Cancer-driven dynamics of immune cells: Mathematical Appendix

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In the main text, paths performed by splenocytes are modeled by means of random walks characterized by (synchronized) discrete time steps and moving on a continuous two-dimensional space (xy). This kind of random walks can be described in terms of a probability distribution $p(\mathbf{r}, t)$ giving the probability that the walker has covered a distance $\mathbf{r}$ in a time t. In fact, we can write

$$p(\mathbf{r}, t+\tau) = \int_{-\infty}^{\infty} p(\mathbf{r}', t)\psi(\mathbf{r}-\mathbf{r}', t)d\mathbf{r}', \tag{1}$$

where $\psi(\mathbf{r}-\mathbf{r}', t)$ is the probability that at time t a step from $\mathbf{r}'$ to $\mathbf{r}$ is performed. For time-homogeneous processes, the width and the direction of a step do not depend on time and the dependence on t can be dropped, i.e. $\psi(\mathbf{r}-\mathbf{r}', t) = \psi(\mathbf{r}-\mathbf{r}')$. Moreover, by exploiting the discreteness of time steps, the time t at which any step occurs is a multiple of τ in such a way that we can write

$$p(\mathbf{r}, n+1) = \int_{-\infty}^{\infty} p(\mathbf{r}', n)\psi(\mathbf{r}-\mathbf{r}')d\mathbf{r}', \tag{2}$$

n being the number of steps performed up to the time considered and $t = \tau n$.

As anticipated (see Results in the main paper), the distribution $\psi(\mathbf{r}-\mathbf{r}')$ qualitatively controls the resulting random walk, possibly giving rise to deterministic walks (e.g. $\psi(\mathbf{r}-\mathbf{r}') = \delta_{\mathbf{r}-\mathbf{r}',\mathbf{k}}$, $\mathbf{k} \neq \mathbf{0}$, corresponding to a ballistic motion), to correlated walks (e.g. $\psi(\mathbf{r}-\mathbf{r}') = f(\mathbf{r}\cdot\mathbf{r}')$, where f is a peaked function, corresponding to a motion with a preferred direction), to completely stochastic walks (e.g. $\psi(\mathbf{r}-\mathbf{r}') = \delta_{|\mathbf{r}-\mathbf{r}'|,\tilde{r}}$, corresponding to an isotropic motion where steps have fixed length $\tilde{r}$), etc [1].

In Euclidean structures, like the two-dimensional substrate considered here, we can decompose $\mathbf{r}$ into its normal coordinates, i.e. $\mathbf{r} = (x, y)$, and, analogously $\mathbf{r}-\mathbf{r}' = (x-x', y-y') \equiv (\Delta x, \Delta y)$. Therefore, Eq. 2 can be rewritten as

$$p((x,y), n+1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p((x',y'), n)\psi(\Delta x, \Delta y)dx'dy', \tag{3}$$

and, assuming that Δx and Δy are independent, $\psi(\Delta x, \Delta y)$ can be factorized as $\psi(\Delta x, \Delta y) = \psi_x(\Delta x)\psi_y(\Delta y)$.

As suggested by Eq. 3, the knowledge of the specific distribution $\psi(\Delta x, \Delta y)$ possibly allows to get an explicit expression for $p((x, y), n)$.

For instance, one can show that, when diffusion is isotropic, i.e. it is equal in the x and y directions, any distribution $\psi(\Delta x, \Delta y)$ fulfilling the central limit theorem asymptotically (resuming the continuous time description) leads to the well-known diffusive limit characterized by the normal distribution [1]

$$p((x,y), t) = \frac{1}{4\pi D t} e^{-\frac{[(x-v_x t)^2 + (y-v_y t)^2]}{4Dt}} \tag{4}$$

where $\mathbf{v} = (v_x, v_y)$ accounts for the presence of a drift, while D is the diffusion-coefficient.
The moments of the distribution are:

$$\langle (x,y) \rangle = (v_x t, v_y t); \tag{5}$$

$$\langle x^2 + y^2 \rangle = v_x^2 t^2 + v_y^2 t^2 + 4Dt, \tag{6}$$

hence, asymptotically, whenever noise is prevailing, we expect to observe a Brownian motion, i.e. $\mathbf{r} \propto \sqrt{t}$, while, whenever there is a real presence of a drift (signal), we expect a ballistic motion, i.e. $\mathbf{r} \propto t$.

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[1] Weiss, G.H., Aspects and applications of random walks, (North-Holl. Press, Amsterdam, 1994).