A mathematical model of chronic pancreatitis

March 30, 2017

Supporting Information (SI)

Parameter estimation

A summary of all the model parameters is given in tables 1 and 2.

In the sequel, in an expression of the form $\frac{X}{K_X+X}$ in the context of activation, the half-saturation parameter K_X is taken to be the steady state of the species X provided X tends to a steady state. Hence in steady state equations this factor is equal to $\frac{1}{2}$.

It is technically hard to assess the level of cytokines in acutal pancreatic tissue. We shall assume that the concentrations in the tissue may be anywhere from 2-fold to 10-fold greater in tissue compared to serum.

Eqn. (1)

- P_0 : The ratio of cells volume to ECM volume varies from less than 1:10 to more than 10:1 [7]. Based on [36], we estimate the ratio between cells volume and ECM in the pancreatic islets to be close to 1:1. However, taken over the entire organ, the ratio of cells volume to ECM volume is smaller, and we take it to be 1:10. The mass of PSC is 4-7% of the total cells mass in the pancreas [4]; we take it to be 5%. Assuming average density of 1 g/ml in the pancreas, we get $P_0 =$ density of PSC=5 × 10⁻³ g/ml.
- $K_{T_{\alpha}}$: There are several reports on the level of TNF- α in serum of patients with chronic pancreatitis (CP) and in healthy control case (C). In [18], the level is 9.88 pg/ml in CP and 10.09 in C; in [24] it is 23 pg/ml in CP and 13 pg/ml, in [36] it is 8.52 pg/ml in C, and in our own clinical tests (see Supplementary Material (SM) it was 8.54 pg/ml in PC and 4.58 pg/ml in C. We assume that the concentration of TNF- α in tissue is larger than in serum and take T_{α} steady state in blood of heathy individuals to be 9 pg/ml, and $T_{\alpha} = K_{T_{\alpha}} = 30$ pg/ml=3 × 10⁻¹¹ g/ml in tissue.
- K_G : The level of serum concentration of healthy individual was reported in [5] to be 17.5 pg/ml. Our clinical data (in SM) show concentrations of

 0.102 ng/ml in CP and 0.363 ng/ml in C. We take larger tissue concentration in healthy steady state, namely, $G = K_G = 60$ ng/ml=6 × 10⁻⁸ g/ml in tissue.

- $K_{T_{\beta}}$: In our clinical tests we found that the level of TGF- β in blood was 6.602 ng/ml for CP and 7.736 ng/ml for C. On the other hand, a larger concentration of 36.75 ng/ml was reported for C in [36]. We assume that concentration of TGF- β in tissue of healthy individuals is larger than in blood, and take $T_\beta = K_{T_\beta} = 8 \times 10^{-7}$ g/ml.
- K_{I_6} : There are several report on the level of IL-6 in CP and C. In [24] it was reported to be 7.3 pg/ml in CP and 3.3 pg/ml in C; in [36] it was reported to be 0.58 pg/ml in C, while our clinical tests give 9.16 pg/ml in CP and 7.14 pg/ml in C. We take the level of IL-6 in healthy tissue to be $I_6 = K_{I_6} = 8$ pg/ml = 8 × 10⁻¹² g/ml.
- λ_{PT_α} and λ_{PI_6} :

Experiments in vivo in [2, 23] reported on the activation of PSC by cytokines TGF-β, TNF- α and IL-6. We assume that TGF-β activates PSC more effectively than TNF- α , but less effectively than IL-6, and take $\lambda_{PT_{\beta}} = 2.2 \times 10^{-2} / \text{day}$, $\lambda_{PT_{\alpha}} = 1.82 \times 10^{-2} / \text{day}$ and $\lambda_{PI_{6}} =$ $3.67 \times 10^{-2} / \text{day}.$

• λ_P and λ_{PG} : Experiments in [3] show that PDGF, and to lesser extend TNF- α , increase the proliferation of APSC. We take $\lambda_P = 4 \times 10^{-3} / \text{day}$, and $\lambda_{PG} = 2.7 \times 10^{-3}$ /day.

