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Supplemental Appendix for:
On the Correct Interpretation of the Hazard Ratio and Proper Communication of Survival Benefit
Andreas Sashegyi et al.

Consider survival distributions for an experimental and a control treatment, $S_E(x) = \exp(-r\theta x)$ and $S_C(x) = \exp(-\theta x)$, $x > 0$, respectively, where $\theta > 0$ is the hazard rate of the control treatment and $0 < r < 1$ is the hazard ratio. Applying Taylor series expansions to the right side of Eq. 1 and letting $p = 1 - r$ yields

$$\frac{\exp(-r\theta x) - \exp(-\theta x)}{1 - \exp(-\theta x)} = \frac{\exp(\theta x[1-r]) - 1}{\exp(\theta x) - 1}$$

$$= \frac{\theta xp + \frac{(\theta xp)^2}{2!} + \frac{(\theta xp)^3}{3!} + \cdots}{\theta x + \frac{(\theta x)^2}{2!} + \frac{(\theta x)^3}{3!} + \cdots}$$

$$= \frac{\theta xp\left[1 + \frac{\theta xp}{2!} + \frac{(\theta xp)^2}{3!} + \cdots\right]}{\theta x\left[1 + \frac{\theta x}{2!} + \frac{(\theta x)^2}{3!} + \cdots\right]}$$

$$< p = 1 - r.$$

To determine the point x_r when relative survival (ratio of control vs experimental arm) equals the hazard ratio, note that $\frac{\exp(-\theta x_r)}{\exp(-r\theta x_r)} = r$ must be satisfied. Thus

$$\theta x_r(1-r) = -\log r$$
$$\therefore x_r = \frac{1}{\theta}\left(\frac{\log r}{r-1}\right).$$

To determine the point of maximal separation of the survival curves, note that this is given by the solution to $\frac{d}{dx}(\exp(-r\theta x) - \exp(-\theta x)) = 0$. Thus,

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$$\begin{aligned}
&\theta \exp(-\theta x) = r\theta \exp(-r\theta x) \\
&-\theta x = \log r - r\theta x \\
&\theta x(r-1) = \log r \\
&\therefore x = \frac{1}{\theta}\left(\frac{\log r}{r-1}\right) = x_r
\end{aligned}$$

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