Electronic Supplementary Material 1 for

Elevated nonlinearity as an indicator of shifts in the dynamics of populations under stress

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Table S1 Pearson correlations between indicators for exploited and unexploited populations in the Northeast Shelf System (NES) and the southern California Current Ecosystem (CCE). p-values are given in parentheses and those < 0.05 are bolded.

Pearson	**NES**		**CCE**		**Both datasets**	
Correlations	*Exploited*	*Unexploited*	*Exploited*	*Unexploited*	*Exploited*	*Unexploited*
CV-AR1	-0.15(0.54)	-0.3(0.43)	**-0.48(0.03)**	0.007(0.98)	-0.15(0.33)	-0.16(0.51)
CV-$\Delta\varrho$	0.32(0.16)	**0.78(0.01)**	**0.51(0.02)**	0.29(0.44)	**0.37(0.02)**	**0.81(<0.001)**
AR1-$\Delta\varrho$	-0.004(0.98)	0.25(0.51)	-0.03(0.89)	0.26(0.49)	0.03(0.87)	0.31(0.21)

Figure S1 Bifurcation analysis, dynamical attractors (a, b), eigenvalues and Lyapunov exponents (c, d) of the deterministic model. In (a, b) we assume that demographic changes shift the growth rate r. Dynamically, a stable equilibrium is replaced by stable but spiraling dynamics, and as growth rate increases, the system starts to oscillate in cycles of increasing periods before becoming chaotic in a series of period-doubling bifurcations. At the boundary value ($r = 0$), the population shifts to extinction at a transcritical bifurcation. In (c, d) we assume a population experiencing exploitation pressure. Dynamically, we have a stable equilibrium that is replaced by an alternative state at the crossing of a fold bifurcation. Theoretically, we measure stability based on the eigenvalue (the rate of return to equilibrium) and the Lyapunov exponent (the rate of divergence from equilibrium after a small perturbation). Eigenvalues crossing unity (-1, +1) signify loss of stability, negative Lyapunov exponents signify convergence (stability), and positive Lyapunov exponents signify divergence (instability and chaotic dynamics).

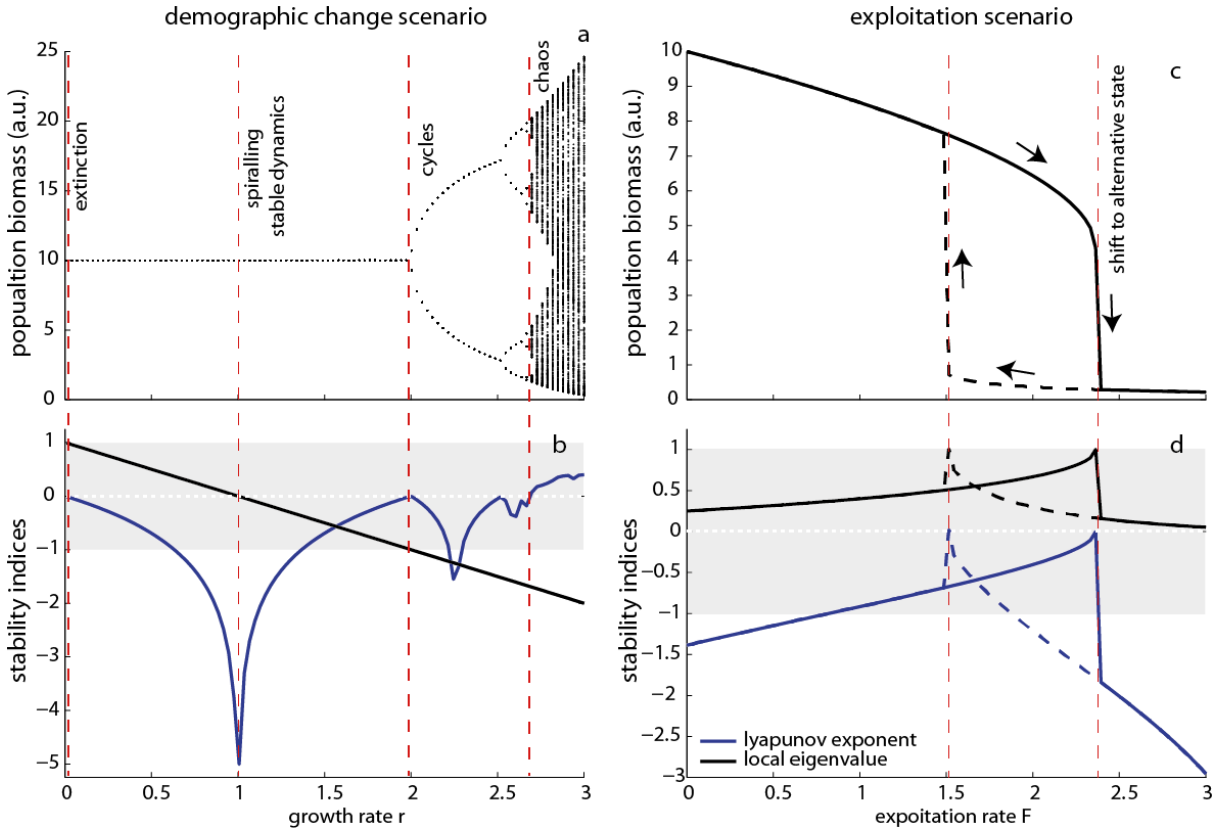

Figure S2 Critical slowing down and nonlinearity indicators for the two scenarios (mean and 95% confidence intervals (CI) based on 1000 simulations) for each level of growth rate *r* and exploitation rate *F*. (a, c, e) In the demographic change scenario, changes are not monotonic but depend on the type of dynamical regime. Red dashed lines indicate the thresholds between extinction, stable, spiraling stable dynamics, cycles, and chaos of the deterministic model. (b, d, f) In the exploitation scenario, all indicators change monotonically before the shift to overexploitation. Red dashed line indicates the threshold at which 50% of the 1000 populations collapse to the alternative overexploited state. Note the strong CI for $\Delta\rho$ in the exploitation scenario. Critical slowing down indicators: coefficient of variation *CV* and autocorrelation at-lag-1 *AR1*. Nonlinearity indicator $\Delta\rho$.