10^{-9}
		c_110min	G2/M	3.9×10^{-9}	G1	2.4×10^{-10}
		c_120min	G2/M	1.4×10^{-3}	S	2.6×10^{-5}
		c_130min	M/G1	7.6×10^{-7}	S	$1.\times10^{-3}$
		c_140min	G1	3.7×10^{-6}	G2/M	1.4×10^{-3}
		c_150min	G1	3.8×10^{-8}	G2/M	1.4×10^{-2}
		c_160min	G1	$4.\times10^{-7}$	G2/M	1.4×10^{-3}
		c_170min	S	2.6×10^{-5}	G2/M	1.4×10^{-2}
		c_180min	S	$1.\times10^{-3}$	None	3.5×10^{-1}
		c_190min	S	$1.\times10^{-3}$	M/G1	2.7×10^{-3}
		c_200min	G2/M	1.4×10^{-3}	M/G1	2.2×10^{-2}
		c_210min	G2/M	1.4×10^{-3}	G1	3.2×10^{-9}
		c_220min	G2/M	1.4×10^{-3}	G1	8.7×10^{-2}
		c_230min	G2/M	9.3×10^{-5}	G1	8.7×10^{-2}
		c_240min	M/G1	2.4×10^{-4}	S	2.2×10^{-1}
		c_250min	M/G1	2.8×10^{-8}	S/G2	1.3×10^{-2}
		c_270min	M/G1	1.6×10^{-5}	G2/M	1.4×10^{-2}
		c_290min	M/G1	2.4×10^{-4}	G2/M	1.4×10^{-2}

```
(* GSVD Sort External Arrays *)

genes = genes1;
genenames = TakeColumns[genenames1, 1];
arrays = arrays1;
arraynames = arraynames1;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{3130}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialarraylets = AppendRows[counter, gsvdphases, genenames, arraylets1];
partialarraylets = Sort[partialarraylets, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialarraylets, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialarraylets, {3, 3}],
   {1, partialgenes}];
partialarraylets = TakeRows[
   TakeColumns[partialarraylets, {4, arrays + 3}],
   {1, partialgenes}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Project Data Onto GSVD Basis and Calculate Contributions of Arraylets to External Arrays *)

partialarraylets = Transpose[partialarraylets];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialarraylets =
  partialarraylets - N[Outer[Times, Dot[partialarraylets, average], average]];
partialarraylets = Transpose[partialarraylets];

arraycontributions = Dot[PseudoInverse[partialarraylets], partialexternalmatrix];
average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
arraycontributions =
  arraycontributions - N[Outer[Times, Dot[arraycontributions, average], average]];
```

```
(* Project Arrays from 6 D Arraylets Subspace Onto 2 D Subspace  *)

externalcoordinates = Table[{
    (-Sqrt[3] * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        2 * Sqrt[3] * (arraycontributions[[4, a]] - arraycontributions[[16, a]]) +
        (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]],
    (3 * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        Sqrt[3] * (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
         (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]]},
   {a, 1, externalarrays}];
```

```
(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

points1 = {Point[externalcoordinates[[10]]], Point[externalcoordinates[[21]]],
   Point[externalcoordinates[[22]]], Point[externalcoordinates[[23]]],
   Point[externalcoordinates[[24]]]};
points2 = {Point[externalcoordinates[[2]]], Point[externalcoordinates[[3]]],
   Point[externalcoordinates[[11]]], Point[externalcoordinates[[12]]],
   Point[externalcoordinates[[13]]]};
points3 = {Point[externalcoordinates[[4]]], Point[externalcoordinates[[14]]],
   Point[externalcoordinates[[15]]], Point[externalcoordinates[[16]]]};
points4 = {Point[externalcoordinates[[5]]]};
points5 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[6]]],
   Point[externalcoordinates[[7]]], Point[externalcoordinates[[8]]],
   Point[externalcoordinates[[9]]], Point[externalcoordinates[[17]]],
   Point[externalcoordinates[[18]]], Point[externalcoordinates[[19]]],
   Point[externalcoordinates[[20]]]};
textcoordinates = externalcoordinates;
Do[
  textcoordinates[[a, 1]] = If[
    textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.085,
    textcoordinates[[a, 1]] + 0.095],
  {a, 1, 9}];
Do[
  textcoordinates[[a, 1]] =
   If[textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.11,
    textcoordinates[[a, 1]] + 0.12],
  {a, 10, externalarrays}];
textcoordinates[[1]] = textcoordinates[[1]] + {0.1, -0.12};
textcoordinates[[3]] = textcoordinates[[3]] + {0, 0.06};
textcoordinates[[4]] = textcoordinates[[4]] - {0.11, 0.12};
textcoordinates[[5]] = textcoordinates[[5]] + {-0.18, 0.085};
textcoordinates[[6]] = textcoordinates[[6]] + {0.17, 0.08};
textcoordinates[[7]] = textcoordinates[[7]] + {0.2, 0.08};
textcoordinates[[8]] = textcoordinates[[8]] + {0.23, 0};
textcoordinates[[9]] = textcoordinates[[9]] + {0.15, -0.15};
textcoordinates[[14]] = textcoordinates[[14]] + {0, -0.06};
textcoordinates[[15]] = textcoordinates[[15]] - {0.23, 0};
textcoordinates[[16]] = textcoordinates[[16]] + {0, 0.1};
textcoordinates[[17]] = textcoordinates[[17]] + {-0.1, 0.12};
textcoordinates[[18]] = textcoordinates[[18]] + {0.23, -0.06};
textcoordinates[[19]] = textcoordinates[[19]] + {0.23, 0.02};
textcoordinates[[22]] = textcoordinates[[22]] + {0.12, -0.12};
textcoordinates[[23]] = textcoordinates[[23]] + {0.24, 0.06};
```

```
texts = Table[Text[a, textcoordinates[[a]]], {a, 1, externalarrays}];
p = Show[
   {Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[0, 0, 1], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points5}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.1, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(b)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/3 ", {0.925, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=-π/3", {0.925, -1.15}]}],
    Graphics[texts],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], 1.25}, {1.25 / Tan[Pi / 3.], -1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], -1.25}, {1.25 / Tan[Pi / 3.], 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Circle[{0, 0}, 0.6,
       {ArcTan[externalcoordinates[[1, 2]] / externalcoordinates[[1, 1]]],
        Pi + ArcTan[externalcoordinates[[2, 2]] / externalcoordinates[[2, 1]]]}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[
       {0.6 * Cos[-0.05 + Pi + ArcTan[externalcoordinates[[2, 2]] / externalcoordinates[[2, 1]]]],
        0.6 * Sin[-0.05 + Pi + ArcTan[externalcoordinates[[2, 2]] / externalcoordinates[[2, 1]]]]},
       {0.6 * Cos[Pi + ArcTan[externalcoordinates[[2, 2]] / externalcoordinates[[2, 1]]]],
        0.6 * Sin[Pi + ArcTan[externalcoordinates[[2, 2]] / externalcoordinates[[2, 1]]]]},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
s2 = Show[p,
   AspectRatio → 1.0,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify GSVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.0000468343}

{-0.000239433}

endph1 = 774;
beginph2 = 775;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.00265793}

{0.999794}

endph2 = 1459;
beginph3 = 1460;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.0003}

{0.999906}

endph3 = 1934;
beginph4 = 1935;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.0009}

{0.99927}

endph4 = 2645;
beginph5 = 2646;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00103}

{1.99935}

(* 3130 yeast genes, 774 in M/G1, 685 in G1, 475 in S, 711 in S/G2, 485 in G2/M. *)
```

```
(* Display GSVD Reconstructed Sorted External Data *)

(* GSVD Reconstruct Sorted External Data *)

subspace = Table[0, {a, 1, arrays}];
Do[subspace[[a]] = 1, {a, 3, 5}];
Do[subspace[[a]] = 1, {a, 14, 16}];
matrix = Dot[partialarraylets, Dot[DiagonalMatrix[subspace], arraycontributions]];

(* Center GSVD Reconstructed Sorted External Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];

(* Create GSVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(c) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 700}, {0, 0}, {0, 1}];
g3 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create GSVD Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.01],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

c_10min	$0.0492085 \text{Cos}[\frac{2\pi x}{3129}] - 0.0485293 \text{Sin}[\frac{2\pi x}{3129}]$
c_30min	$-0.0723436 \text{Cos}[\frac{2\pi x}{3129}] + 0.0667448 \text{Sin}[\frac{2\pi x}{3129}]$
c_50min	$-0.138921 \text{Cos}[\frac{2\pi x}{3129}] - 0.0208362 \text{Sin}[\frac{2\pi x}{3129}]$
c_70min	$-0.0308964 \text{Cos}[\frac{2\pi x}{3129}] - 0.0619406 \text{Sin}[\frac{2\pi x}{3129}]$
c_80min	$-0.0358439 \text{Cos}[\frac{2\pi x}{3129}] - 0.0609898 \text{Sin}[\frac{2\pi x}{3129}]$
c_90min	$0.0109438 \text{Cos}[\frac{2\pi x}{3129}] - 0.0866357 \text{Sin}[\frac{2\pi x}{3129}]$
c_100min	$0.0253978 \text{Cos}[\frac{2\pi x}{3129}] - 0.0546163 \text{Sin}[\frac{2\pi x}{3129}]$
c_110min	$0.0442143 \text{Cos}[\frac{2\pi x}{3129}] - 0.020275 \text{Sin}[\frac{2\pi x}{3129}]$
c_120min	$0.0683384 \text{Cos}[\frac{2\pi x}{3129}] + 0.0564395 \text{Sin}[\frac{2\pi x}{3129}]$
c_130min	$0.0527318 \text{Sin}[\frac{2\pi x}{3129}]$
c_140min	$-0.0400991 \text{Cos}[\frac{2\pi x}{3129}] + 0.109573 \text{Sin}[\frac{2\pi x}{3129}]$
c_150min	$-0.0566555 \text{Cos}[\frac{2\pi x}{3129}] + 0.0301401 \text{Sin}[\frac{2\pi x}{3129}]$
c_160min	$-0.108313 \text{Cos}[\frac{2\pi x}{3129}] + 0.0683256 \text{Sin}[\frac{2\pi x}{3129}]$
c_170min	$-0.0589913 \text{Cos}[\frac{2\pi x}{3129}]$
c_180min	$-0.0169827 \text{Cos}[\frac{2\pi x}{3129}]$
c_190min	$-0.0101811 \text{Cos}[\frac{2\pi x}{3129}] - 0.0750546 \text{Sin}[\frac{2\pi x}{3129}]$
c_200min	$-0.0400349 \text{Sin}[\frac{2\pi x}{3129}]$
c_210min	$0.0336404 \text{Cos}[\frac{2\pi x}{3129}] - 0.0569504 \text{Sin}[\frac{2\pi x}{3129}]$
c_220min	$0.0210032 \text{Cos}[\frac{2\pi x}{3129}]$
c_230min	$0.0551835 \text{Cos}[\frac{2\pi x}{3129}] + 0.0309614 \text{Sin}[\frac{2\pi x}{3129}]$
c_240min	$0.0344608 \text{Cos}[\frac{2\pi x}{3129}] + 0.0467684 \text{Sin}[\frac{2\pi x}{3129}]$
c_250min	$0.0501874 \text{Cos}[\frac{2\pi x}{3129}] + 0.0421348 \text{Sin}[\frac{2\pi x}{3129}]$
c_270min	$0.0406502 \text{Cos}[\frac{2\pi x}{3129}] + 0.0221476 \text{Sin}[\frac{2\pi x}{3129}]$
c_290min	$0.143793 \text{Cos}[\frac{2\pi x}{3129}] + 0.010278 \text{Sin}[\frac{2\pi x}{3129}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(d) Expression Level";
framex = Table[{0.5 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];

Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.5 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.5 * n < 0, 0,
      If[matrix[[n, a]] + 0.5 * n > 12.5, 12.5,
       matrix[[n, a]] + 0.5 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.5 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 12.5}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 700}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 3,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g4 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];
```

```
(* Create GSVD Basis External Array Correlations Bar Chart Display *)

partialarraylets = Transpose[partialarraylets];
partialarraylets = Drop[partialarraylets, {17, 18}];
partialarraylets = Drop[partialarraylets, {6, 13}];
partialarraylets = Drop[partialarraylets, {1, 2}];
partialarraylets = Transpose[partialarraylets];

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];

u = Transpose[SingularValues[partialarraylets][[1]]];
gsvdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[gsvdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.318979

limit = 0.35;
```

```
gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(b) GSVD Basis Correlation";
labely = " ";
g = BarChart[
   Table[gsvdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 5.3}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 4}, {0, 0}, {0, 1}];
q4 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* SVD Sort External Arrays *)

genes = genes2;
genenames = TakeColumns[genenames2, 1];
arrays = arrays2;
arraynames = arraynames2;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{3325}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialeigenarrays = AppendRows[counter, svdphases, genenames, eigenarrays];
partialeigenarrays = Sort[partialeigenarrays, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialeigenarrays, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialeigenarrays, {3, 3}],
   {1, partialgenes}];
partialeigenarrays = TakeRows[
   TakeColumns[partialeigenarrays, {4, arrays + 3}],
   {1, partialgenes}];
partialeigenarrays = TakeColumns[partialeigenarrays, {1, 11}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Center Gene Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];

(* Center Array Data *)

partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
(* Create SVD Basis External Array Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialeigenarrays][[1]]];
svdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 11}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[svdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.338071

limit = 0.35;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{a + 1, externalarrays - a}, {a, 0, externalarrays - 1}];
labelx = "(a) SVD Basis Correlation";
labely = "Arrays";
g = BarChart[
   Table[svdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - limit / 5., c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 5.3}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 4}, {0, 0}, {0, 1}];
q3 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Project Data Onto SVD Basis *)

externalarraycorrelations = Dot[PseudoInverse[partialeigenarrays], partialexternalmatrix];
partialexternalmatrix = Dot[partialeigenarrays, externalarraycorrelations];

(* Calculate Contributions of Eigenarrays to External Arrays *)

partialexternalmatrix = Transpose[partialexternalmatrix];
externalcoordinates = Table[
   {externalarraycorrelations[[1, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]],
    externalarraycorrelations[[2, a]] /
     Sqrt[Dot[partialexternalmatrix[[a]], partialexternalmatrix[[a]]]]},
   {a, 1, externalarrays}];
partialexternalmatrix = Transpose[partialexternalmatrix];

(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

points1 = {Point[externalcoordinates[[10]]], Point[externalcoordinates[[21]]],
   Point[externalcoordinates[[22]]], Point[externalcoordinates[[23]]],
   Point[externalcoordinates[[24]]]};
points2 = {Point[externalcoordinates[[2]]], Point[externalcoordinates[[3]]],
   Point[externalcoordinates[[11]]], Point[externalcoordinates[[12]]],
   Point[externalcoordinates[[13]]]};
points3 = {Point[externalcoordinates[[4]]], Point[externalcoordinates[[14]]],
   Point[externalcoordinates[[15]]], Point[externalcoordinates[[16]]]};
points4 = {Point[externalcoordinates[[5]]]};
points5 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[6]]],
   Point[externalcoordinates[[7]]], Point[externalcoordinates[[8]]],
   Point[externalcoordinates[[9]]], Point[externalcoordinates[[17]]],
   Point[externalcoordinates[[18]]], Point[externalcoordinates[[19]]],
   Point[externalcoordinates[[20]]]};
textcoordinates = externalcoordinates;
Do[
  textcoordinates[[a, 1]] = If[
    textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.085,
    textcoordinates[[a, 1]] + 0.095],
  {a, 1, 9}];
Do[
  textcoordinates[[a, 1]] =
   If[textcoordinates[[a, 1]] > 0,
    textcoordinates[[a, 1]] - 0.11,
    textcoordinates[[a, 1]] + 0.12],
  {a, 10, externalarrays}];
textcoordinates[[2]] = textcoordinates[[2]] - {0.1, 0.12};
textcoordinates[[3]] = textcoordinates[[3]] - {0.2, 0};
textcoordinates[[4]] = textcoordinates[[4]] - {0.2, 0};
textcoordinates[[5]] = textcoordinates[[5]] - {0.2, 0};
textcoordinates[[6]] = textcoordinates[[6]] + {0.22, 0.04};
textcoordinates[[7]] = textcoordinates[[7]] + {0.12, 0.12};
textcoordinates[[8]] = textcoordinates[[8]] + {0.12, -0.12};
textcoordinates[[12]] = textcoordinates[[12]] + {0, 0.04};
textcoordinates[[14]] = textcoordinates[[14]] - {0.25, 0};
textcoordinates[[16]] = textcoordinates[[16]] + {0.14, 0.1};
textcoordinates[[17]] = textcoordinates[[17]] - {0.23, 0};
textcoordinates[[18]] = textcoordinates[[18]] + {0.23, 0};
textcoordinates[[19]] = textcoordinates[[19]] + {0.08, -0.1};
textcoordinates[[21]] = textcoordinates[[21]] + {0.23, 0};
textcoordinates[[22]] = textcoordinates[[22]] + {0.1, 0.12};
textcoordinates[[23]] = textcoordinates[[23]] - {0.23, 0};
textcoordinates[[24]] = textcoordinates[[24]] + {0.23, 0};
```

```
texts = Table[Text[a, textcoordinates[[a]]], {a, 1, externalarrays}];
p = Show[
   {Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[0, 0, 1], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points5}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.2, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(a)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/2", {0.2, 1.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[texts],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5, {0. * Pi, 2 * Pi}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{0, -1.25}, {0, 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Circle[{0, 0}, 0.6,
       {ArcTan[externalcoordinates[[1, 2]] / externalcoordinates[[1, 1]]],
        ArcTan[externalcoordinates[[19, 2]] / externalcoordinates[[19, 1]]]}]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[
       {0.6 * Cos[-0.05 + ArcTan[externalcoordinates[[19, 2]] / externalcoordinates[[19, 1]]]],
        0.6 * Sin[-0.05 + ArcTan[externalcoordinates[[19, 2]] / externalcoordinates[[19, 1]]]]},
       {0.6 * Cos[ArcTan[externalcoordinates[[19, 2]] / externalcoordinates[[19, 1]]]],
        0.6 * Sin[ArcTan[externalcoordinates[[19, 2]] / externalcoordinates[[19, 1]]]]},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {{{0, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
s1 = Show[p,
   AspectRatio -> 1.,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify SVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.000221307}

{-0.000407097}

endph1 = 709;
beginph2 = 710;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.000187677}

{0.999788}

endph2 = 1366;
beginph3 = 1367;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.00101}

{0.999626}

endph3 = 1865;
beginph4 = 1866;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.00001}

{0.999004}

endph4 = 2686;
beginph5 = 2687;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00032}

{1.99995}

(* 3325 yeast genes, 709 in M/G1, 657 in G1, 499 in S, 821 in S/G2, 639 in G2/M. *)
```

```
(* Display SVD Reconstructed and Sorted External Data *)

(* SVD Reconstruct Sorted External Data *)

matrix = partialexternalmatrix;

(* Create SVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[2, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(a) Arrays";
labely = ColumnForm[{" ", "Genes", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 700}, {0, 0}, {0, 1}];
g1 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];
```

```
(* Create Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.02],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[2, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

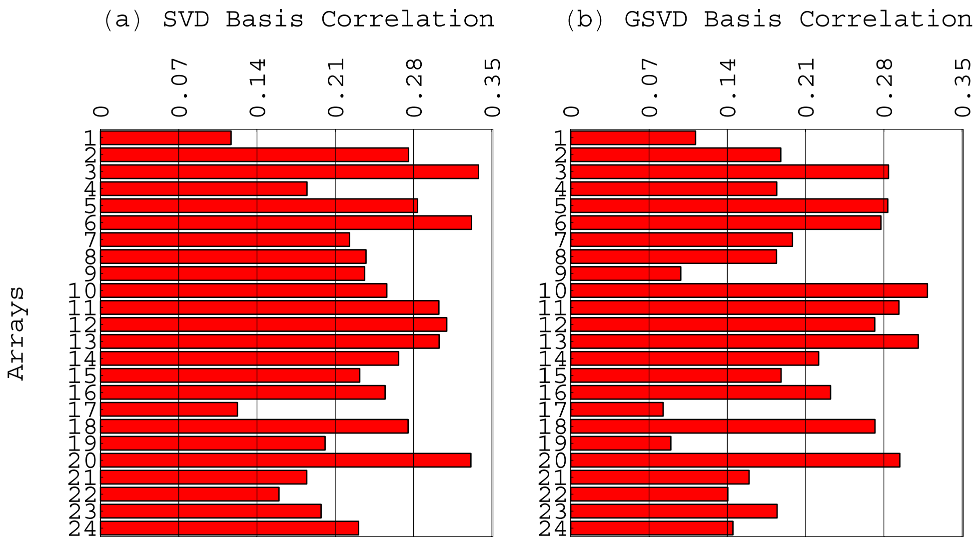
c_10min	$0.0228943 - 0.0742822 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_30min	$0.024142 - 0.0488941 \mathrm{Cos}[\frac{\pi x}{1662}]$
c_50min	$-0.133288 \mathrm{Cos}[\frac{\pi x}{1662}]$
c_70min	$-0.0294803 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.0800656 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_80min	$-0.0382924 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.0476319 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_90min	$-0.0254685 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.122499 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_100min	$0.0489285 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.0591578 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_110min	$0.0234884 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.0685371 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_120min	$0.07527 \mathrm{Cos}[\frac{\pi x}{1662}]$
c_130min	$-0.0201401 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.0231013 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_140min	$-0.024612 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.122708 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_150min	$-0.070311 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.0280971 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_160min	$-0.0623872 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.121001 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_170min	$-0.0566786 \mathrm{Cos}[\frac{\pi x}{1662}]$
c_180min	$0.0206253 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_190min	$-0.0894523 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_200min	$-0.0311128 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_210min	$0.0306943 \mathrm{Cos}[\frac{\pi x}{1662}] - 0.0699062 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_220min	$0.0293425 \mathrm{Cos}[\frac{\pi x}{1662}]$
c_230min	$0.149017 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.0280899 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_240min	$0.0321831 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.0433264 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_250min	$0.0308584 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.0505389 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_270min	$0.0421422 \mathrm{Sin}[\frac{\pi x}{1662}]$
c_290min	$-0.0399727 + 0.101134 \mathrm{Cos}[\frac{\pi x}{1662}] + 0.132049 \mathrm{Sin}[\frac{\pi x}{1662}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(b) Expression Level";
framex = Table[{0.5 * a, externalarraynames[[2, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.5 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.5 * n < 0, 0,
      If[matrix[[n, a]] + 0.5 * n > 12.5, 12.5,
       matrix[[n, a]] + 0.5 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.5 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 12.5}, {95, -partialgenes + 1 - 95}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1400}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 700}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 3,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];

g2 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];
```

