Supplementary Materials for "Testing a Single Regression Coefficient in High Dimensional Linear Models"

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Abstract

This supplemental material includes two sections. Section 1 presents simulation results for less sparse regression models. Specifically, Tables 1 and 2 report simulation results for Example 1, which contains 50 and 100 non-zero coefficients, respectively. In addition, Tables 3 and 4 report simulation results for Example 2, while Tables 5 and 6 report simulation results for Example 3. Section 2 presents simulation results for the predictors, which follow a polynomial decay structure.

KEY WORDS: Less Sparse Models; Simulation Results

Section 1: Simulation Results For Less Sparse Models

In this section, we present simulation results for Examples 1–3 under less sparse scenarios (i.e., $d_0 = 50$ and 100). Specifically, Tables 1–2, Tables 3–4, and Tables 5–6 report simulation results for Example 1, Example 2, and Example 3, respectively.

Table S1: Simulation results for Example 1 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 50$.

p	n	Methods	ES	EP	FDR	TR	FR
1000	100	MUR	0.054	0.347	0.684	0.318	0.002
		LDPE	0.148	0.289	0.327	0.260	0.015
		CPS	0.056	0.182	0.106	0.177	0.000
	200	MUR	0.052	0.526	0.593	0.618	0.006
		LDPE	0.127	0.442	0.285	0.484	0.011
		CPS	0.051	0.329	0.086	0.358	0.000
	500	MUR	0.053	0.831	0.605	0.853	0.015
		LDPE	0.083	0.634	0.226	0.695	0.007
		CPS	0.052	0.473	0.063	0.508	0.000
2000	100	MUR	0.058	0.285	0.764	0.264	0.005
		LDPE	0.183	0.207	0.418	0.229	0.032
		CPS	0.054	0.138	0.148	0.126	0.000
	200	MUR	0.053	0.476	0.638	0.507	0.015
		LDPE	0.144	0.418	0.322	0.451	0.022
		CPS	0.054	0.260	0.116	0.303	0.000
	500	MUR	0.051	0.754	0.592	0.792	0.033
		LDPE	0.090	0.549	0.253	0.553	0.016
		CPS	0.051	0.437	0.077	0.451	0.000

Table S2: Simulation results for Example 1 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 100$.

p	n	Methods	ES	EP	FDR	TR	FR
1000	100	MUR	0.056	0.277	0.613	0.294	0.008
		LDPE	0.184	0.228	0.286	0.253	0.022
		CPS	0.054	0.098	0.125	0.137	0.000
	200	MUR	0.053	0.459	0.586	0.476	0.015
		LDPE	0.143	0.392	0.223	0.416	0.009
		CPS	0.054	0.258	0.094	0.274	0.000
	500	MUR	0.053	0.774	0.558	0.753	0.020
		LDPE	0.108	0.510	0.182	0.547	0.004
		CPS	0.054	0.418	0.080	0.452	0.000
2000	100	MUR	0.057	0.243	0.774	0.259	0.009
		LDPE	0.201	0.195	0.506	0.201	0.025
		CPS	0.055	0.083	0.152	0.113	0.000
	200	MUR	0.053	0.422	0.679	0.451	0.014
		LDPE	0.151	0.361	0.331	0.380	0.016
		CPS	0.053	0.227	0.117	0.254	0.000
	500	MUR	0.052	0.707	0.624	0.731	0.018
		LDPE	0.120	0.468	0.206	0.509	0.005
		CPS	0.052	0.382	0.084	0.411	0.000

Table S3: Simulation results for Example 2 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 50$.

p	n	Test	ES	EP	FDR	TR	FR
1000	100	MUR	0.057	0.509	0.339	0.522	0.008
		LDPE	0.177	0.423	0.206	0.458	0.027
		CPS	0.055	0.246	0.127	0.266	0.000
	200	MUR	0.055	0.722	0.341	0.749	0.011
		LDPE	0.139	0.613	0.153	0.653	0.016
		CPS	0.052	0.405	0.118	0.418	0.000
	500	MUR	0.051	0.906	0.332	0.925	0.017
		LDPE	0.092	0.789	0.128	0.805	0.008
		CPS	0.050	0.661	0.077	0.690	0.000
2000	100	MUR	0.059	0.439	0.426	0.458	0.010
		LDPE	0.193	0.366	0.228	0.332	0.031
		CPS	0.056	0.139	0.130	0.182	0.000
	200	MUR	0.054	0.665	0.399	0.689	0.015
		LDPE	0.144	0.548	0.177	0.574	0.018
		CPS	0.052	0.339	0.123	0.327	0.000
	500	MUR	0.051	0.813	0.342	0.852	0.020
		LDPE	0.095	0.706	0.130	0.735	0.009
		CPS	0.050	0.612	0.082	0.651	0.000

Table S4: Simulation results for Example 2 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 100$.

