SOME ARITHMETICAL CONSIDERATIONS ON THE PROGRESS OF EPIDEMICS.

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In 1866, during the height of the ravages of the cattle plague, a statement made by Mr. Lowe in the House to the effect that there was no reason why the terrible law of increase which had prevailed hitherto should not prevail henceforth was controverted by Dr. Farr, in a letter to a daily paper, in which he showed that the real law of this epidemic implied that the ratio of increase went on rapidly decreasing, until the ratio itself was a decreasing one. Applying the law of gradually decreasing increase obtained from the earlier weeks of the epidemic, he calculated its probable course, which, as will be seen from Plate I, very nearly resembled its actual course. Now, it seemed to me, that it would be very interesting to ascertain if by the use of this method we could learn something more about other epidemics, if, in fact, epidemics usually followed this course of gradually decreasing increase, and if we could at any early part of their course, predicate the period at about which they would culminate, and the rate of their subsequent decline. With this view I have endeavoured to investigate a few epidemics of which the progress is known, and to ascertain whether they followed a similar course to that taken by the cattle-plague, and whether this method of calculating the probable course were applicable also to them.

Perhaps it will be as well, in the first place, to describe as briefly as possible the method of calculation which I have received at second hand, through a member of the society, who had obtained it from Dr. Farr. I will take the instance of the cattle plague. Take nine weeks of the early course of the epidemic, in three groups of three weeks each: find the deaths per week in each group: find the number by which

you must multiply the first average to obtain the second, and the number by which you must multiply the second average to obtain the third : or, as a simpler process, take the difference between the logarithms of the first and second averages, and between the logarithms of the second and third averages. The first of these differences we will call δ^1, and the difference between these two differences, which should, to bear out this theory, be a negative quantity, we will call δ^2. We have now the data for constructing our series. The average of the first three weeks is our starting-point, and represents the centre week of those three. The next number in the series is obtained by adding to the logarithm of our first number, a number composed of $\frac{\delta 1}{3}-\frac{\delta 2}{9}$, remembering that δ^2 is a negative quantity; we continue to add to the logarithm for each place in the series a number gradually diminished by the addition in each place of $\frac{\delta 2}{9}$: after a time the number to be added becomes negative, and our series gradually diminishes. It is obvious that this method of calculation will not produce a gradually diminishing series unless δ^2 be a negative quantity, that is, unless the data from which you calculate the probable course of an epidemic be taken from a period at which the epidemic is advancing with a gradually decreasing rate of increase. Now the question is, can we generally find, early in the course of an epidemic, data for calculating its probable course? I will give you now the instances in which I have endeavoured to apply this method. I have selected, as far as possible, such epidemics as are but little affected by sanitary measures, most of my instances being from the weekly returns of deaths in London from scarlet fever.

I will begin, however, with the cattle plague. It will be seen from Plate I, that a calculation based upon data obtained from nine weeks' reported attacks ending December 30, 1865, produced a curve for the epidemic which reached its acme in the fourth and fifth weeks of 1866, and then gradually declined. Now the largest number of reported cases appeared in the sixth week of 1866, and the real decline of the epidemic was more rapid than the calculated decline, a result very reasonably attributed by Dr. Farr to the effect of slaughtering in the latter weeks of the epidemic. On the whole, the epidemic seems to have followed the course indicated by its commencement. I may remark that the calculation could not include the first week of reported attacks, ending October 25, as in that case δ^2 would

have been a positive quantity, and the calculated series would have gone on increasing indefinitely. Supposing, on the other hand, that the calculation had been commenced with the week ending November 11, a very different curve would have been obtained (indicated in blue ink on the Plate), arriving at its summit in the week ending January 20, and after the first nine weeks, very little resembling the real course of the epidemic.

In the case of the cholera deaths in London, in 1849, it will be seen in Plate II that using nine weeks of the epidemic, commencing with the second, a curve is obtained, which neither in height nor in position of highest point, well represents the course of the epidemic. In this case the first week had to be neglected, as otherwise δ^2 would have been a positive quantity.

In the case of the cholera deaths in London in 1854, it will be seen by Plate III that the calculated series very nearly resembles the real course, but then it must be remembered that the nine weeks from which the data were obtained bring us to the acme of the epidemic, which indeed only lasted eighteen weeks altogether.