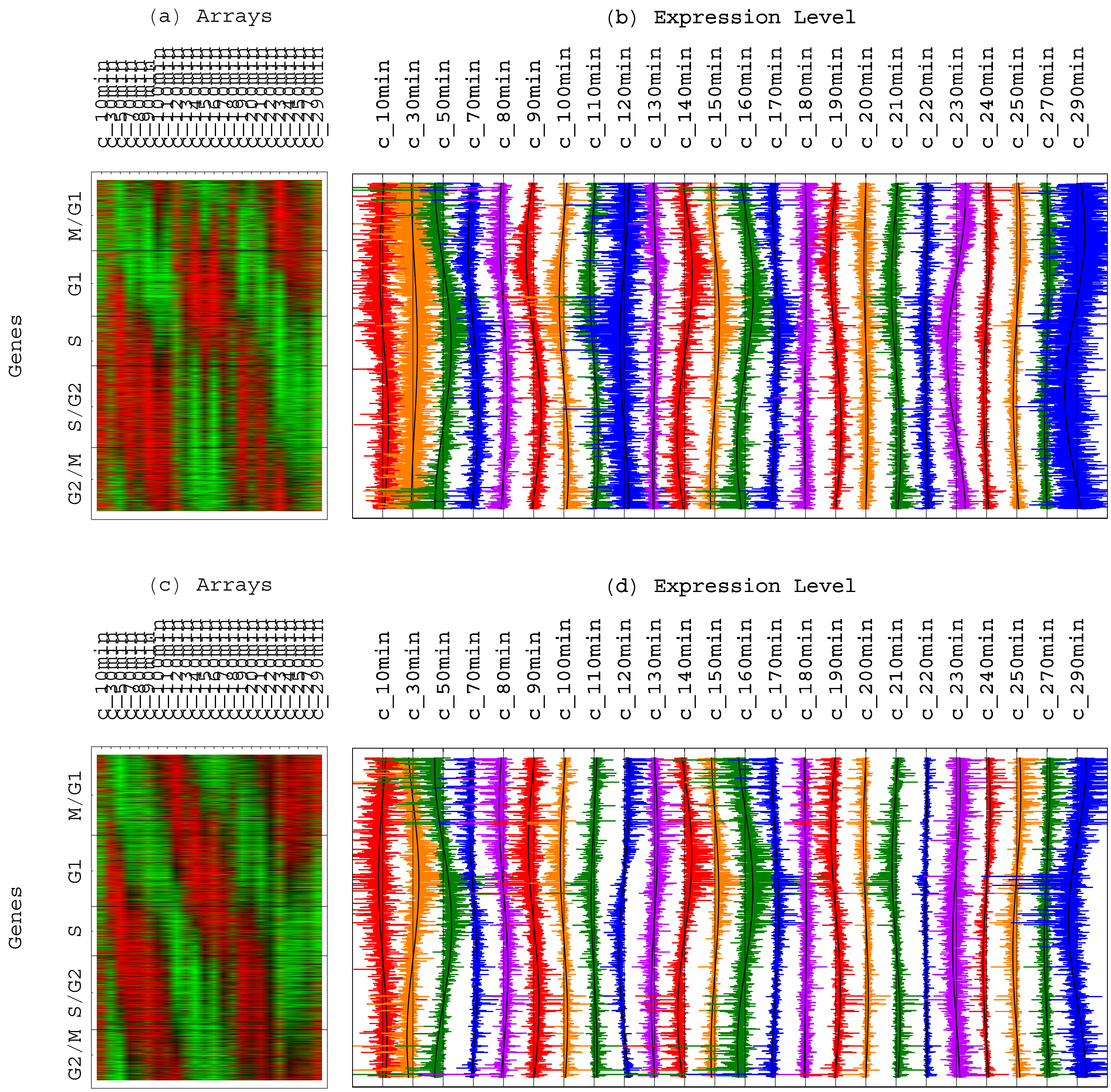
(* Display Correlations of External Arrays with SVD and GSVD Bases *)

```
Show[GraphicsArray[{q3, q4}],
  GraphicsSpacing -> -0.12];
```



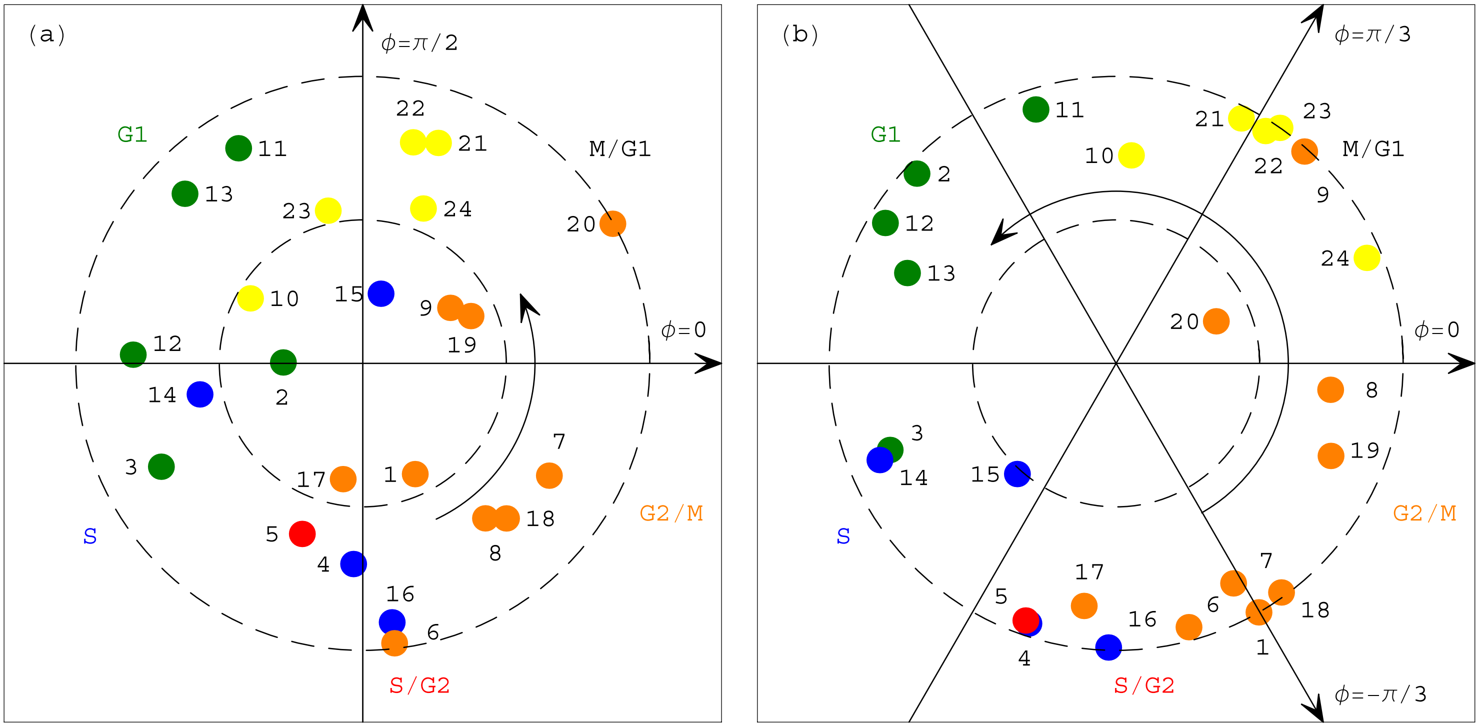
(* Display SVD and GSVD Sorted External Data *)

```
q1 = Show[{
     Graphics[{Rectangle[{0, 0}, {60, 75}, g1]}],
     Graphics[{Rectangle[{60, 0}, {192, 75}, g2]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
q2 = Show[{
     Graphics[{Rectangle[{0, 0}, {60, 75}, g3]}],
     Graphics[{Rectangle[{60, 0}, {192, 75}, g4]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
Show[GraphicsArray[{{q1}, {q2}}],
  GraphicsSpacing → -0.24];
```