Eqn. (2)

• A_{P_0} : From the steady state of PSC for a healthy pancreas, we have $A_{P_0} =$ $d_{P_0}P_0$. Taking $P_0 = 5 \times 10^{-3}$ g/ml and using the value $d_{P_0} = 1.66 \times 10^{-2}$ day⁻¹ [33], we get $A_{P_0} = 8.3 \times 10^{-5}$ g/cm³ day⁻¹.

Eqn. (3)

• d_{CM} : The degradation rate of MCP-1 is $d_C = 1.73 \text{ day}^{-1}$ [6]. MCP-1 chemoattracts macrophages, so some of the cytokines get internalized by macrophage [21, 27]. We assume that the rate of internalization is the same as the rate of degradation when C is at half-saturation

$$
d_C K_C = d_{CM} \frac{K_C}{K_C + K_C} M_0.
$$

and $M_0 = 5 \times 10^{-5}$ [10], we get $d_{CM} = \frac{2d_C K_C}{M_0} = 2.08 \times 10^{-5}$ day⁻¹.

• K_C : In our clinical tests (SM) we found that the serum concentration of MCP-1 in CP is 54.66 pg/ml and in C is 58.89 pg/ml. We assume a larger concentration of MCP-1 in tissue of healthy individuals and, accordingly, take $K_C = 300 \text{pg/ml} = 3 \times 10^{-10} \text{ g/ml}.$

• λ_{CP} : According to the experiments of MCP-1 production by TNF- α in [30], we have the following linear relation

 $\lambda_{CP} P - d_C C = 0.$

More precisely, the concentration of MCP-1 is 1.8 ng/ml when TNF- α is 100 ng/ml. Assuming that in this experiment the density of P exceeded its half-saturation K_P , we take $\lambda_{CP} = 2 \times 10^{-7}$ day⁻¹.

Eqn. (6)

• $\lambda_{\rho T_{\beta}}$: We take $\lambda_{\rho P} = 0.0432 \text{ day}^{-1}$. It was reported in [3] that collagen synthesis with TGF- $\beta = 1$ ng/ml, was increased to three fold.

We assume that with such a small amount of T_β (compared to K_{T_β}) the production would increase only 5% the collagen in the pancreas, so that

$$
\lambda_{\rho T_{\beta}} \frac{1}{20} = 3\lambda_{\rho P},\tag{1}
$$

or $\lambda_{\rho T_{\beta}} = 3\lambda_{\rho P} = 2.673 \text{ day}^{-1}$.

• ρ^* : The steady states of ρ^* , Q^* and Q_r^* of ρ , Q and Q_r are determined by solving the following steady state equations in healthy pancreas

$$
\label{eq:1D1V:0} \left\{ \begin{aligned} &\lambda_{\rho P}P_0\big(1-\frac{\rho}{\rho_0}\big)-d_{\rho Q}Q\rho-d_{\rho}\rho=0,\\ &\lambda_{QP_0}P_0-d_{QQ_r}Q_rQ-d_{Q}Q=0,\\ &\lambda_{Q_rP_0}P_0-d_{Q_rQ}QQ_r-d_{Q_r}Q_r=0. \end{aligned} \right.
$$

where all the parameters are given in Table 1 and 2. By direct computation, we find that $\rho^* = 3.22 \times 10^{-4}$ g/cm³, $Q^* = 4.18 \times 10^{-6}$ g/cm³ and $Q_r^* = 4.24 \times 10^{-11} \text{ g/cm}^3.$

Eqn. (7)

In steady state in health, $\lambda_{T_\beta} = \frac{d_{T_\beta} T_\beta}{P}$ $\frac{\beta^{1\beta}}{P}$, where $d_{T_{\beta}} = 3.33 \times 10^2/\text{day}$ [31], $T_{\beta} =$ 8×10^{-7} g/ml and $P = P_0 = 5 \times 10^{-3}$ g/ml, so that $\frac{d_{T_\beta} T_\beta}{P} = 5.328 \times 10^{-1} / \text{day}$. P is proliferating, so in steady state P is larger than P_0 and hence λ_{T_β} should be smaller; we take $\lambda_{T_{\beta}} = 6.7 \times 10^{-2} / \text{day}$.

