Complex structures from patterned cell sheets

Supplementary Material

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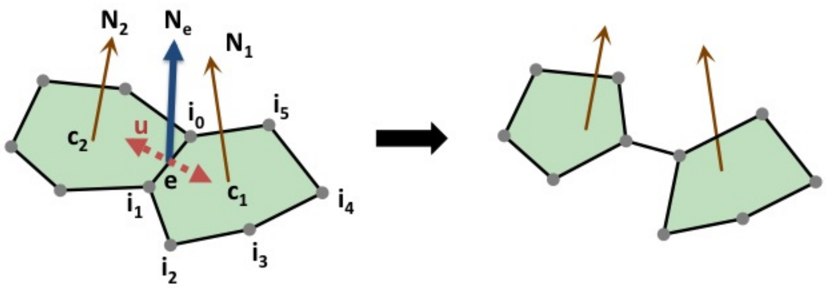
S1: Defining T1 transition for non-coplanar cells



Supplementary Figure 1: Schematic of a T1 transition. Different terms shown in the figure are described in greater detail in section S1.

Epithelial sheet is assumed to evolve in an over-damped setting. This leads to the dynamics of the vertex i, with position vector $\mathbf{x}_i$, governed by the following equation

$$\eta \frac{d\mathbf{x}_i}{dt} = \mathbf{F}_i = -\nabla_i E, \tag{1}$$

where η is the mobility coefficient and $\mathbf{F}_i$ is the force acting on the vertex i.

Keeping this in mind, we define a threshold length $L_t = 2\sigma_e dt/\eta$, where dt is the time step, e is a particular edge, σ_e is the correponding line tension coefficient and η is the mobility coefficient. This approximates the threshold length below which the edge will vanish in next time step. Our main assumption is that the velocity of both the vertices that comprises the edge is approximately equal to σ/η. If the length of the edge is below the threshold length, we implement a T1 transition event before the next time step that leads to cell neighbor exchange.

Let us say that the edge e, defined by two vertices i_0 and i_1, is shared by two adjacent cells C_1 and C_2 with corresponding unit normal vectors $\mathbf{N}_1$ and $\mathbf{N}_2$ (Fig. S4). The unit normal vectors are defined in the following way

$$\begin{aligned} \mathbf{A}_1 &= \frac{1}{2}\sum_{i=0}^{n-1}(\mathbf{r}_i \times \mathbf{r}_{i+1}), \\ \mathbf{N}_1 &= \frac{\mathbf{A}_1}{|\mathbf{A}_1|}, \end{aligned} \tag{2}$$

where n is the number of vertices in a cell and $\mathbf{r}_i$ is the position vector of vertex i.

We then define a direction vector $\mathbf{u}$ in the following way

$$\mathbf{u} = \mathbf{N}_e \times (\mathbf{r}_{i0} - \mathbf{r}_{i1}), \tag{3}$$

where $\mathbf{N}_e$ is the average normal vector ($\mathbf{N}_e = \frac{1}{2}(\mathbf{N}_1 + \mathbf{N}_2)$).

The direction vector is perpendicular to both the edge e and the vector that defines the average plane of adjacent cells. New edge (with length $= 1.1 * L_t$) is defined along the vector $\mathbf{u}$ with similar mid-point as the old edge e, followed by exchange of cell neighbors (Fig. S1). It should also be noted here that the vertices are free to move in three-dimensions, so each cell can be non-planar as well and the normal vector only approximates the plane of each cell.