(a) Arrays
(b) Expression Level
Genes
M/G1
G1
S
S/G2
G2/M
c_10min c_30min c_50min c_70min c_80min c_90min c_100min c_110min c_120min c_130min c_140min c_150min c_160min c_170min c_180min c_190min c_200min c_210min c_220min c_230min c_240min c_250min c_270min c_290min
(c) Arrays
(d) Expression Level
Genes
M/G1
G1
S
S/G2
G2/M
c_10min c_30min c_50min c_70min c_80min c_90min c_100min c_110min c_120min c_130min c_140min c_150min c_160min c_170min c_180min c_190min c_200min c_210min c_220min c_230min c_240min c_250min c_270min c_290min

```
(* Display Mapping of Cdc15 Time Course Data *)
```

```
Show[GraphicsArray[{s1, s2}],
    GraphicsSpacing -> 0];
```

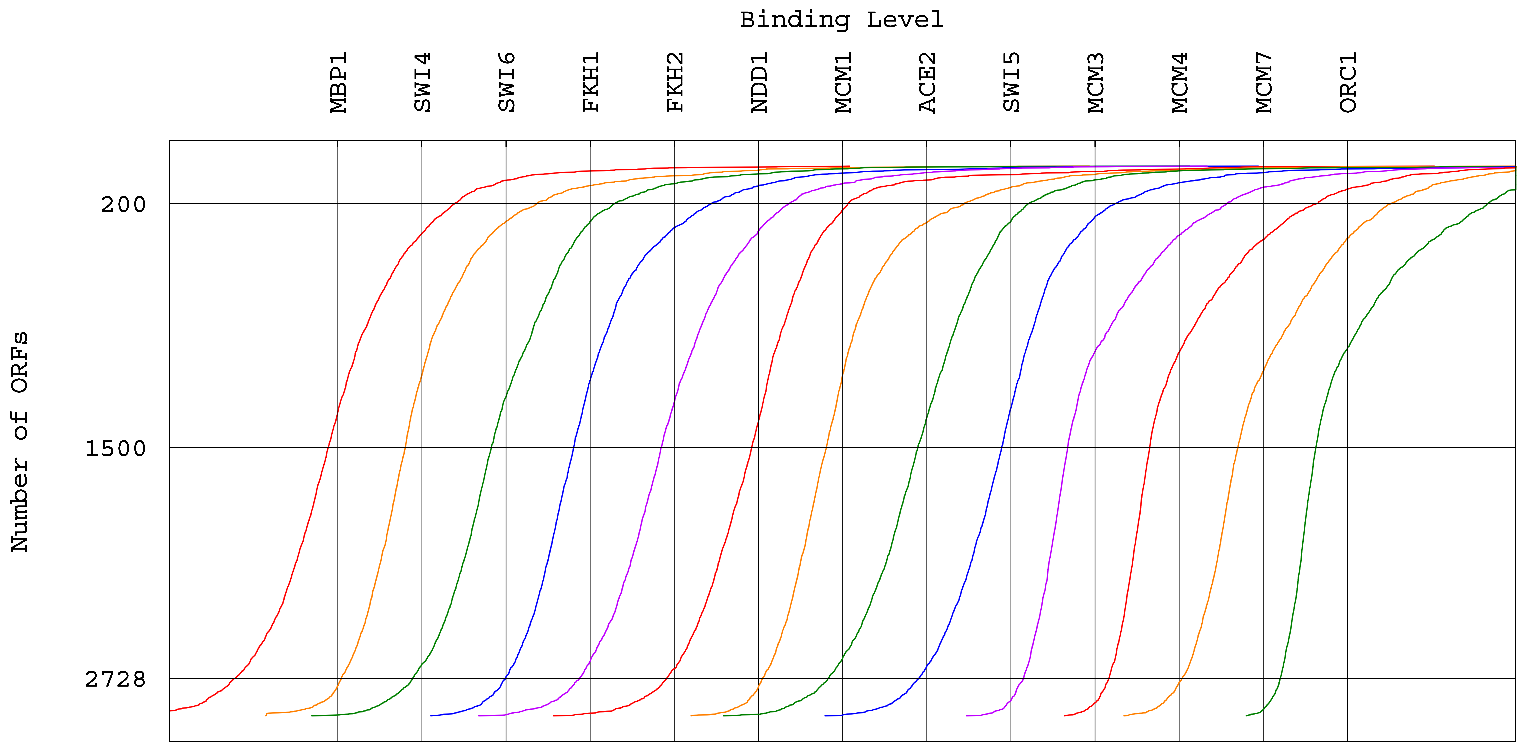


```
(* Map Yeast Protein Binding Data *)

(* Read Data *)

stream = "Desktop/Networks/Data/Protein_Bind.txt.nb";
externalmatrix = ReadList[stream, Word, RecordLists -> True, NullWords -> True];
{externalgenes, externalarrays} = Dimensions[externalmatrix] - {1, 3}
Clear[stream];

{2928, 13}

externalgenenames = TakeRows[
   TakeColumns[externalmatrix, {1, 3}],
   {2, externalgenes + 1}];
externalarraynames = TakeColumns[
   TakeRows[externalmatrix, {1, 1}],
   {4, externalarrays + 3}];
externalmatrix = TakeColumns[
   TakeRows[externalmatrix, {2, externalgenes + 1}],
   {4, externalarrays + 3}];
externalmatrix = ToExpression[externalmatrix];

sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[externalarraynames[[1, a]]
      ]]],
    {a, 1, externalarrays}]];
size = Sort[sizes, OrderedQ[{#2, #1}] &][[1]];
Do[
  Do[externalarraynames[[1, a]] = StringJoin[ToString[externalarraynames[[1, a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, externalarrays}];

(* Convert to Ratios *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
externalmatrix =
  externalmatrix / N[Outer[Times, Dot[externalmatrix, average], average]];

(* Display Sorted External Arrays *)

arraypatterns = Transpose[externalmatrix];

(* Center External Arrays *)

average = Table[1, {a, 1, externalgenes}];
average = N[average / Sqrt[Dot[average, average]]];
arraypatterns = arraypatterns - N[Outer[Times, Dot[arraypatterns, average], average]];

(* Normalize External Arrays *)

Do[
 arraypatterns[[a]] =
  arraypatterns[[a]] / Sqrt[Dot[arraypatterns[[a]], arraypatterns[[a]]]],
 {a, 1, externalarrays}]
```

```
(* Sort External Arrays *)

Do[
 arraypatterns[[a]] = Sort[arraypatterns[[a]], OrderedQ[{{#2}, {#1}}] &],
 {a, 1, externalarrays}]

(* Create Sorted External Arrays Graph Display *)

p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "Binding Level";
labely = ColumnForm[
   {" ", "Number of ORFs", " ", " ", " ", " ", " ", " ", " ", " "},
   Center];
framex = Table[{0.02 * a, externalarraynames[[1, a]]},
   {a, 1, externalarrays}];
framey = {{-200, "200"}, {-1500, "1500"}, {-externalgenes + 200, "2728"}};

Do[{
   coordinates = Table[
     If[arraypatterns[[n, a]] + 0.02 * n < -0.02, -0.02,
      If[arraypatterns[[n, a]] + 0.02 * n > 0.3, 0.3,
       arraypatterns[[n, a]] + 0.02 * n]],
     {a, 1, externalgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, externalgenes}],
   line = Line[coordinates],
   g = Show[
     Graphics[{color[[Mod[n, 5] + 1]], line}],
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {{{0.02 * n, RGBColor[0, 0, 0]}}, {{-200, RGBColor[0, 0, 0]},
        {-1500, RGBColor[0, 0, 0]}, {-externalgenes + 200, RGBColor[0, 0, 0]}}},
     PlotRange -> {{-0.02, 0.3}, {135, -externalgenes + 1 - 135}},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 500}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 250}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → 1 / 1.2 / GoldenRatio,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
```

(* Display Sorted External Arrays *)

```
Show[Table[p[[a]], {a, 1, externalarrays}],
  DisplayFunction → $DisplayFunction];
```



```
(* Estimate Significance of Association of External Data with the Cell Cycle *)

(* Use Microarray Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {2}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table1 = AppendRows[
   Table[{externalarraynames[[1, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Use Traditional Classification of Yeast Genes *)

most = 200;
genenames = TakeColumns[externalgenenames, {3}];
stages = {"M/G1", "G1", "S", "S/G2", "G2/M", "None"};
numbers = Flatten[Table[{Count[Flatten[genenames], stages[[a]]]}, {a, 1, Dimensions[stages][[1]]}]];
genelet = Table[{a}, {a, 1, externalarrays}];
probability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
parallelprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiannotation = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];
antiprobability = Table[{0}, {a, 1, Dimensions[genelet][[1]]}];

Do[{
  arraylet = TakeColumns[Sort[
     AppendRows[TakeColumns[externalmatrix, genelet[[c]]], genenames],
     OrderedQ[{{#2}, {#1}}] &], {2}],
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {1, most}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  parallelannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  parallelprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]},
  table = Table[{
     stages[[a]],
     numbers[[a]],
     Count[Flatten[TakeRows[arraylet, {externalgenes - most + 1, externalgenes}]], stages[[a]]]},
    {a, 1, Dimensions[stages][[1]]}],
  probability = Table[{
     Sum[N[Binomial[table[[a, 2]], b] * Binomial[externalgenes - table[[a, 2]], most - b] /
         Binomial[externalgenes, most]], {b, table[[a, 3]], most}],
     stages[[a]]},
    {a, 1, Dimensions[stages][[1]]}],
  antiannotation[[c]] = {Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 2]]},
  antiprobability[[c]] = {ScientificForm[Sort[probability, OrderedQ[{{#1}, {#2}}] &][[1, 1]], 2]}},
 {c, 1, Dimensions[genelet][[1]]}]

table2 = AppendRows[
   Table[{externalarraynames[[1, a]]}, {a, 1, externalarrays}],
   parallelannotation,
   parallelprobability,
   antiannotation,
   antiprobability];
```

```
(* Display Significance of Association of External Arrays with the Cell Cycle *)

headerx = {{
    ColumnForm[{" ", " ", " "}, Left],
    ColumnForm[{" ", " ", "Classification"}, Left],
    ColumnForm[{" ", "External", "Array"}, Left],
    ColumnForm[{"Most Likely", "Parallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Parallel", "Association"}, Left],
    ColumnForm[{"Most Likely", "Antiparallel", "Association"}, Left],
    ColumnForm[{"P-Value of", "Antiparallel", "Association"}, Left]},
   {" ", " ", " ", " ", " ", " ", " "}};
spacerx = {{" ", " ", " ", " ", " "}};
headery = Table[" ", {a, 1, 2 * externalarrays + 1}, {b, 1, 2}];
headery[[1]] = {"(a)", "Microarray"};
headery[[externalarrays + 2]] = {"(b)", "Traditional"};
association =
  AppendColumns[headerx,
   AppendRows[headery,
    AppendColumns[table1, spacerx, table2]]];
TableForm[association, TableSpacing → {1, 1}]
```

