S1 Likelihood-based inference framework

The weekly reported mortality cases are obtained as

$$Z_i = \int_{t_i}^{t_{i+1}} \phi \gamma_{h1} I_{h1} dt \tag{1}$$

where ϕ denotes the case-fatality ratio. We assume that the weekly observed mortality cases C_i is a random sample from a Negative-binomial (NB) distribution

$$C_i \sim \text{NB}\left(n = \frac{1}{\tau}, p = \frac{1}{1 + Z_i \tau}\right) \tag{2}$$

where n and p denote the size and probability of the NB distribution, and τ denotes an over-dispersion parameter which will be estimated.

The following likelihood function is for one administrative unit

$$L(\theta_1 | C_{1,\ldots,N}) = \prod_{i=1}^{N} l_i \tag{3}$$

where θ_1 denotes the parameter vector, l_i is the weekly density associated with C_i and Z_i, and N denotes the number of weeks. This function is then applied to three cities: London boroughs, Birmingham and Liverpool, and the overall likelihood is

$$L_{\text{overall}} = \prod_{j=1}^{3} L_j \tag{4}$$

We use iterated filtering based plug-and-play inference framework [1–7] to estimate θ_1 via maximizing L_{overall}.

References

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