Appendix A: The Estimation of Cumulative Incidence Function

Let us consider a competing risk time-to-event study with *N* independent observations concerning the situation where multiple causes of failure (*K*) are possible. Let w_i be the inverse probability (IP) weight for the i^{th} subject ($i = 1, 2, \ldots, N$). Let $0 < t_1 < t_2 < \ldots < t_j$ denote the ordered distinct event time points at which failures of any cause occur. Let d_{kj} denote the number of patients failing from cause *k* ($k = 1, 2, \ldots, K$) at t_J and R_{kj} be the corresponding set of individuals. Let $d_j = \sum_{k=1}^{K} d_{kj}$ denote the total number of failures from any cause. Let n_j be the number of individuals at risk at time t_j and R_j be the corresponding set of individuals. Thus, the weighted number of events failing from cause *k* is $d_{kj}^W = \sum_{i \in R_{kj}} w_i$, the weighted total number of failures from any cause is $d_j^W = \sum_{k=1}^{K} d_{kj}^W$, and the weighted number of individuals at risk is $n_j^W = \sum_{i \in R_j} w_i$ at time t_j.

The overall unweighted survival probability $S(x)$, can be estimated by the Kaplan-Meier estimator as $\hat{S}(x) = \prod_{j:\, t_j \le x} \left(1 - \frac{d_j}{n_j}\right) = \prod_{j:\, t_j \le x} \left(1 - \sum_{k=1}^{K} \widehat{\lambda_k}\ (t_j)\right)$, where the unweighted cause specific hazard function $\widehat{\lambda_k}\left(t_j\right) = {}^{d_{kj}}\!/_{n_j}$. The unweighted cumulative incidence $I_k(x)$ of cause *k* at time *x* is estimated $\widehat{I_k}(x) = \sum_{j:\, t_j \le x} \widehat{\lambda_k}\, (t_j)\, \hat{S}\, (t_{j-1})$.

The overall inverse probability of treatment (IPT)-weighted survival probability $S^W(x)$, can be estimated by the weighted Kaplan-Meier estimator as $\widehat{S^W}(x) = \prod_{j:\, t_j \le x} \left(1 - \frac{d_j^W}{n_j^W}\right) = \prod_{j:\, t_j \le x} \left(1 - \sum_{k=1}^{K} \widehat{\lambda_j^W}\ (t_j)\right)$, where the IPT-weighted cause specific hazard function $\widehat{\lambda_k^W}\left(t_j\right) = {}^{d_{kj}^W}\!/_{n_j^W}$. The weighted cumulative incidence $I_k^W\,(x)$ of cause *k* at time *x* is estimated $\widehat{I_k^W}(x) = \sum_{j:\, t_j \le x} \widehat{\lambda_k^W}\, (t_j)\, \widehat{S^W}\, (t_{j-1})$.