	Classification	External Array	Most Likely Parallel Association	P-Value of Parallel Association	Most Likely Antiparallel Association	P-Value of Antiparallel Association
(a)	Microarray	MBP1	G1	1.6×10^{-14}	None	$4. \times 10^{-3}$
		SWI4	G1	1.5×10^{-17}	None	1.2×10^{-1}
		SWI6	G1	4.7×10^{-32}	G2/M	7.3×10^{-2}
		FKH1	S/G2	7.2×10^{-4}	None	3.5×10^{-1}
		FKH2	G2/M	3.9×10^{-11}	None	8.3×10^{-2}
		NDD1	G2/M	$2. \times 10^{-19}$	G1	9.5×10^{-2}
		MCM1	G2/M	1.2×10^{-12}	G1	$4. \times 10^{-3}$
		ACE2	M/G1	1.1×10^{-3}	G2/M	8.4×10^{-3}
		SWI5	M/G1	1.3×10^{-15}	G1	4.5×10^{-5}
		MCM3	None	4.5×10^{-4}	G1	7.9×10^{-10}
		MCM4	None	1.3×10^{-2}	G1	1.2×10^{-8}
		MCM7	None	1.3×10^{-2}	G1	7.9×10^{-10}
		ORC1	None	$4. \times 10^{-3}$	G1	4.3×10^{-13}
(b)	Traditional	MBP1	G1	2.7×10^{-10}	None	9.3×10^{-2}
		SWI4	G1	2.7×10^{-7}	None	9.3×10^{-2}
		SWI6	G1	4.8×10^{-19}	G2/M	4.4×10^{-2}
		FKH1	S/G2	$4. \times 10^{-2}$	S	3.9×10^{-1}
		FKH2	G2/M	3.7×10^{-6}	None	2.7×10^{-2}
		NDD1	G2/M	$5. \times 10^{-9}$	M/G1	3.3×10^{-1}
		MCM1	G2/M	1.6×10^{-7}	G1	3.3×10^{-2}
		ACE2	M/G1	1.1×10^{-1}	S	7.8×10^{-2}
		SWI5	M/G1	6.2×10^{-4}	G2/M	6.2×10^{-5}
		MCM3	None	2.7×10^{-2}	G1	$5. \times 10^{-4}$
		MCM4	None	$4. \times 10^{-3}$	G1	2.4×10^{-3}
		MCM7	None	2.7×10^{-2}	G1	$5. \times 10^{-4}$
		ORC1	None	2.2×10^{-1}	G1	$5. \times 10^{-4}$

```
(* GSVD Sort External Arrays *)

genes = genes1;
genenames = TakeColumns[genenames1, 1];
arrays = arrays1;
arraynames = arraynames1;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{2139}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialarraylets = AppendRows[counter, gsvdphases, genenames, arraylets1];
partialarraylets = Sort[partialarraylets, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialarraylets, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialarraylets, {3, 3}],
   {1, partialgenes}];
partialarraylets = TakeRows[
   TakeColumns[partialarraylets, {4, arrays + 3}],
   {1, partialgenes}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];
```

```
(* Project Data Onto GSVD Basis and Calculate Contributions of Arraylets to External Arrays *)

partialarraylets = Transpose[partialarraylets];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialarraylets =
  partialarraylets - N[Outer[Times, Dot[partialarraylets, average], average]];
partialarraylets = Transpose[partialarraylets];

arraycontributions = Dot[PseudoInverse[partialarraylets], partialexternalmatrix];
average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
arraycontributions =
  arraycontributions - N[Outer[Times, Dot[arraycontributions, average], average]];

(* Project GSVD Reconstructed Replication Initiation Protein Data Onto
   GSVD Reconstructed Transcription Factor Data *)

partialexternalmatrix1 = TakeColumns[partialexternalmatrix, {1, 9}];
partialexternalmatrix2 = TakeColumns[partialexternalmatrix, {10, 13}];
arraycontributions12 = Transpose[Dot[
    Dot[PseudoInverse[partialexternalmatrix1], partialarraylets],
    Dot[PseudoInverse[partialarraylets], partialexternalmatrix2]]];
```

```
(* Project Arrays from 6 D Arraylets Subspace Onto 2 D Subspace  *)

externalcoordinates = Table[{
    (-Sqrt[3] * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        2 * Sqrt[3] * (arraycontributions[[4, a]] - arraycontributions[[16, a]]) +
        (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
           (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
          (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]],
    (3 * (arraycontributions[[3, a]] - arraycontributions[[15, a]]) +
        Sqrt[3] * (3 * arraycontributions[[5, a]] + Sqrt[3] * arraycontributions[[14, a]])) / 6 /
     Sqrt[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) ^ 2 / 3 +
       (arraycontributions[[4, a]] - arraycontributions[[16, a]]) ^ 2 / 3 +
       (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3]) ^ 2 +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
           (arraycontributions[[4, a]] - arraycontributions[[16, a]])] / Sqrt[3] +
       Abs[(arraycontributions[[3, a]] - arraycontributions[[15, a]]) / Sqrt[3] *
          (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])] +
       Abs[(arraycontributions[[4, a]] - arraycontributions[[16, a]]) / Sqrt[3] *
          (arraycontributions[[5, a]] + arraycontributions[[14, a]] / Sqrt[3])]]},
   {a, 1, externalarrays}];

(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

points1 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[2]]],
   Point[externalcoordinates[[3]]]};
points2 = {Point[externalcoordinates[[4]]]};
points3 = {Point[externalcoordinates[[5]]], Point[externalcoordinates[[6]]],
   Point[externalcoordinates[[7]]]};
points4 = {Point[externalcoordinates[[8]]], Point[externalcoordinates[[9]]]};
points5 = {Point[externalcoordinates[[10]]], Point[externalcoordinates[[11]]],
   Point[externalcoordinates[[12]]], Point[externalcoordinates[[13]]]};
```

```
p = Show[
   {Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[0.75, 0, 1], PointSize[0.035], points5}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.1, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(b)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/3 ", {0.925, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=-π/3", {0.925, -1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MBP1", externalcoordinates[[1]] + {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI4", externalcoordinates[[2]] + {0.13, -0.13}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI6", externalcoordinates[[3]] - {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["FKH1", externalcoordinates[[4]] + {0.2, 0.02}]}],
    Graphics[{RGBColor[0, 0, 0], Text["FKH2", externalcoordinates[[5]] - {0.08, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["NDD1", externalcoordinates[[6]] + {0, 0.13}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM1", externalcoordinates[[7]] - {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ACE2", externalcoordinates[[8]] + {0, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI5", externalcoordinates[[9]] - {0, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM3", externalcoordinates[[10]] + {0.275, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM4", externalcoordinates[[11]] + {0.06, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM7", externalcoordinates[[12]] + {0.16, 0.09}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ORC1", externalcoordinates[[13]] + {0.03, 0.13}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], 1.25}, {1.25 / Tan[Pi / 3.], -1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25 / Tan[Pi / 3.], -1.25}, {1.25 / Tan[Pi / 3.], 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
m2 = Show[p,
   AspectRatio → 1.0,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify GSVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.0000468343}

{-0.000239433}

endph1 = 528;
beginph2 = 529;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.00415166}

{0.999706}

endph2 = 1038;
beginph3 = 1039;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.00032}

{0.99936}

endph3 = 1366;
beginph4 = 1367;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.0009}

{0.99987}

endph4 = 1849;
beginph5 = 1850;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00019}

{1.99914}

(* 2139 yeast genes, 528 in M/G1, 510 in G1, 328 in S, 483 in S/G2, 290 in G2/M. *)
```

```
(* Display GSVD Reconstructed Sorted External Data *)

(* GSVD Reconstruct Sorted External Data *)

partialarraylets = Transpose[partialarraylets];
partialarraylets = Drop[partialarraylets, {17, 18}];
partialarraylets = Drop[partialarraylets, {6, 13}];
partialarraylets = Drop[partialarraylets, {1, 2}];
partialarraylets = Transpose[partialarraylets];

arraycontributions = Drop[arraycontributions, {17, 18}];
arraycontributions = Drop[arraycontributions, {6, 13}];
arraycontributions = Drop[arraycontributions, {1, 2}];

matrix = Dot[partialarraylets, arraycontributions];

(* Center GSVD Reconstructed Sorted External Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
matrix = matrix - N[Outer[Times, Dot[matrix, average], average]];
matrix = Transpose[matrix];

(* Create GSVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(c) Samples";
labely = ColumnForm[{"ORFs", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 340}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 150}, {0, 0}, {0, 1}];
g3 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction -> Identity];

(* Create GSVD Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.005],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[1, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

MBP1	$-0.0388423\,\mathrm{Cos}[\frac{\pi x}{1069}] + 0.0282288\,\mathrm{Sin}[\frac{\pi x}{1069}]$
SWI4	$-0.0807485\,\mathrm{Cos}[\frac{\pi x}{1069}] + 0.0405207\,\mathrm{Sin}[\frac{\pi x}{1069}]$
SWI6	$-0.0771488\,\mathrm{Cos}[\frac{\pi x}{1069}] + 0.0407178\,\mathrm{Sin}[\frac{\pi x}{1069}]$
FKH1	$-0.00949282\,\mathrm{Sin}[\frac{\pi x}{1069}]$
FKH2	$-0.0125451\,\mathrm{Sin}[\frac{\pi x}{1069}]$
NDD1	$0.0257796\,\mathrm{Cos}[\frac{\pi x}{1069}] - 0.0142828\,\mathrm{Sin}[\frac{\pi x}{1069}]$
MCM1	$0.035563\,\mathrm{Cos}[\frac{\pi x}{1069}]$
ACE2	$0.0164531\,\mathrm{Cos}[\frac{\pi x}{1069}] + 0.0307157\,\mathrm{Sin}[\frac{\pi x}{1069}]$
SWI5	$0.0516627\,\mathrm{Sin}[\frac{\pi x}{1069}]$
MCM3	$0.0406182\,\mathrm{Cos}[\frac{\pi x}{1069}] - 0.0130613\,\mathrm{Sin}[\frac{\pi x}{1069}]$
MCM4	$0.0280758\,\mathrm{Cos}[\frac{\pi x}{1069}] - 0.0389401\,\mathrm{Sin}[\frac{\pi x}{1069}]$
MCM7	$0.0231392\,\mathrm{Cos}[\frac{\pi x}{1069}] - 0.0247272\,\mathrm{Sin}[\frac{\pi x}{1069}]$
ORC1	$0.0242502\,\mathrm{Cos}[\frac{\pi x}{1069}] - 0.0834609\,\mathrm{Sin}[\frac{\pi x}{1069}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(d) Binding Level";
framex = Table[{0.2 * a, externalarraynames[[1, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.2 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.2 * n < 0, 0,
      If[matrix[[n, a]] + 0.2 * n > 2.8, 2.8,
       matrix[[n, a]] + 0.2 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.2 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 2.8}, {65, -partialgenes + 1 - 65}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 330}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 142.5}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 2.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
matrix = Transpose[matrix];

g4 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];
```

```
(* Display Projection Onto GSVD Cell Cycle Subspace *)

(* Center GSVD Sorted External Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];

(* Create GSVD Sorted External Data 2 D Red & Green Raster Display *)

contrast = 1.5 * 1.5;
displaying = Table[
   If[contrast * partialexternalmatrix[[i, j]] > 0,
    If[contrast * partialexternalmatrix[[i, j]] < 1,
     {contrast * partialexternalmatrix[[i, j]], 0}, {1, 0}],
    If[contrast * partialexternalmatrix[[i, j]] > -1,
     {0, -contrast * partialexternalmatrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
labelx = "Samples";
labely = ColumnForm[{" ", "                ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p5 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create GSVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1,
     {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1,
     {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
labelx = "Samples";
labely = ColumnForm[{" ", "                 ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.8, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p6 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create GSVD Sorted Arraylets 2 D Red & Green Raster Display *)

contrast = 10 * 15;
displaying = Table[
   If[contrast * partialarraylets[[i, j]] > 0,
    If[contrast * partialarraylets[[i, j]] < 1,
     {contrast * partialarraylets[[i, j]], 0}, {1, 0}],
    If[contrast * partialarraylets[[i, j]] > -1,
     {0, -contrast * partialarraylets[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, 6}];
framex = {{0.5, "3    "}, {1.5, "4    "}, {2.5, "5    "}, {3.5, "14   "}, {4.5, "15   "}, {5.5, "16   "}};
labelx = "Arraylets";
labely = ColumnForm[{" ", "                ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, 6}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p7 = Show[g,
   AspectRatio -> GoldenRatio * 1.2 * 2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create Arraylets Contributions to External Arrays 2 D Red & Green Raster Display *)

contrast = 0.5;
displaying = Table[
   If[contrast * arraycontributions[[i, j]] > 0,
    If[contrast * arraycontributions[[i, j]] < 1,
     {contrast * arraycontributions[[i, j]], 0}, {1, 0}],
    If[contrast * arraycontributions[[i, j]] > -1,
     {0, -contrast * arraycontributions[[i, j]]}, {0, 1}]],
   {i, 1, 6}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = {{0.5, "16"}, {1.5, "15"}, {2.5, "14"}, {3.5, "5"}, {4.5, "4"}, {5.5, "3"}};
labely = ColumnForm[{" ", "Arraylets", " "}, Center];
labelx = ColumnForm[{"Samples", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 6, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 3, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.9}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1.6}, {0, 0}, {0, 1}];
p8 = Show[g,
   AspectRatio → 1.05 / 2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Display GSVD Correlations *)

(* Create GSVD Basis External Array Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialarraylets][[1]]];
gsvdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 6}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[gsvdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.267766

limit = 0.3;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{13 - a + 1, externalarraynames[[1, a]]}, {a, 1, 13}];
labelx = "(b) GSVD Basis Correlation";
labely = " ";
g = BarChart[
   Table[gsvdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 0.077, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 3}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1.5}, {0, 0}, {0, 1}];
q4 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create GSVD Transcription Factor Basis Replication Initiation Protein Sample
   Correlations Bar Chart Display *)

partialexternalmatrix1 = TakeColumns[partialexternalmatrix, {1, 9}];
partialexternalmatrix2 = TakeColumns[partialexternalmatrix, {10, 13}];

u = Transpose[SingularValues[partialexternalmatrix1][[1]]];
gsvdcorrelation12 =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix2][[a]], u]^2)[[b]], {b, 1, 9}] /
     Dot[Transpose[partialexternalmatrix2][[a]], Transpose[partialexternalmatrix2][[a]]]],
   {a, 1, 4}];
Sort[gsvdcorrelation12, OrderedQ[{{#2}, {#1}}] &][[1]]

0.708433

limit = 0.75;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{13 - a + 1, externalarraynames[[1, a]]}, {a, 10, 13}];
labelx = "(b) GSVD Correlation";
labely = " ";
g = BarChart[
   Table[gsvdcorrelation12[[4 - a]], {a, 0, 4 - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, 4 + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 0.2, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.88}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.44}, {0, 0}, {0, 1}];
q6 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Display GSVD Pseudoinverse Correlations *)

(* Create GSVD Correlations Red and Green Raster Display *)

contrast = 0.5;
displaying = Table[
   If[contrast * arraycontributions[[i, j]] > 0,
    If[contrast * arraycontributions[[i, j]] < 1,
     {contrast * arraycontributions[[i, j]], 0}, {1, 0}],
    If[contrast * arraycontributions[[i, j]] > -1,
     {0, -contrast * arraycontributions[[i, j]]}, {0, 1}]],
   {i, 1, 6}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = {{0.5, "16"}, {1.5, "15"}, {2.5, "14"}, {3.5, "5"}, {4.5, "4"}, {5.5, "3"}};
labely = ColumnForm[{"(c) Arraylets"}, Center];
labelx = ColumnForm[{"Samples"}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 6, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 2.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 2}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1}, {0, 0}, {0, 1}];
f3 = Show[g,
   PlotRange -> All,
   AspectRatio → 1 / GoldenRatio,
   DisplayFunction → Identity];
```

```
(* Create GSVD Correlations Graph Display *)

p = Table[0, {a, 1, 6}];
color = {
   RGBColor[0, 0.5, 0],
   RGBColor[1, 0, 0],
   RGBColor[0, 0, 1]
  };
framex = Table[{a - 1, "     "}, {a, 1, externalarrays}];

matrixplot = {
   Sqrt[Dot[
       arraycontributions[[1]],
       arraycontributions[[1]]]] /
     2 * Sin[Pi * (x + 1) / 4],
   -Sqrt[Dot[
        arraycontributions[[2]],
        arraycontributions[[2]]]] /
     2 * Sin[Pi * (x + 1) / 4],
   Sqrt[Dot[
       arraycontributions[[3]],
       arraycontributions[[3]]]] /
     2 * Cos[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions[[4]],
        arraycontributions[[4]]]] /
     2 * Cos[Pi * (x - 1) / 4],
   -Sqrt[Dot[
        arraycontributions[[5]],
        arraycontributions[[5]]]] /
     2 * Sin[Pi * (x + 2) / 4],
   Sqrt[Dot[
       arraycontributions[[6]],
       arraycontributions[[6]]]] /
     2 * Cos[Pi * x / 4]};
```

```
labelx = " ";
labely = ColumnForm[{"(d) Correlation"}, Center];
framey = Table[a, {a, -5, 5, 5}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 3] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions[[n, a]]},
     {a, 1, externalarrays}],
   points = Table[Point[coordinates[[a]]], {a, 1, externalarrays}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 3] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 3] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     GridLines -> {{{8.5, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
     FrameTicks -> {None, framey, framex, None},
     PlotRange -> {-7.5, 5.01},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 2.3, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 2.5}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 1.25}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     PlotRange -> All,
     AspectRatio → 1 / GoldenRatio,
     DisplayFunction → Identity]
  }, {n, 1, 3}];
f4 = Show[Table[p[[a]], {a, 1, 3}],
   DisplayFunction → Identity];
```

```
labelx = " ";
labely = ColumnForm[{"(e) Correlation"}, Center];
framey = Table[a, {a, -2.5, 2.5, 2.5}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 3] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions[[n, a]]},
     {a, 1, externalarrays}],
   points = Table[Point[coordinates[[a]]], {a, 1, externalarrays}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 3] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 3] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     GridLines -> {{{8.5, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
     FrameTicks -> {None, framey, framex, None},
     PlotRange -> {-2.5, 3.5},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 2.3, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 1.2}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 0.6}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     PlotRange -> All,
     AspectRatio → 1 / GoldenRatio,
     DisplayFunction → Identity]
  }, {n, 4, 6}];
f5 = Show[Table[p[[a]], {a, 4, 6}],
   DisplayFunction → Identity];
```

```
(* Display GSVD Pseudoinverse Correlations
   Between Replication Initiation Proteins and Transcription Factors *)

(* Create GSVD Correlations Red and Green Raster Display *)

contrast = 50;
displaying = Table[
   If[contrast * arraycontributions12[[i, j]] > 0,
    If[contrast * arraycontributions12[[i, j]] < 1,
     {contrast * arraycontributions12[[i, j]], 0}, {1, 0}],
    If[contrast * arraycontributions12[[i, j]] > -1,
     {0, -contrast * arraycontributions12[[i, j]]}, {0, 1}]],
   {i, 1, 4}, {j, 1, 9}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, 9}];
framey = Table[{13 - a + 0.5, externalarraynames[[1, a]]}, {a, 10, 13}];
labely = ColumnForm[{"(d) Samples"}, Center];
labelx = ColumnForm[{"Samples"}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 4, 1, -1}, {j, 1, 9}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 1.73, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1.34}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.67}, {0, 0}, {0, 1}];
h4 = Show[g,
   AspectRatio → 1 / GoldenRatio,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create GSVD Correlations Graph Display *)

p = Table[0, {a, 1, 4}];
color = {
   RGBColor[0, 0, 1],
   RGBColor[1, 0, 0]
  };
framex = Table[{a - 1, "     "}, {a, 1, 9}];

matrixplot = {
   -Sqrt[Dot[
        arraycontributions12[[1]],
        arraycontributions12[[1]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[2]],
        arraycontributions12[[2]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[3]],
        arraycontributions12[[3]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[4]],
        arraycontributions12[[4]]]] /
     2 * Sin[Pi * x / 4]};
```

```
labelx = " ";
labely = ColumnForm[{"(e) Correlation"}, Center];
framey = Table[a, {a, -0.025, 0.025, 0.025}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 2] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions12[[n, a]]},
     {a, 1, 9}],
   points = Table[Point[coordinates[[a]]], {a, 1, 9}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 2] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 2] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
     PlotRange -> {-0.035, 0.035},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 1.5, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 0.0233}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 0.0116}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → 1 / GoldenRatio,
     PlotRange → All,
     DisplayFunction → Identity],
  }, {n, 1, 2}];
h5 = Show[Table[p[[a]], {a, 1, 2}]];
```

```
labelx = " ";
labely = ColumnForm[{"(f) Correlation"}, Center];
framey = Table[a, {a, -0.05, 0.05, 0.05}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 2] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions12[[n, a]]},
     {a, 1, 9}],
   points = Table[Point[coordinates[[a]]], {a, 1, 9}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 2] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 2] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
     PlotRange -> {-0.055, 0.055},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 1.5, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 0.0367}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 0.0183}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio -> 1 / GoldenRatio,
     PlotRange → All,
     DisplayFunction → Identity],
  }, {n, 3, 4}];
h6 = Show[Table[p[[a]], {a, 3, 4}]];
```

```
(* SVD Sort External Arrays *)

genes = genes2;
genenames = TakeColumns[genenames2, 1];
arrays = arrays2;
arraynames = arraynames2;

externalgenenames = TakeColumns[externalgenenames, 1];
list = Flatten[Intersection[genenames, externalgenenames]];
{partialgenes} = Dimensions[list]

{2227}

counter = Table[{Position[genenames, list[[a]]][[1, 1]], list[[a]]}, {a, 1, partialgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
counter = Sort[counter, OrderedQ[{#1, #2}] &];
list = Flatten[TakeColumns[counter, {2}]];

counter = Table[Flatten[Position[list, genenames[[a, 1]]]], {a, 1, genes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialeigenarrays = AppendRows[counter, svdphases, genenames, eigenarrays];
partialeigenarrays = Sort[partialeigenarrays, OrderedQ[{#1, #2}] &];
partialphases = TakeRows[
   TakeColumns[partialeigenarrays, {2, 2}],
   {1, partialgenes}];
partialgenenames = TakeRows[
   TakeColumns[partialeigenarrays, {3, 3}],
   {1, partialgenes}];
partialeigenarrays = TakeRows[
   TakeColumns[partialeigenarrays, {4, arrays + 3}],
   {1, partialgenes}];
partialeigenarrays = TakeColumns[partialeigenarrays, {1, 9}];

counter = Table[Flatten[Position[list, externalgenenames[[a, 1]]]], {a, 1, externalgenes}];
counter = ReplaceAll[counter, {} -> {Null}];
partialexternalmatrix = AppendRows[counter, externalgenenames, externalmatrix];
partialexternalmatrix = Sort[partialexternalmatrix, OrderedQ[{#1, #2}] &];
partialexternalgenenames = TakeRows[
   TakeColumns[partialexternalmatrix, {2, 2}],
   {1, partialgenes}];
partialexternalmatrix = TakeRows[
   TakeColumns[partialexternalmatrix, {3, externalarrays + 2}],
   {1, partialgenes}];

(* Center Gene Data *)

average = Table[1, {a, 1, externalarrays}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];

(* Center Array Data *)

partialexternalmatrix = Transpose[partialexternalmatrix];
average = Table[1, {a, 1, partialgenes}];
average = N[average / Sqrt[Dot[average, average]]];
partialexternalmatrix =
  partialexternalmatrix - N[Outer[Times, Dot[partialexternalmatrix, average], average]];
partialexternalmatrix = Transpose[partialexternalmatrix];
```

```
(* Create SVD Basis External Array Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialeigenarrays][[1]]];
svdcorrelation =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix][[a]], u]^2)[[b]], {b, 1, 9}] /
     Dot[Transpose[partialexternalmatrix][[a]], Transpose[partialexternalmatrix][[a]]]],
   {a, 1, externalarrays}];
Sort[svdcorrelation, OrderedQ[{{#2}, {#1}}] &][[1]]

0.340345

limit = 0.35;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{13 - a + 1, externalarraynames[[1, a]]}, {a, 1, 13}];
labelx = "(a) SVD Basis Correlation";
labely = "Samples";
g = BarChart[
   Table[svdcorrelation[[externalarrays - a]], {a, 0, externalarrays - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, externalarrays + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 0.09, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 3}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1.5}, {0, 0}, {0, 1}];
q3 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create SVD Transcription Factor Basis Replication Initiation Protein Sample
   Correlations Bar Chart Display *)

u = Transpose[SingularValues[partialexternalmatrix1][[1]]];
svdcorrelation12 =
  Table[
   Sqrt[
    Sum[(Dot[Transpose[partialexternalmatrix2][[a]], u]^2)[[b]], {b, 1, 9}] /
     Dot[Transpose[partialexternalmatrix2][[a]], Transpose[partialexternalmatrix2][[a]]]],
   {a, 1, 4}];
Sort[svdcorrelation12, OrderedQ[{{#2}, {#1}}] &][[1]]

0.738166

limit = 0.75;

gridx = Table[a, {a, 0, limit, limit / 5.}];
framex = gridx;
sizes = Flatten[
   Table[
    Dimensions[
     Characters[
      ToString[framex[[a]]
      ]]], {a, 1, 6}]];
Do[
  Do[framex[[a]] = StringJoin[ToString[framex[[a]]], " "],
   {b, 1, size - sizes[[a]]}],
  {a, 1, 6}];
framex = Table[{gridx[[a]], framex[[a]]}, {a, 1, 6}];
gridx = Table[{gridx[[a]], RGBColor[0, 0, 0]}, {a, 1, 6}];
framey = Table[{13 - a + 1, externalarraynames[[1, a]]}, {a, 10, 13}];
labelx = "(a)  SVD Basis Correlation";
labely = "Samples";
g = BarChart[
   Table[svdcorrelation12[[4 - a]], {a, 0, 4 - 1}],
   BarOrientation -> Horizontal,
   PlotRange -> {{0, limit + 0.001}, {0.5, 4 + 0.5}},
   AspectRatio -> 1,
   Axes -> False,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {gridx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 0.2, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.88}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.44}, {0, 0}, {0, 1}];
q5 = Show[g,
   AspectRatio -> 1.05,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Project Data Onto SVD Basis *)

externalarraycorrelations = Dot[PseudoInverse[partialeigenarrays], partialexternalmatrix];
matrix = Dot[partialeigenarrays, externalarraycorrelations];

(* Project SVD Reconstructed Replication Initiation Protein Data Onto
   SVD Reconstructed Transcription Factor Data *)

partialexternalmatrix1 = TakeColumns[partialexternalmatrix, {1, 9}];
partialexternalmatrix2 = TakeColumns[partialexternalmatrix, {10, 13}];
arraycontributions12 = Transpose[Dot[
    Dot[PseudoInverse[partialexternalmatrix1], partialeigenarrays],
    Dot[PseudoInverse[partialeigenarrays], partialexternalmatrix2]]];

(* Calculate Contributions of Eigenarrays to External Arrays *)

matrix = Transpose[matrix];
externalcoordinates = Table[
   {externalarraycorrelations[[1, a]] /
     Sqrt[Dot[matrix[[a]], matrix[[a]]]],
    externalarraycorrelations[[2, a]] /
     Sqrt[Dot[matrix[[a]], matrix[[a]]]]},
   {a, 1, externalarrays}];
matrix = Transpose[matrix];

(* Create Parameter Graph of Arrays Projected Onto 2 D Subspace *)

points1 = {Point[externalcoordinates[[1]]], Point[externalcoordinates[[2]]],
   Point[externalcoordinates[[3]]]};
points2 = {Point[externalcoordinates[[4]]]};
points3 = {Point[externalcoordinates[[5]]], Point[externalcoordinates[[6]]],
   Point[externalcoordinates[[7]]]};
points4 = {Point[externalcoordinates[[8]]], Point[externalcoordinates[[9]]]};
points5 = {Point[externalcoordinates[[10]]], Point[externalcoordinates[[11]]],
   Point[externalcoordinates[[12]]], Point[externalcoordinates[[13]]]};
```

```
p = Show[
   {Graphics[{RGBColor[0, 0.5, 0], PointSize[0.035], points1}],
    Graphics[{RGBColor[1, 0, 0], PointSize[0.035], points2}],
    Graphics[{RGBColor[1, 0.5, 0], PointSize[0.035], points3}],
    Graphics[{RGBColor[1, 1, 0], PointSize[0.035], points4}],
    Graphics[{RGBColor[0.75, 0, 1], PointSize[0.035], points5}],
    Graphics[{RGBColor[1, 0.5, 0], Text["G2/M", {1.075, -0.52}]}],
    Graphics[{RGBColor[0, 0, 0], Text["M/G1", {0.9, 0.75}]}],
    Graphics[{RGBColor[1, 0, 0], Text["S/G2", {0.2, -1.12}]}],
    Graphics[{RGBColor[0, 0, 1], Text["S", {-0.95, -0.6}]}],
    Graphics[{RGBColor[0, 0.5, 0], Text["G1", {-0.8, 0.8}]}],
    Graphics[{RGBColor[0, 0, 0], Text["(a)", {-1.1, 1.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=π/2", {0.2, 1.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ϕ=0", {1.12, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MBP1", externalcoordinates[[1]] - {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI4", externalcoordinates[[2]] - {0.15, 0.09}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI6", externalcoordinates[[3]] - {0.1, 0.1}]}],
    Graphics[{RGBColor[0, 0, 0], Text["FKH1", externalcoordinates[[4]] - {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["FKH2", externalcoordinates[[5]] + {0, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["NDD1", externalcoordinates[[6]] - {0.24, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM1", externalcoordinates[[7]] + {-0.04, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ACE2", externalcoordinates[[8]] + {0.02, 0.12}]}],
    Graphics[{RGBColor[0, 0, 0], Text["SWI5", externalcoordinates[[9]] + {0, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM3", externalcoordinates[[10]] - {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM4", externalcoordinates[[11]] + {-0.24, 0.06}]}],
    Graphics[{RGBColor[0, 0, 0], Text["MCM7", externalcoordinates[[12]] - {0, 0.15}]}],
    Graphics[{RGBColor[0, 0, 0], Text["ORC1", externalcoordinates[[13]] + {0.2, 0}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 0.5, {0. * Pi, 2 * Pi}]}],
    Graphics[{RGBColor[0, 0, 0], Dashing[{0.03, 0.02}], Circle[{0, 0}, 1]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{-1.25, 0}, {1.25, 0},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}],
    Graphics[{RGBColor[0, 0, 0], Arrow[{0, -1.25}, {0, 1.25},
       HeadCenter -> 0.5, HeadLength -> 0.035, HeadWidth -> 0.75]}]},
   AspectRatio -> 1,
   PlotRange -> {{-1.25, 1.25}, {-1.25, 1.25}},
   Frame -> True,
   FrameTicks -> False,
   FrameLabel -> {None, None, None, None},
   GridLines -> {{{0, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
   DisplayFunction -> Identity];
p = FullGraphics[p];
p[[1, 2]] = p[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {-1.18, 0}, {0, 0}, {0, 1}];
m1 = Show[p,
   AspectRatio -> 1.,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Classify SVD Gene Phases into Cell Cycle Phases *)

ph1 = 0;
ph2 = -1 / 2.;
ph3 = -1.;
ph4 = -4 / 3.;
ph5 = -5 / 3.;

endph5 = partialgenes;
beginph1 = 1;
partialphases[[endph5]] - ph1
partialphases[[beginph1]] - ph1

{0.000221307}

{-0.000407097}

endph1 = 470;
beginph2 = 471;
partialphases[[endph1]] - ph2
partialphases[[beginph2]] - ph2

{0.000187677}

{0.998634}

endph2 = 913;
beginph3 = 914;
partialphases[[endph2]] - ph3
partialphases[[beginph3]] - ph3

{1.00122}

{0.998991}

endph3 = 1261;
beginph4 = 1262;
partialphases[[endph3]] - ph4
partialphases[[beginph4]] - ph4

{1.00001}

{0.999004}

endph4 = 1798;
beginph5 = 1799;
partialphases[[endph4]] - ph5
partialphases[[beginph5]] - ph5

{2.00349}

{1.99843}

(* 2227 yeast genes, 470 in M/G1, 443 in G1, 348 in S, 537 in S/G2, 429 in G2/M. *)
```

```
(* Display SVD Reconstructed and Sorted External Data *)

(* Create SVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1,
     {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1,
     {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = {
   {partialgenes - endph1 / 2, "M/G1"},
   {partialgenes - (endph1 + endph2) / 2, "G1"},
   {partialgenes - (endph2 + endph3) / 2, "S"},
   {partialgenes - (endph3 + endph4) / 2, "S/G2"},
   {(partialgenes - endph4) / 2, "G2/M"}};
gridy = {
   {partialgenes - endph1 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph2 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph3 + 0.5, {RGBColor[0, 0, 0]}},
   {partialgenes - endph4 + 0.5, {RGBColor[0, 0, 0]}}};
labelx = "(a) Samples";
labely = ColumnForm[{"ORFs", " ", " ", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, gridy},
   DisplayFunction -> Identity];
```

```
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.5, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 340}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 150}, {0, 0}, {0, 1}];
g1 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction → Identity];

(* Create SVD Reconstructed Sorted External Data Graph Display *)

matrix = Transpose[matrix];
matrixplot = Table[0, {a, 1, externalarrays}];
Do[
 matrixplot[[a]] = Chop[TrigFit[matrix[[a]], 1, {x, partialgenes - 1}], 0.01],
 {a, 1, externalarrays}]
table = Table[
   {externalarraynames[[1, a]], matrixplot[[a]]}, {a, 1, externalarrays}];
TableForm[table]
```