I now come to the scarlet fever epidemics, where I get my figures from the "Weekly Deaths from Scarlet Fever in London in the Thirty-two years 1840-71." I have taken the years in which scarlet fever deaths rose considerably above the average; and I have endeavoured, with the help of the charts which I now show, to find whether at any early period of these epidemics, their course furnished data for constructing a curve resembling the real course of the epidemic.

First: in 1848 (see Plate IV), although the fever appears to have become epidemic about the twenty-first week, its course was at first so erratic that we have no data for calculation. On using the nine weeks from the 28th to the 36th inclusive, we get a curve indicated by a blue line, which certainly resembles the course of the epidemic. On using the nine weeks from the thirty-second to the fortieth, inclusive, we get the curve represented by a red line, which is far apart from the actual course during the early part of its decline, but comes very close to it at the end. Both these curves pointed to very nearly the same time as the acme of the epidemic, in from the forty-second to the forty-fifth week, the highest actual numbers being in the forty-first and the forty-third weeks.

In 1854 (see Plate V), the early course of the epidemic

was so erratic that any calculation made would lead to a never-ending epidemic till we used the nine weeks from the thirty-fifth to the forty-third, the latter being very near the apex of the curve. Here, the calculated series singularly resembles the actual course of the epidemic in its decline.

In 1858 (see Plate VI), the calculation based upon the nine weeks from the twenty-ninth to the thirty-seventh gave the blue curve, while one based upon the nine weeks commencing with the thirtieth gave the red curve, which, although never attaining the same height as the actual epidemic, culminates half-way between the two highest points of the epidemic, and keeps extremely close to it through most of its decline. Here, with data supplied by the thirty-eighth week, it might have been predicted that the epidemic would probably commence to decline about the forty-fourth or forty-fifth week, and that it would decline at a certain indicated rate.

In 1869 (see Plate VII), calculating on the basis of the first nine weeks in which the death-rate rapidly rose (the twenty-ninth to the thirty-seventh), we get the red curve, which very fairly represents the course of the epidemic; by using a later series of nine weeks (the thirty-second to the fortieth), we get the blue curve, which in some respects more nearly represents the actual course.

Take the case of the epidemic of scarlet fever in the latter half of 1874. The average number of scarlet fever deaths per week in the first twenty-one weeks of the year was unusually low, only 17.7. After this time, it was never under 30, and slowly and steadily increased up to the forty-third week, when it reached 128. Now, taking our epidemic as commencing at the twenty-second week, let us try and apply Dr. Farr's method. If we commence our series of groups with the twenty-second week, we get an indefinitely ascending scale. Commencing with the twenty-third week we get a similar result, the same with the twenty-fourth and twenty-fifth. With the twenty-sixth we get a very short series, which never gives a greater number than 66, and begins to descend at the thirty-sixth week. With the twenty-seventh we are again met by the positive second difference, meaning a continuous ascent, and it is not till we begin with the twenty-eighth week, that is to say, it is only when we use the nine weeks representing the seventh to the fifteenth of our epidemic, that we can calculate anything like an approximation to its course. We then get a

curve which reaches its highest point at the same time as the actual epidemic, and which closely resembles in its descent the descent of the epidemic.

Let us just enquire whether in any other way it might have been possible to predict the culminating point of this scarlet fever epidemic. On looking back at the weekly returns of deaths from scarlet fever in London we find that in those years in which scarlet fever may be said to have been epidemic, the highest death-rate occurred in the forty-first week in 1844, in the forty-first week in 1848, in the forty-fourth week in 1852, in the forty-seventh week in 1854, in the forty-seventh week in 1858, in the fortieth week in 1863, in the forty-second week in 1868, in the forty-eighth week in 1869, in the forty-first week in 1870. In the thirty-two years from 1840–71, the forty-third week had the highest number of deaths recorded, and the forty-first the second highest.

And now, you may fairly ask, what is the result of these calculations. Well, I may say, very little result at all. It is difficult to say how, in the cases where this method produces a curve like that of the epidemic, we are to ascertain exactly at what period of the epidemic we are to take our data. Even when we find an epidemic growing with a gradually diminishing ratio of increase, we are not certain that that ratio will continue. On the whole, I am rather disappointed; for I thought at first that it was likely that some indication might be found pretty constantly of the period in an epidemic when a calculation of its course might be made. There is no doubt that this mode of calculation represents correctly the course during the nine periods of data-giving observation; and therefore that it would hold good for the entire epidemic if it continued to progress on the same conditions as at first. But it appears that it is by no means universally applicable, and that in many of the cases where the method is applicable, the prediction could hardly be made sufficiently early to be of much value.