Mathematical constraints on F_{ST}: biallelic markers in arbitrarily many populations SUPPLEMENTARY FILE S2: MS commands

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Conditioning on one segregating site

For $K = 2$

```
./ms 200 100000 -s 1 -I 2 100 100 x #replace x by the desired 4Nm value
```

For $K = 7$

```
./ms 700 100000 -s 1 -I 7 100 100 100 100 100 100 100 x #island; replace x by 4Nm
./ms 700 100000 -s 1 -I 7 100 100 100 100 100 100 100 -ma 0 x/2 x/2 0 0 0 0 x/2 0 0 x/2 0 0 0 x/3 0 0 x/3 x/3
    0 0 0 x/3 x/3 0 0 x/3 0 0 0 x/3 0 0 x/3 x/3 0 0 0 x/2 x/2 0 0 0 0 0 0 x 0 0 #rectangular stepping-stone
./ms 700 100000 -s 1 -I 7 100 100 100 100 100 100 100 -ma 0 x 0 0 0 0 0 x/2 0 x/2 0 0     0 0 0 x/2 0 x/2 0 0
    0 0 0 x/2 0 x/2 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 x 0 #linear stepping-stone
```

For $K = 40$

```
./ms 4000 100000 -s 1 -I 40 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
    100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 x  #island; replace x by 4Nm
./ms 4000 100000 -s 1 -I 40 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
    100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 -ma 0 x/2 0 0 0 0 0 0 x/2 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3
    0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 x/3
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 0 0 0 0 0 0 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0
    0 x/4 0 x/4 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0
    0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0
    0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0
    0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4
    0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0
    x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0
    0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4
    0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/4 0 0 0 0 0 0 x/4 0 x/4 0 0 0 0 0 0 x/4 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 0 0
    0 0 0 0 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 x/3 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3
    0 0 0 0 0 0 x/3 0 x/3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/3 0 0 0 0 0 0 x/3 0 x/3
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 0 0 0 0 0 x/2 0 #rect. stepping-stone
```

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```
./ms 4000 100000 -s 1 -I 40 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
    100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 -ma 0 x 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2
    0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0
    x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
    0 0 0 0 0 0 0 x/2 0 x/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x/2 0 x/2
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 x 0 #linear stepping-stone
```

Using an infinitely-many-sites model under island migration

For $K = 2$

```
./ms 200 100000 -t 0.08513492 -I 2 100 100 x #replace x by 4Nm; 0.08513492 is the local mutation rate theta/2
```

For $K = 7$

```
./ms 700 100000 -t 0.02004286 -I 7 100 100 100 100 100 100 100 x #0.02004286 is the local mutation rate theta/7
```

For $K = 40$

```
./ms 4000 100000 -t 0.002818127 -I 40 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100
    100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 100 x #0.002818127 is theta/40
```