MBP1	$-0.0428963 \operatorname{Cos}[\frac{\pi x}{1113}] + 0.0465378 \operatorname{Sin}[\frac{\pi x}{1113}]$
SWI4	$-0.0898583 \operatorname{Cos}[\frac{\pi x}{1113}] + 0.0611395 \operatorname{Sin}[\frac{\pi x}{1113}]$
SWI6	$-0.0762082 \operatorname{Cos}[\frac{\pi x}{1113}] + 0.0682877 \operatorname{Sin}[\frac{\pi x}{1113}]$
FKH1	$-0.0151408 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0256583 \operatorname{Sin}[\frac{\pi x}{1113}]$
FKH2	$-0.0112308 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0109385 \operatorname{Sin}[\frac{\pi x}{1113}]$
NDD1	$0.0290329 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0294006 \operatorname{Sin}[\frac{\pi x}{1113}]$
MCM1	$0.0547816 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0212513 \operatorname{Sin}[\frac{\pi x}{1113}]$
ACE2	$-0.0139835 \operatorname{Cos}[\frac{\pi x}{1113}] + 0.0259147 \operatorname{Sin}[\frac{\pi x}{1113}]$
SWI5	$-0.0111965 + 0.0308045 \operatorname{Sin}[\frac{\pi x}{1113}]$
MCM3	$0.014907 + 0.038586 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0301169 \operatorname{Sin}[\frac{\pi x}{1113}]$
MCM4	$0.0555616 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0223721 \operatorname{Sin}[\frac{\pi x}{1113}]$
MCM7	$0.0269092 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0386775 \operatorname{Sin}[\frac{\pi x}{1113}]$
ORC1	$0.0135928 + 0.043864 \operatorname{Cos}[\frac{\pi x}{1113}] - 0.0542691 \operatorname{Sin}[\frac{\pi x}{1113}]$

```
p = Table[0, {a, 1, externalarrays}];
color = {
   RGBColor[0.75, 0, 1],
   RGBColor[1, 0, 0],
   RGBColor[1, 0.5, 0],
   RGBColor[0, 0.5, 0],
   RGBColor[0, 0, 1]};
labelx = "(b) Binding Level";
framex = Table[{0.2 * a, externalarraynames[[1, a]]},
   {a, 1, externalarrays}];
```

```
Do[{
   graph = ParametricPlot[{matrixplot[[n]] + 0.2 * n, -x},
     {x, 0, partialgenes - 1},
     PlotStyle -> {RGBColor[0, 0, 0]},
     DisplayFunction -> Identity],
   coordinates = Table[
     If[matrix[[n, a]] + 0.2 * n < 0, 0,
      If[matrix[[n, a]] + 0.2 * n > 2.8, 2.8,
       matrix[[n, a]] + 0.2 * n]],
     {a, 1, partialgenes}],
   coordinates = Table[{coordinates[[a]], -a + 1}, {a, 1, partialgenes}],
   line = Line[coordinates],
   g = Show[{
      Graphics[{color[[Mod[n, 5] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, None, labelx, None},
     FrameTicks -> {None, None, framex, None},
     GridLines -> {{{0.2 * n, RGBColor[0, 0, 0]}}, None},
     PlotRange -> {{0, 2.8}, {65, -partialgenes + 1 - 65}},
     DisplayFunction → Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 330}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 142.5}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → GoldenRatio * 1.2 / 2.5,
     PlotRange -> All,
     DisplayFunction → Identity]
  }, {n, 1, externalarrays}];
matrix = Transpose[matrix];

g2 = Show[Table[p[[a]], {a, 1, externalarrays}],
   DisplayFunction → Identity];
```

```
(* Display Projection Onto SVD Cell Cycle Subspace *)

(* Create SVD Sorted External Data 2 D Red & Green Raster Display *)

contrast = 1.5 * 1.5;
displaying = Table[
   If[contrast * partialexternalmatrix[[i, j]] > 0,
    If[contrast * partialexternalmatrix[[i, j]] < 1,
     {contrast * partialexternalmatrix[[i, j]], 0}, {1, 0}],
    If[contrast * partialexternalmatrix[[i, j]] > -1,
     {0, -contrast * partialexternalmatrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
labelx = "Samples";
labely = ColumnForm[{" ", "                ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p1 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create SVD Recosntructed Sorted External Data 2 D Red & Green Raster Display *)

contrast = 10 * 1.5;
displaying = Table[
   If[contrast * matrix[[i, j]] > 0,
    If[contrast * matrix[[i, j]] < 1, {contrast * matrix[[i, j]], 0}, {1, 0}],
    If[contrast * matrix[[i, j]] > -1, {0, -contrast * matrix[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
labelx = "Samples";
labely = ColumnForm[{" ", "                ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {1., 0.}] →
    Text[a, {b - 1.8, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p2 = Show[g,
   AspectRatio -> GoldenRatio * 1.2,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create SVD Sorted Eigenarrays 2 D Red & Green Raster Display *)

contrast = 10 * 15;
displaying = Table[
   If[contrast * partialeigenarrays[[i, j]] > 0,
    If[contrast * partialeigenarrays[[i, j]] < 1,
     {contrast * partialeigenarrays[[i, j]], 0}, {1, 0}],
    If[contrast * partialeigenarrays[[i, j]] > -1,
     {0, -contrast * partialeigenarrays[[i, j]]}, {0, 1}]],
   {i, 1, partialgenes}, {j, 1, 9}];
framex = Table[{a - 0.5, StringJoin[ToString[a], "    "]}, {a, 1, 9}];
labelx = "Eigenarrays";
labely = ColumnForm[{" ", "                ORFs", " ", " ", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, partialgenes, 1, -1}, {j, 1, 9}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, None, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   GridLines -> {None, None},
   DisplayFunction -> Identity];

g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 400}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 200}, {0, 0}, {0, 1}];
p3 = Show[g,
   AspectRatio -> GoldenRatio * 1.2 * 1.5,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Create Eigenarrays Contributions to External Arrays 2 D Red & Green Raster Display *)

contrast = 0.5;
displaying = Table[
   If[contrast * externalarraycorrelations[[i, j]] > 0,
    If[contrast * externalarraycorrelations[[i, j]] < 1,
     {contrast * externalarraycorrelations[[i, j]], 0}, {1, 0}],
    If[contrast * externalarraycorrelations[[i, j]] > -1,
     {0, -contrast * externalarraycorrelations[[i, j]]}, {0, 1}]],
   {i, 1, 9}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = Table[{a + 0.5, ToString[9 - a]}, {a, 0, 8}];
labely = ColumnForm[{" ", "Eigenarrays", " "}, Center];
labelx = ColumnForm[{"Samples", " ", " "}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 9, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 3, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 0.8}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1.6}, {0, 0}, {0, 1}];
p4 = Show[g,
   AspectRatio → 1.05 / 1.5,
   PlotRange -> All,
   DisplayFunction → Identity];
```

```
(* Display SVD Pseudoinverse Correlations *)

(* Create SVD Correlations Red and Green Raster Display *)

contrast = 0.5;
displaying = Table[
   If[contrast * externalarraycorrelations[[i, j]] > 0,
    If[contrast * externalarraycorrelations[[i, j]] < 1,
     {contrast * externalarraycorrelations[[i, j]], 0}, {1, 0}],
    If[contrast * externalarraycorrelations[[i, j]] > -1,
     {0, -contrast * externalarraycorrelations[[i, j]]}, {0, 1}]],
   {i, 1, 9}, {j, 1, externalarrays}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, externalarrays}];
framey = Table[{a + 0.5, ToString[9 - a]}, {a, 0, 8}];
labely = ColumnForm[{"(a) Eigenarrays"}, Center];
labelx = ColumnForm[{"Samples"}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 9, 1, -1}, {j, 1, externalarrays}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 2, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 2}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 1}, {0, 0}, {0, 1}];
f1 = Show[g,
   PlotRange -> All,
   AspectRatio → 1 / GoldenRatio * 1.25,
   DisplayFunction → Identity];

(* Create SVD Correlations Graph Display *)

p = Table[0, {a, 1, 2}];
color = {
   RGBColor[0, 0.5, 0],
   RGBColor[1, 0, 0],
   RGBColor[0, 0, 1]};
labelx = ColumnForm[{" "}, Center];
labely = ColumnForm[{"(b) Correlation"}, Center];
framex = Table[{a - 1, "    "}, {a, 1, externalarrays}];
framey = Table[a, {a, -5, 5, 5}];

matrixplot = {
   -Sqrt[Dot[
        externalarraycorrelations[[1]],
        externalarraycorrelations[[1]]]] /
     2 * Sin[Pi * (x + 1) / 4],
   Sqrt[Dot[
       externalarraycorrelations[[2]],
       externalarraycorrelations[[2]]]] /
     2 * Cos[Pi * x / 4]};
```

```
Do[{
   graph = Plot[matrixplot[[n]],
      {x, 0, 8},
      PlotStyle -> {color[[Mod[n, 3] + 1]], Dashing[{0.03, 0.02}]},
      DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
       externalarraycorrelations[[n, a]]},
      {a, 1, externalarrays}],
   points = Table[Point[coordinates[[a]]], {a, 1, externalarrays}],
   line = Line[coordinates],
   g = Show[
      {Graphics[{color[[Mod[n, 3] + 1]], PointSize[0.022], points}],
       Graphics[{color[[Mod[n, 3] + 1]], line}],
       graph},
      Frame -> True,
      FrameLabel -> {None, labely, labelx, None},
      GridLines -> {{{8.5, RGBColor[0, 0, 0]}}, {{0, RGBColor[0, 0, 0]}}},
      FrameTicks -> {None, framey, framex, None},
      PlotRange -> {-5.001, 5.001},
      DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
      Text[labely, {b_, c_}, {1., 0.}] →
       Text[labely, {b - 1.75, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
      Text[labelx, {b_, c_}, {0., -1.}] →
       Text[labelx, {b, c + 1.5}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
      Text[a_, {b_, c_}, {0., -1.}] →
       Text[a, {b, c + 0.75}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
      PlotRange -> All,
      AspectRatio → 1 / GoldenRatio * 1.25,
      DisplayFunction → Identity]
  }, {n, 1, 2}];
f2 = Show[Table[p[[a]], {a, 1, 2}],
   DisplayFunction → Identity];
```

