Appendix

We introduce the following notation for the bivariate distribution (3)

$$\boldsymbol{x_i} = \begin{pmatrix} y_i \\ z_i \end{pmatrix}, \qquad \boldsymbol{\lambda}_i = \begin{pmatrix} \theta_i \\ u_i \end{pmatrix}, \qquad \boldsymbol{S}_i = \begin{pmatrix} s_i^2 & \rho s_i \\ \rho s_i & 1 \end{pmatrix}$$

The joint distribution of y_i and z_i is $(y_i, z_i) = 2\frac{1}{\sqrt{2\pi|S_i|}} e^{-\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{\lambda}_i)' \boldsymbol{S}_i^{-1} (\boldsymbol{x}_i - \boldsymbol{\lambda}_i)}$. Note that $f(y_i, z_i)$ is the probability density function of a bivariate normal multiplied by 2 because the truncation constraint in the positive z_i values reduces the area of a bivariate normal by half.

We are interested in the marginal distribution of z_i, $f_{z_i}(y_i, z_i) = \int_{-\infty}^{+\infty} f(y_i, z_i) dz$.

Cartinhour found a closed form expression for the marginal distributions of a truncated multivariate normal distribution [36]. More specifically, he found that the marginal distribution is a truncated normal density multiplied by a skew function. Following his formula the marginal distribution of z_i is

$$f_{z_i}(y_i, z_i) = 2\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_i - u_i)^2} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\left|s_i^2\right|}} \; e^{-\frac{\frac{1}{2}\left(y_i - \mu_i + \rho s_i (z_i - u_i)\right)^2}{s_i^2}} dx$$

It is easily seen that the first part of the equation $(2\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_i - u_i)^2})$ refers to the probability density function of a normally distributed random variable truncated on $[0, +\infty)$ whereas the second part (induced by the a skewed distribution [36]) is the area under the curve of a normally distributed random variable on its entire sample space and is equal to one. Consequently, the marginal distribution of z_i is a truncated normal distribution, i.e. $z_i \sim N(u_i, 1) I_{z_{i>0}}$. If there were truncation constraints for y_i, the truncated normal for z_i would not hold as the integral limits would refer to the part of the sample space y_i is truncated at and the integral would not be equal to one.