

1 **New insights on the complex dynamics of two-phase flow in porous media under**
2 **intermediate-wet conditions**

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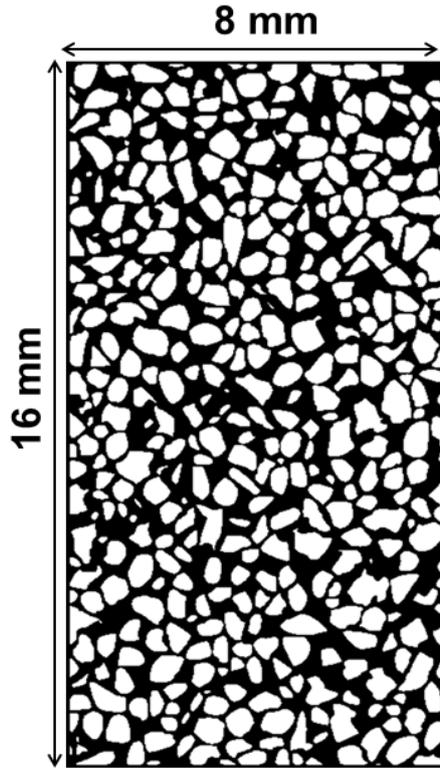
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17 **Supplementary information**

18 The porous medium used for numerical modelling in the present study is shown in Fig. S1.



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20 **Figure S1.** The porous medium (numerical domain) used for simulation of immiscible
21 displacement under different wettability conditions. Black and white colours represent void
22 and solid phase, respectively. The porous medium is based on the pore-scale image obtained
23 by X-ray tomography of a sand pack³².

24 In this research direct numerical simulation was performed using C++ library called
25 OpenFoam (Open Field Operation and Manipulation).The pore scale dynamics of immiscible
26 displacement is governed by mass and momentum equation, as follows³⁰;

27 Mass balance

$$\nabla \cdot \mathbf{u} = 0 \tag{1.1}$$

28 Momentum balance

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)) + f_{sa} \quad (1.2)$$

29 where \mathbf{u} is the velocity vector (m s^{-1}), ρ is the density (kg m^{-3}), p is the pressure ($\text{kg m}^{-1} \text{s}^{-2}$),

30 μ is the viscosity ($\text{kg m}^{-1} \text{s}^{-1}$) and f_{sa} is the capillary force ($\text{kg m}^{-2} \text{s}^{-2}$) acting as a source term.

31 Volume of fluid (VOF) was employed as an interface capturing approach. It defines the

32 distribution of phases using volume indicator function γ expressed as;

$$\gamma \in \begin{cases} (0,1) & \text{Interface;} \\ (1) & \text{Defending fluid;} \\ (0) & \text{Invading fluid;} \end{cases}$$

33 The volume indicator function evolves according to advection transport as shown in Eq.

34 (1.3):

$$\frac{\partial \gamma}{\partial t} + \nabla \cdot (\gamma \mathbf{u}) + \nabla \cdot (\gamma (1 - \gamma) \mathbf{u}_r) = 0 \quad (1.3)$$

35 where \mathbf{u}_r is the relative velocity between two fluids (m s^{-1}), also known as “compression

36 velocity”. In each computational cell, ρ and μ is a linear function of γ and varies according to

37 phase present.

$$\rho = \gamma \rho_i + (1 - \gamma) \rho_d \quad (1.4)$$

$$\mu = \gamma \mu_i + (1 - \gamma) \mu_d \quad (1.5)$$

38 In Eq. 1.4 and 1.5, ρ_i is the density of invading phase (kg m^{-3}), ρ_d is the density of defending

39 phase (kg m^{-3}), μ_i refers to viscosity of invading phase ($\text{kg m}^{-1} \text{s}^{-1}$) and μ_d is the viscosity of

40 defending phase ($\text{kg m}^{-1} \text{s}^{-1}$). The term f_{sa} was described using Continuum Surface Force

41 (CSF) method³⁰ as follows;

$$f_{sa} = \sigma_{12}\kappa\nabla\gamma \delta_s \quad (1.6)$$

42 where δ_s is the Dirac Function and κ is the curvature of the interface (m^{-1}) computed as;

$$\kappa = \nabla \cdot \left(\frac{\nabla\gamma}{|\nabla\gamma|} \right) \quad (1.7)$$

43 f_{sa} is only active at the interface and vanishes at limits of γ . All equations are discretized at
 44 each computational cell using finite volume method and solved numerically to obtain
 45 variables p, u and γ .

46 The post-processing of numerical results was conducted with Paraview; an open-source
 47 visualization application. For curvature calculation shown in Fig. 1, the contour map of
 48 interface with $\gamma=0.5$ is extracted. Then the built-in Paraview plugin was used to compute
 49 distribution of curvature at each interface. Finally, the curvature of whole interface was
 50 represented by the median value.

51 **Table S1.** Physical description of the porous medium as given in Norouzi Rad et al.³²

Grain size range	Average grain size	Average pore size	Porosity (-)
(mm)	(mm)	(mm)	
0.36-0.77	0.60	0.16	0.37

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