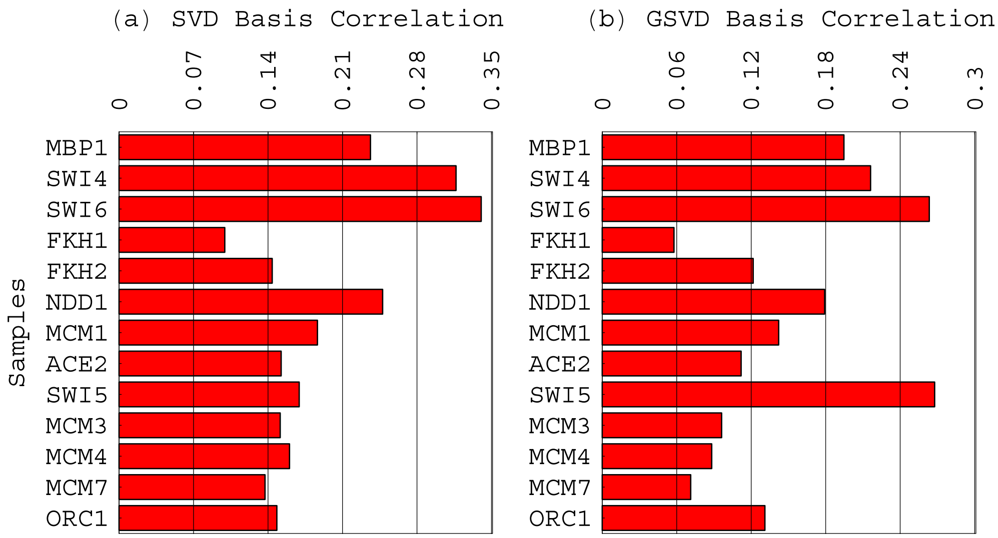
```
(* Display SVD Pseudoinverse Correlations
   Between Replication Initiation Proteins and Transcription Factors *)

(* Create SVD Correlations Red and Green Raster Display *)

contrast = 40;
displaying = Table[
   If[contrast * arraycontributions12[[i, j]] > 0,
    If[contrast * arraycontributions12[[i, j]] < 1,
     {contrast * arraycontributions12[[i, j]], 0}, {1, 0}],
    If[contrast * arraycontributions12[[i, j]] > -1,
     {0, -contrast * arraycontributions12[[i, j]]}, {0, 1}]],
   {i, 1, 4}, {j, 1, 9}];
framex = Table[{a - 0.5, externalarraynames[[1, a]]}, {a, 1, 9}];
framey = Table[{13 - a + 0.5, externalarraynames[[1, a]]}, {a, 10, 13}];
labely = ColumnForm[{"(d) Samples"}, Center];
labelx = ColumnForm[{"Samples"}, Center];
g = Show[
   Graphics[
    RasterArray[
     Table[
      RGBColor[displaying[[i, j, 1]], displaying[[i, j, 2]], 0],
      {i, 4, 1, -1}, {j, 1, 9}]]],
   AspectRatio -> 1,
   Frame -> True,
   FrameTicks -> {None, framey, framex, None},
   FrameLabel -> {None, labely, labelx, None},
   DisplayFunction -> Identity];
g = FullGraphics[g];
g[[1, 2]] = g[[1, 2]] /.
   Text[labely, {b_, c_}, {1., 0.}] →
    Text[labely, {b - 1.73, c}, {0, 0}, {0, 1}];
g[[1, 2]] = g[[1, 2]] /.
   Text[labelx, {b_, c_}, {0., -1.}] →
    Text[labelx, {b, c + 1.34}, {0, -1}, {1, 0}];
g[[1, 2]] = g[[1, 2]] /.
   Text[a_, {b_, c_}, {0., -1.}] →
    Text[a, {b, c + 0.67}, {0, 0}, {0, 1}];
h1 = Show[g,
   AspectRatio → 1 / GoldenRatio,
   PlotRange -> All,
   DisplayFunction → Identity];

(* Create SVD Correlations Graph Display *)

p = Table[0, {a, 1, 4}];
color = {
   RGBColor[0, 0, 1],
   RGBColor[1, 0, 0]
  };
framex = Table[{a - 1, "    "}, {a, 1, 9}];
```

```
matrixplot = {
   -Sqrt[Dot[
        arraycontributions12[[1]],
        arraycontributions12[[1]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[2]],
        arraycontributions12[[2]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[3]],
        arraycontributions12[[3]]]] /
     2 * Sin[Pi * x / 4],
   -Sqrt[Dot[
        arraycontributions12[[4]],
        arraycontributions12[[4]]]] /
     2 * Sin[Pi * x / 4]};

labelx = " ";
labely = ColumnForm[{"(e) Correlation"}, Center];
framey = Table[a, {a, -0.025, 0.025, 0.025}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 2] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions12[[n, a]]},
     {a, 1, 9}],
   points = Table[Point[coordinates[[a]]], {a, 1, 9}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 2] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 2] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
     PlotRange -> {-0.04, 0.035},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 1.5, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 0.0249}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 0.0124}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio → 1 / GoldenRatio,
     PlotRange → All,
     DisplayFunction → Identity],
  }, {n, 1, 2}];
h2 = Show[Table[p[[a]], {a, 1, 2}]];
```

```
labelx = " ";
labely = ColumnForm[{"(f) Correlation"}, Center];
framey = Table[a, {a, -0.05, 0.04, 0.025}];
Do[{
   graph = Plot[matrixplot[[n]],
     {x, 0, 8},
     PlotStyle -> {color[[Mod[n, 2] + 1]], Dashing[{0.03, 0.02}]},
     DisplayFunction -> Identity],
   coordinates = Table[{a - 1,
      arraycontributions12[[n, a]]},
     {a, 1, 9}],
   points = Table[Point[coordinates[[a]]], {a, 1, 9}],
   line = Line[coordinates],
   g = Show[
     {Graphics[{color[[Mod[n, 2] + 1]], PointSize[0.022], points}],
      Graphics[{color[[Mod[n, 2] + 1]], line}],
      graph},
     Frame -> True,
     FrameLabel -> {None, labely, labelx, None},
     FrameTicks -> {None, framey, framex, None},
     GridLines -> {None, {{0, RGBColor[0, 0, 0]}}},
     PlotRange -> {-0.055, 0.04},
     DisplayFunction -> Identity],
   g = FullGraphics[g],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labely, {b_, c_}, {1., 0.}] →
      Text[labely, {b - 1.5, c}, {0, 0}, {0, 1}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[labelx, {b_, c_}, {0., -1.}] →
      Text[labelx, {b, c + 0.0317}, {0, -1}, {1, 0}],
   g[[1, 2]] = g[[1, 2]] /.
     Text[a_, {b_, c_}, {0., -1.}] →
      Text[a, {b, c + 0.0158}, {0, 0}, {0, 1}],
   p[[n]] = Show[g,
     AspectRatio -> 1 / GoldenRatio,
     PlotRange → All,
     DisplayFunction → Identity],
  }, {n, 3, 4}];
h3 = Show[Table[p[[a]], {a, 3, 4}]];
```