Eqn. (8)

In steady state in health

$$
\lambda_{T_{\alpha}} = \frac{d_{T_{\alpha}} T_{\alpha}}{M},
$$

where $d_{T_{\alpha}} = 55.45/\text{day}$ [25], $T_{\alpha} = 3 \times 10^{-11} \text{g/ml}$, and $M = M_0 = 5 \times 10^{-5} \text{g/ml}$ [10], so that $\frac{d_{T_{\alpha}} T_{\alpha}}{M} = 3.3 \times 10^{-5} / \text{day}$. In chronic pancreatic case the concentration of T_{α} is higher, so we take $\lambda_{T_{\alpha}} = 9.98 \times 10^{-5} / \text{day}$.

Eqn. (9)

• λ_{I_6} : In steady state in health, $\lambda_{I_6} = \frac{d_{I_6} I_6}{P}$ where $d_{I_6} = 0.173/\text{day}$ [19], $I_6 = 4 \times 10^{-12}$ g/ml and $P = P_0 = 5 \times 10^{-3}$ so that $\frac{d_{I_6} I_6}{P} = 1.4 \times 10^{-10}$ /day. In chronic pancreatic case P is larger than P_0 , so we take the smaller value $\lambda_{I_6} = 7 \times 10^{-11} / \text{day}.$

Eqn. (10)

• λ_G : In steady state in health, $\lambda_G = \frac{d_G G}{P}$ where $d_G = 2.84/\text{day}$ [34], $G = 6 \times 10^{-8}$ g/ml and $P = P_0 = 5 \times 10^{-3}$ g/ml. Hence $\frac{d_G G}{P} = 4 \times$ $10^{-5}/\text{day}$. For the chronic pancreatitis case P is larger than P_0 , so we take $\lambda_G = 2 \times 10^{-7}$ day⁻¹.

Eqn. (11)

• λ_{QP_0} and λ_{QP} : We take $\lambda_{QP_0} = 3.025 \times 10^{-5}$ day⁻¹ as in [16]. In [26], it was reported that the amount of MMP activated by P is 30% more than that by P_0 . Accordingly, we have

$$
\lambda_{QP} = 1.3 \lambda_{QP_0} = 3.93 \times 10^{-5} \text{ day}^{-1}.
$$

• λ_{QT_β} and λ_{QI_6} : According to [26], T_β increases the production of Q by P more than I_6 does, we assume that T_β increases this production by approximately 1-fold and I_6 increases it by approximately $\frac{1}{2}$ -fold, and take $\lambda_{QT_\beta} = 7.6 \times 10^{-5} \text{ day}^{-1}$. and $\lambda_{QI_6} = 3.72 \times 10^{-5} \text{ day}^{-1}$.

Clinical data

Peripheral venous blood was collected into sodium heparin tubes by venipuncture from human patients. All studies were conducted under an IRB-approved protocol from patients with clinically confirmed chronic calcific pancreatitis (17 patients), non-calcific pancreatitis (9), a family history of pancreatic disease (non-pancreatitis) (12), and no disease(7). Blood samples were centrifuged at 1200 x g for 10 minutes to obtain plasma, and was stored at -80 $^{\circ}C$. Plasma samples were batch analyzed using a custom Luminex Multiplex Cytokine Kits (Procarta Cytokine Assay Kit, Affymetrix). Analyte concentrations were calculated based on a standard curve for each analyte and represent the average of two batched duplicates. Additional single-plex ELISA kits were used to analyze the concentration of additional factors, including $TGF-\beta$ and IL-6 (R&D) Systems, Inc.), as per manufacturer instructions.