p	n	Test	ES	EP	FDR	TR	FR
1000	100	MUR	0.058	0.442	0.397	0.461	0.008
		LDPE	0.203	0.365	0.254	0.368	0.030
		CPS	0.054	0.144	0.139	0.182	0.000
	200	MUR	0.056	0.661	0.338	0.685	0.012
		LDPE	0.145	0.504	0.196	0.539	0.015
		CPS	0.051	0.301	0.106	0.344	0.000
	500	MUR	0.054	0.852	0.299	0.863	0.015
		LDPE	0.103	0.707	0.156	0.734	0.009
		CPS	0.050	0.584	0.082	0.611	0.000
2000	100	MUR	0.059	0.368	0.411	0.403	0.011
		LDPE	0.227	0.301	0.276	0.312	0.032
		CPS	0.055	0.118	0.146	0.154	0.000
	200	MUR	0.055	0.581	0.354	0.602	0.016
		LDPE	0.148	0.429	0.205	0.451	0.019
		CPS	0.053	0.266	0.119	0.289	0.000
	500	MUR	0.052	0.813	0.312	0.829	0.023
		LDPE	0.099	0.619	0.157	0.657	0.009
		CPS	0.051	0.551	0.089	0.561	0.000

Table S5: Simulation results for Example 3 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 50$.

p	n	Test	ES	EP	FDR	TR	FR
1000	100	MUR	1.000	1.000	0.999	1.000	1.000
		LDPE	0.138	0.438	0.307	0.456	0.009
		CPS	0.058	0.297	0.138	0.311	0.000
	200	MUR	1.000	1.000	0.999	1.000	1.000
		LDPE	0.107	0.731	0.267	0.754	0.006
		CPS	0.050	0.502	0.112	0.532	0.000
	500	MUR	1.000	1.000	0.999	1.000	1.000
		LDPE	0.091	0.903	0.185	0.933	0.003
		CPS	0.051	0.722	0.076	0.774	0.000
2000	100	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.148	0.387	0.284	0.405	0.008
		CPS	0.057	0.229	0.129	0.251	0.000
	200	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.119	0.643	0.218	0.650	0.004
		CPS	0.056	0.459	0.097	0.477	0.000
	500	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.095	0.822	0.172	0.851	0.002
		CPS	0.051	0.684	0.069	0.755	0.000

Table S6: Simulation results for Example 3 with $\alpha = 5\%$, $q = 5\%$ and $d_0 = 100$.

p	n	Test	ES	EP	FDR	TR	FR
1000	100	MUR	1.000	1.000	0.998	1.000	1.000
		LDPE	0.145	0.378	0.326	0.403	0.012
		CPS	0.059	0.229	0.129	0.258	0.000
	200	MUR	1.000	1.000	0.998	1.000	1.000
		LDPE	0.127	0.619	0.249	0.652	0.008
		CPS	0.051	0.477	0.112	0.502	0.000
	500	MUR	1.000	1.000	0.998	1.000	1.000
		LDPE	0.102	0.811	0.202	0.792	0.005
		CPS	0.052	0.674	0.072	0.706	0.000
2000	100	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.159	0.329	0.287	0.342	0.009
		CPS	0.055	0.175	0.130	0.207	0.000
	200	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.134	0.542	0.238	0.570	0.005
		CPS	0.057	0.408	0.103	0.428	0.000
	500	MUR	1.000	1.000	0.997	1.000	1.000
		LDPE	0.092	0.705	0.168	0.734	0.003
		CPS	0.050	0.599	0.070	0.633	0.000

Section 2: Simulation Results for Example 1 with Predictors exhibiting Polynomial Decay

Consider a linear regression model with predictors X_i being generated from a multivariate normal distribution with mean 0 and covariance $\Sigma = I_p + uu^\top$, where $u = (u_1, \cdots, u_p)^\top \in \mathbb{R}^p$, $u_j = \delta j^{-2}$ for $j = 1, \cdots, p$, and δ is a finite constant. The regression coefficients β, the number of non-zero coefficients d_0, and the variance of ε_i (σ_i^2), are the same as those in Example 1 of the manuscript. Since the simulation results for $d_0 = 10$, 50 and 100 and various δ values yield a similar pattern, we only present the results for $d_0 = 10$ and $\delta = 1$ in Table S7 given below.

Table S7: Simulation results for Example 1 when the predictors exhibit polynomial decay and $\alpha = 5\%$, $q = 5\%$, $\delta = 1$ and $d_0 = 10$.

p	n	Methods	ES	EP	FDR	TR	FR
1000	100	MUR	0.057	0.893	0.426	0.906	0.003
		LDPE	0.069	0.795	0.285	0.764	0.030
		CPS	0.054	0.431	0.094	0.429	0.000
	200	MUR	0.055	1.000	0.392	1.000	0.010
		LDPE	0.058	0.905	0.263	0.882	0.009
		CPS	0.052	0.729	0.076	0.706	0.000
	500	MUR	0.051	1.000	0.325	1.000	0.016
		LDPE	0.052	1.000	0.143	1.000	0.002
		CPS	0.055	0.955	0.056	0.982	0.000
2000	100	MUR	0.059	0.794	0.524	0.802	0.006
		LDPE	0.072	0.622	0.347	0.643	0.035
		CPS	0.052	0.385	0.102	0.379	0.000
	200	MUR	0.051	1.000	0.462	1.000	0.018
		LDPE	0.059	0.853	0.261	0.821	0.010
		CPS	0.055	0.682	0.082	0.652	0.000
	500	MUR	0.054	1.000	0.452	1.000	0.025
		LDPE	0.053	1.000	0.155	1.000	0.004
		CPS	0.056	0.902	0.056	0.911	0.000