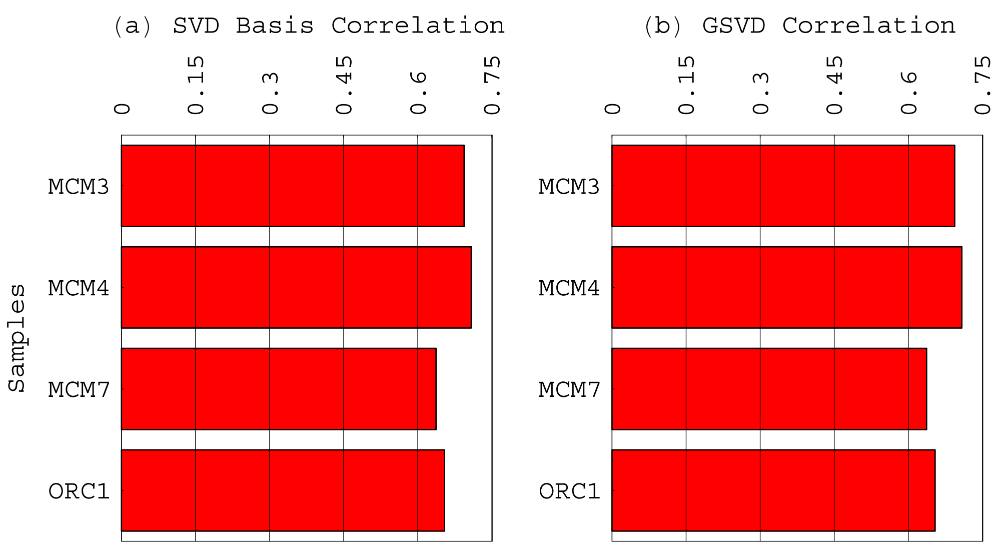
```
(* Display Correlations of External Arrays with SVD and GSVD Bases *)
```

```
Show[GraphicsArray[{q3, q4}], GraphicsSpacing -> -0.06];
```



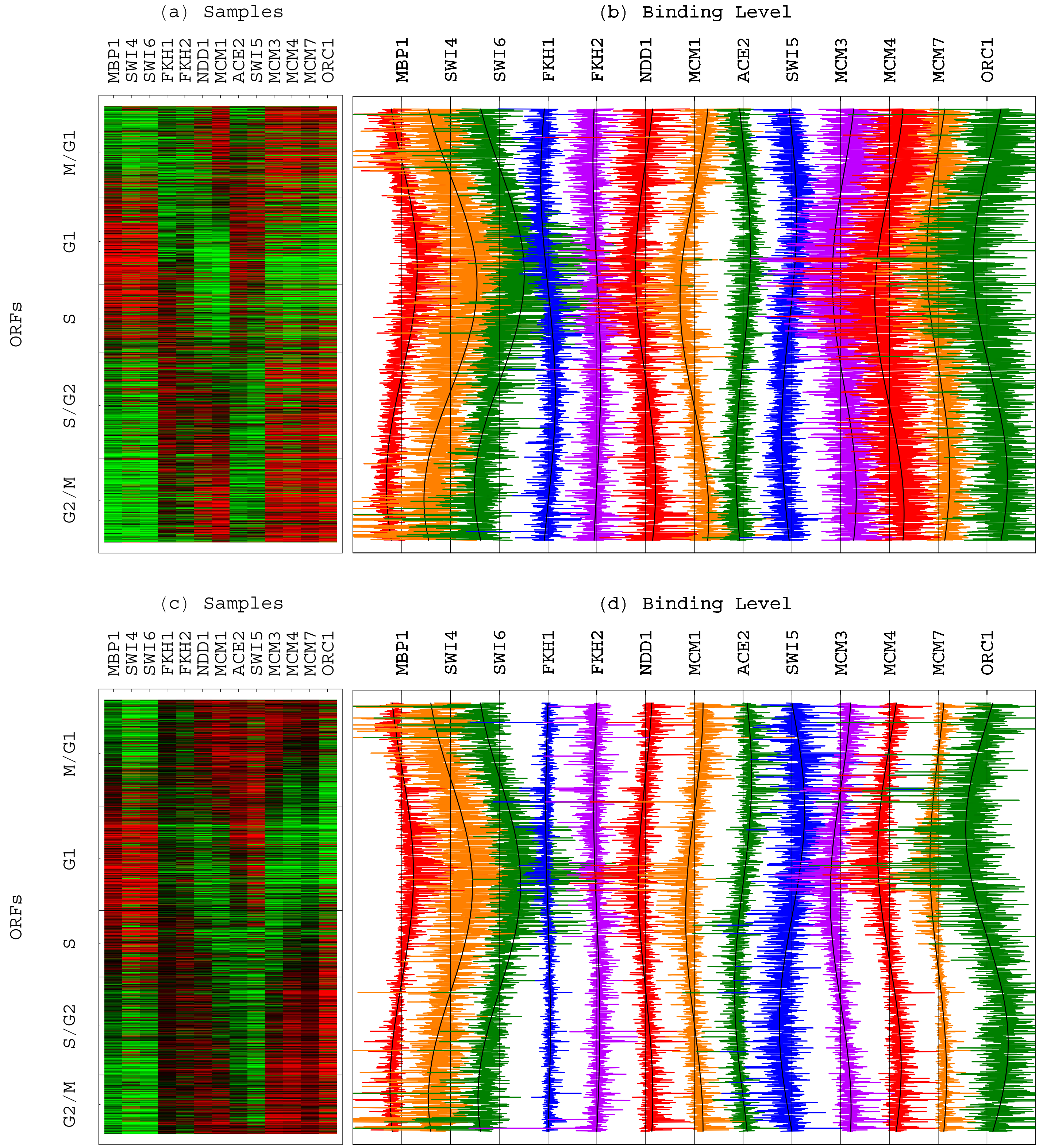

```
(* Display Correlations Between SVD- and GSVD-
   Reconstructed Replication Initiation Proteins and Transcription Factors  *)
```

```
Show[GraphicsArray[{q5, q6}], GraphicsSpacing → -0.06];
```



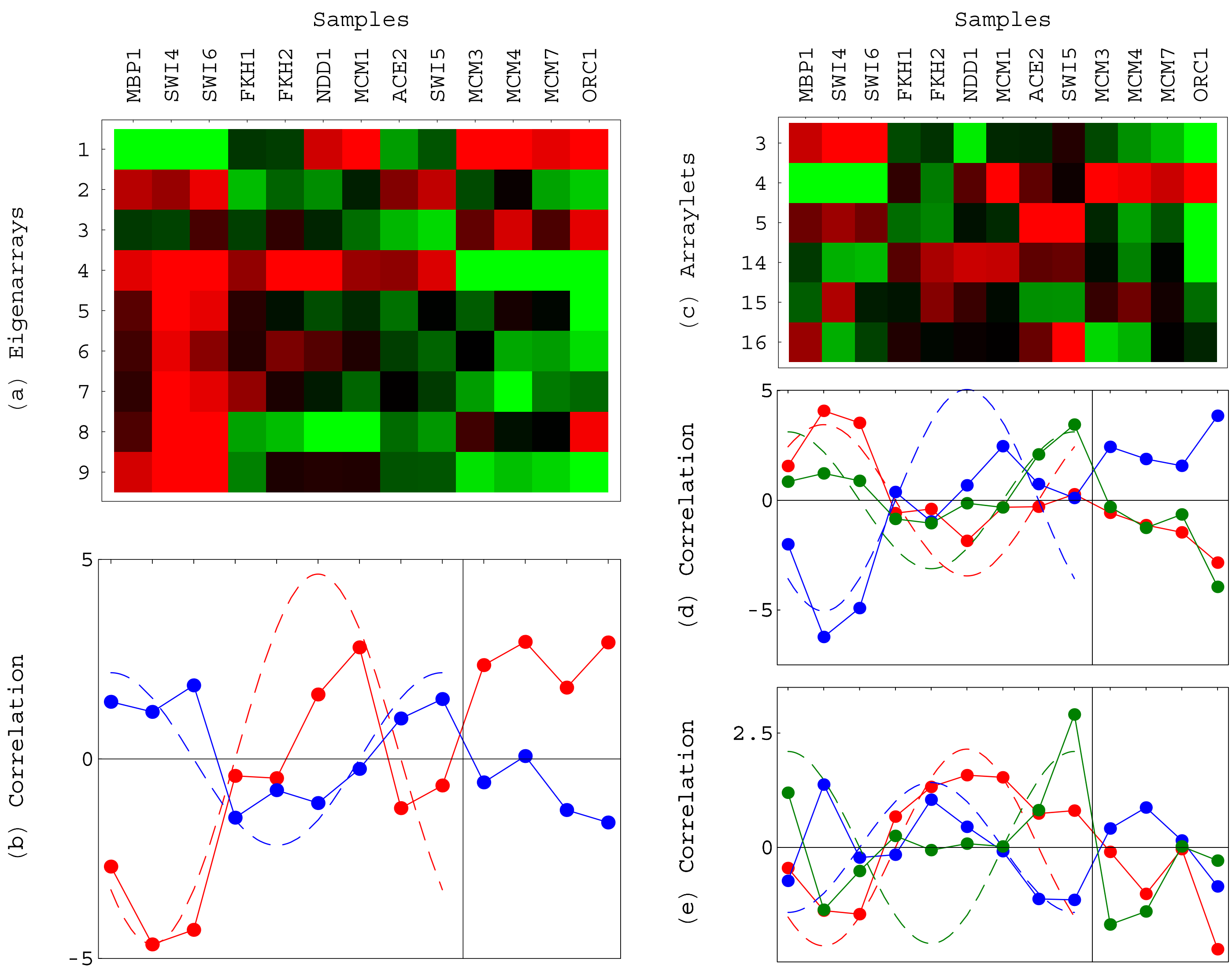
```
(* Display SVD and GSVD Reconstructed and Sorted External Data  *)
```

```
q1 = Show[{
    Graphics[{Rectangle[{0, 0}, {52, 75}, g1]}],
    Graphics[{Rectangle[{50, 0}, {158, 75}, g2]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
q2 = Show[{
    Graphics[{Rectangle[{0, 0}, {52, 75}, g3]}],
    Graphics[{Rectangle[{50, 0}, {158, 75}, g4]}]},
   PlotRange -> All,
   DisplayFunction → Identity];
Show[GraphicsArray[{{q1}, {q2}}],
  GraphicsSpacing → -0.13];
```