1 Tables

References

[1] D. G. Alleva, C. J. Burger, and K. D. Elgert. Tumor-induced regulation of suppressor macrophage nitric oxide and TNF-alpha production. Role

Parameter	Description	Value
d_{P}	degradation rate of activated PSC	4.15×10^{-2} day ⁻¹ [33]
d_{P_0}	degradation rate of quiescent PSC	1.66×10^{-2} day ⁻¹ [33]
d_{M_1}	death rate of M1 macrophage	$0.02 \mathrm{day}^{-1}$ [8]
d_{M_2}	death rate of M2 macrophage	0.015 day^{-1} [9]
d_C	degradation rate of MCP-1	$1.73 \mathrm{day}^{-1}$ [6]
d_{CM_1}	degradation rate of MCP-1 due to macrophage	2.08×10^{-5} day $^{-1}$ [6, 21, 27] & estimated
d_{I_6}	degradation rate of IL-6	0.173 day^{-1} [19]
$d_{T_{\beta}}$	degradation rate of TGF- β	3.33×10^2 day ⁻¹ [31]
$d_{T_{\alpha}}$	degradation rate of TNF- α	55.45 day^{-1} [25]
d_{ρ}	degradation rate of ECM	$0.37 \mathrm{day}^{-1}$ [34]
\mathfrak{d}_G	degradation rate of PDGF	3.84 day ⁻¹ [34]
d_{QQ_r}	binding rate of MMP to TIMP	4.98×10^8 $cm^3 g^{-1}$ day ⁻¹ [10]
d_{Q_rQ}	binding rate of TIMP to MMP	1.04×10^9 cm^3g^{-1} day ⁻¹ [10]
d_Q	degradation rate of MMP	$4.32 \text{ day}^{-1}[10, 15]$
d_{Q_r}	degradation rate of TIMP	21.6 day^{-1} [10, 35]
$d_{\rho Q}$	degradation rate of ECM due to MMP	2.59×10^7 cm^3g^{-1} day ⁻¹ [10, 16]
χ_G	chemotactic sensitivity parameter	$\frac{10 \text{ cm}^5 g^{-1} \text{ day}^{-1} [15, 16]}{2}$
χ_C	chemotactic sensitivity parameter	$10 \ cm^5 g^{-1}$ day ⁻¹ [15, 16]
A_{P_0}	PSC source	8.3×10^{-5} g/cm ³ day ⁻¹ estimated
K_{I_6}	IL-6 half saturation	8×10^{-12} g/cm^3 [29, 24, 36], SM* $\&$ estimated
$K_{T_{\alpha}}$	TNF- α half saturation	3×10^{-11} g/cm ³ [18, 36], SM [*] & estimated
$K_{T_{\beta}}$	TGF- β half saturation	8×10^{-7} g/cm ³ [36], SM [*] & estimated
K_G	PDGF half saturation	6×10^{-8} g/cm^3 [5], SM* & estimated
K_C	MCP-1 half saturation	3×10^{-10} g/cm^3 SM* $\&$ estimated
K_P	PSC half saturation	3×10^{-3} g/cm^3 [14] & estimated
P_0	inactive PSC density	$5\times10^{-3}~g/cm^3$
M_0	source of macrophages	$5\times10^{-5}~g/cm^3$
ρ_0	ECM density	10^{-3} gcm ⁻³ [17]
ρ^*	ECM density	3.22×10^{-4} gcm^{-3} estimated
Q_r^* Q_r^*	MMP density	4.18×10^{-6} gcm^{-3} estimated
	TIMP density	4.24×10^{-11} gcm^{-3} estimated
	influx rate	1

Table 2: Parameters' description and value

SM[∗] : data from supplementary material.

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