(a) Samples
(b) Binding Level
(c) Samples
(d) Binding Level
ORFs
M/G1
G1
S
S/G2
G2/M
MBP1
SWI4
SWI6
FKH1
FKH2
NDD1
MCM1
ACE2
SWI5
MCM3
MCM4
MCM7
ORC1

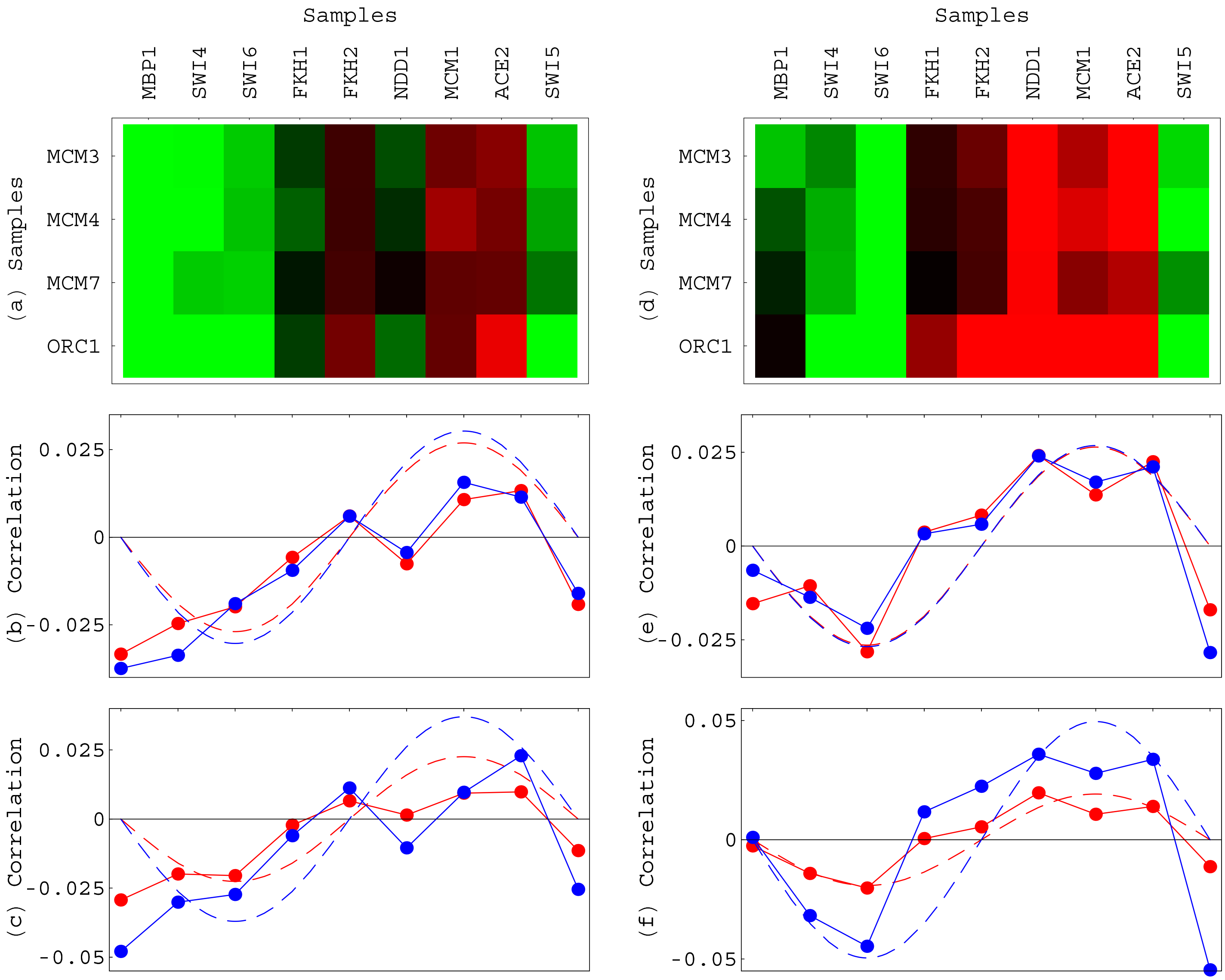
```
(* Display SVD and GSVD Pseudoinverse Correlations *)
```

```
s1 = Show[GraphicsArray[{{f1}, {f2}}],
   GraphicsSpacing → -0.1,
   DisplayFunction → Identity];
s2 = Show[GraphicsArray[{{f3}, {f4}, {f5}}],
   GraphicsSpacing → -0.2,
   DisplayFunction → Identity];
Show[GraphicsArray[{s1, s2}],
  GraphicsSpacing → -0.075];
```



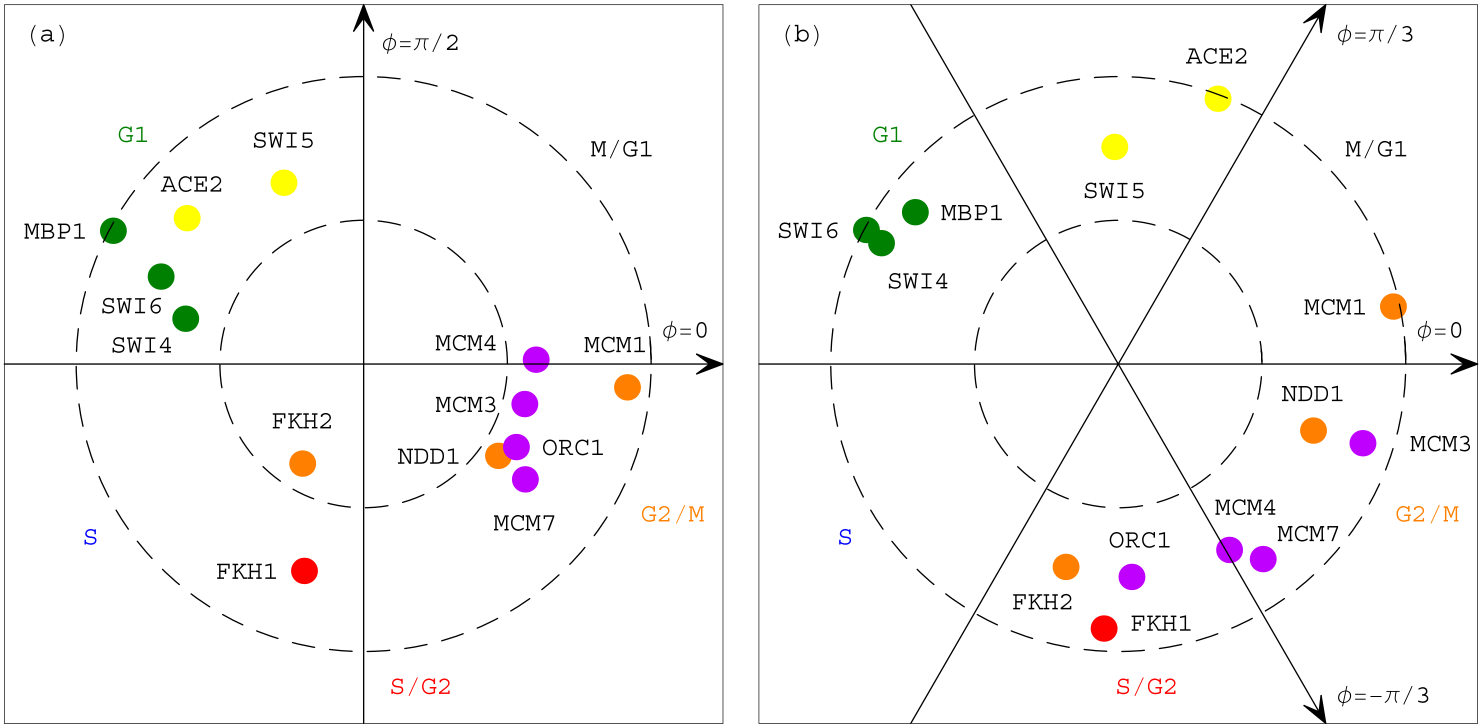
```
(* Display SVD and GSVD Pseudoinverse Correlations
    Between Replication Initiation Proteins and Transcription Factors  *)
```

```
s1 = Show[GraphicsArray[{{h1}, {h2}, {h3}}],
   GraphicsSpacing → -0.25,
   DisplayFunction → Identity];
s2 = Show[GraphicsArray[{{h4}, {h5}, {h6}}],
   GraphicsSpacing → -0.25,
   DisplayFunction → Identity];
Show[GraphicsArray[{s1, s2}],
  GraphicsSpacing → -0.05];
```



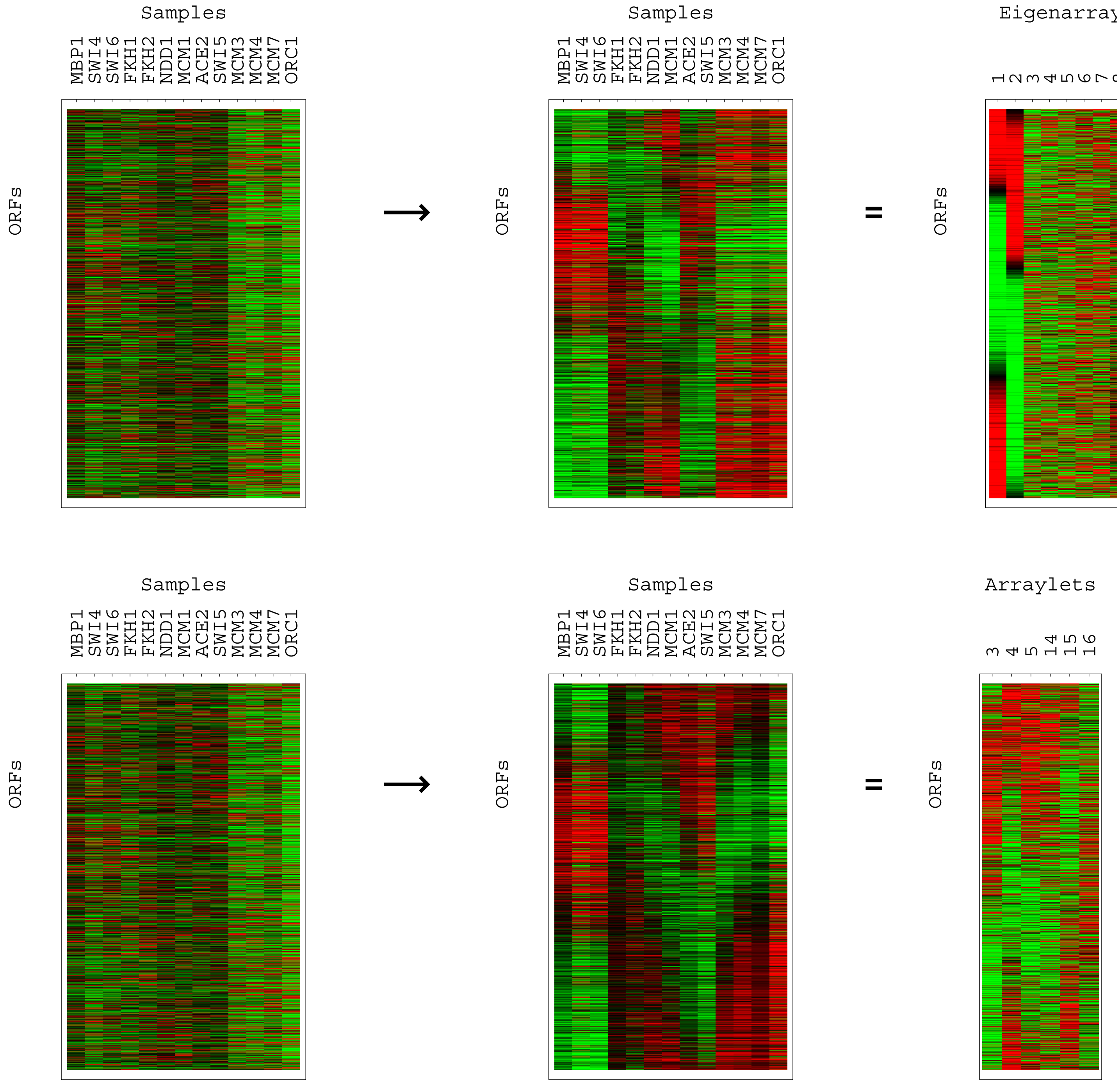
```
(* Display Mapping of Protein Binding Data *)
```

```
Show[GraphicsArray[{m1, m2}],
    GraphicsSpacing -> 0];
```



```
(* Display Projections Onto SVD and GSVD Cell Cycle Subspaces  *)
```

```
equal = Show[Graphics[
            Text[StyleForm["=", FontSize → 20, FontWeight -> Bold], {0, 0}]
        ], DisplayFunction -> Identity];
times = Show[Graphics[
            Text[StyleForm["×", FontSize -> 20, FontWeight -> Bold], {0, 0}]
        ], DisplayFunction -> Identity];
rightarrow = Show[Graphics[
            Text[StyleForm["⟶", FontSize → 20, FontWeight -> Bold], {0, 0}]
        ], DisplayFunction -> Identity];

Show[{
   Graphics[{Rectangle[{0, 0}, {100, 200}, p1]}],
   Graphics[{Rectangle[{0, -220}, {100, -20}, p5]}],
   Graphics[{Rectangle[{100, 35}, {130, 200}, rightarrow]}],
   Graphics[{Rectangle[{100, -185}, {130, -20}, rightarrow]}],
   Graphics[{Rectangle[{120, 0}, {220, 200}, p2]}],
   Graphics[{Rectangle[{120, -220}, {220, -20}, p6]}],
   Graphics[{Rectangle[{220, 35}, {240, 200}, equal]}],
   Graphics[{Rectangle[{220, -185}, {240, -20}, equal]}],
   Graphics[{Rectangle[{230, 0}, {305, 200}, p3]}],
   Graphics[{Rectangle[{230, -220}, {292, -20}, p7]}],
   Graphics[{Rectangle[{305, 35}, {325, 200}, times]}],
   Graphics[{Rectangle[{305, -185}, {325, -20}, times]}],
   Graphics[{Rectangle[{315, 75}, {435, 200}, p4]}],
   Graphics[{Rectangle[{315, -126}, {435, -20}, p8]}]
  },
  PlotRange -> All];
```


Samples
MBP1
SWI4
SWI6
FKH1
FKH2
NDD1
MCM1
ACE2
SWI5
MCM3
MCM4
MCM7
ORC1
ORFs
Eigenarray
Arraylets
3
4
5
14
15
16