Supporting Information to "Variable selection for zero-inflated and overdispersed data with application to health care demand in Germany"

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List of Tables

Table 1: Simulation results with example 2, $\rho = 0.4, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.806 (0.47)	0.971 (0.15)	0.907 (0.092)	0.768 (0.104)
NB-MCP	0.826 (0.522)	0.971 (0.118)	0.944 (0.072)	0.805 (0.116)
NB-SCAD	0.826 (0.54)	0.979 (0.101)	0.904 (0.097)	0.799 (0.109)
ZINB-LASSO	0.308 (0.228)	0.979 (0.12)	0.949 (0.067)	2.424 (0.578)
ZINB-MCP	0.136 (0.155)	0.933 (0.171)	**0.984** (0.045)	2.32 (0.782)
ZINB-SCAD	**0.102** (0.106)	0.962 (0.133)	0.979 (0.047)	**2.262** (0.65)
BE(0.1573)	0.598 (0.249)	**1** (0)	0.755 (0.131)	2.777 (1.028)
BE(0.05)	0.281 (0.248)	0.995 (0.047)	0.916 (0.091)	2.471 (0.818)
BE(0.01)	0.105 (0.11)	0.973 (0.114)	0.982 (0.038)	2.312 (0.653)
ZINB-ORACLE	0.063 (0.057)	1 (0)	1 (0)	2.26 (0.576)
		Zero component		
ZINB-LASSO	0.292 (0.32)	0.578 (0.287)	0.932 (0.087)	
ZINB-MCP	0.521 (0.448)	**0.786** (0.201)	0.765 (0.179)	
ZINB-SCAD	**0.279** (0.269)	0.521 (0.255)	0.956 (0.069)	
BE(0.1573)	0.609 (0.395)	0.743 (0.206)	0.761 (0.143)	
BE(0.05)	0.412 (0.362)	0.597 (0.232)	0.929 (0.076)	
BE(0.01)	0.348 (0.283)	0.358 (0.245)	**0.983** (0.04)	
ZINB-ORACLE	0.095 (0.096)	1 (0)	1 (0)	

Table 2: Simulation results with example 3, $\rho = 0.4, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.787 (0.989)	0.82 (0.348)	0.889 (0.108)	0.456 (0.078)
NB-MCP	2.313 (1.342)	0.851 (0.281)	0.891 (0.089)	0.483 (0.072)
NB-SCAD	2.312 (1.353)	0.866 (0.275)	0.86 (0.109)	0.481 (0.076)
ZINB-LASSO	0.289 (0.246)	0.938 (0.181)	0.924 (0.082)	2.29 (0.689)
ZINB-MCP	**0.077** (0.093)	0.923 (0.196)	**0.982** (0.034)	2.197 (0.76)
ZINB-SCAD	0.168 (0.209)	0.923 (0.196)	0.962 (0.06)	2.172 (0.694)
BE(0.1573)	0.562 (0.225)	**0.974** (0.111)	0.769 (0.124)	2.789 (0.961)
BE(0.05)	0.267 (0.264)	0.954 (0.163)	0.928 (0.071)	2.401 (0.861)
BE(0.01)	0.099 (0.126)	0.918 (0.2)	0.979 (0.04)	**2.11** (0.684)
ZINB-ORACLE	0.049 (0.048)	1 (0)	1 (0)	2.141 (0.656)
		Zero component		
ZINB-LASSO	0.385 (0.288)	0.634 (0.302)	0.954 (0.065)	
ZINB-MCP	**0.367** (0.284)	0.791 (0.193)	0.917 (0.085)	
ZINB-SCAD	0.377 (0.263)	0.572 (0.242)	0.97 (0.047)	
BE(0.1573)	0.624 (0.254)	**0.835** (0.187)	0.778 (0.14)	
BE(0.05)	0.423 (0.278)	0.729 (0.221)	0.927 (0.084)	
BE(0.01)	0.408 (0.288)	0.508 (0.23)	**0.979** (0.04)	
ZINB-ORACLE	0.084 (0.067)	1 (0)	1 (0)	

Table 3: Simulation results with example 1, $\rho = 0.8, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.009 (0.5)	0.622 (0.359)	0.819 (0.094)	0.316 (0.041)
NB-MCP	1.409 (0.773)	0.459 (0.357)	0.901 (0.058)	0.331 (0.043)
NB-SCAD	1.425 (0.729)	0.459 (0.364)	0.88 (0.067)	0.328 (0.044)
ZINB-LASSO	**0.118** (0.095)	**0.837** (0.285)	0.887 (0.076)	2.229 (0.659)
ZINB-MCP	0.158 (0.203)	0.566 (0.377)	**0.957** (0.045)	**2.069** (0.59)
ZINB-SCAD	0.191 (0.218)	0.648 (0.368)	0.93 (0.069)	2.138 (0.637)
BE(0.1573)	0.64 (0.289)	0.742 (0.317)	0.722 (0.137)	2.846 (0.908)
BE(0.05)	0.353 (0.297)	0.668 (0.339)	0.868 (0.101)	2.478 (0.792)
BE(0.01)	0.177 (0.222)	0.611 (0.359)	0.949 (0.059)	2.101 (0.595)
ZINB-ORACLE	0.018 (0.017)	1 (0)	1 (0)	2.108 (0.566)
		Zero component		
ZINB-LASSO	**0.191** (0.134)	0.365 (0.239)	0.952 (0.056)	
ZINB-MCP	0.234 (0.154)	0.487 (0.25)	0.95 (0.044)	
ZINB-SCAD	0.252 (0.188)	0.349 (0.223)	**0.973** (0.039)	
BE(0.1573)	0.53 (0.284)	**0.634** (0.241)	0.756 (0.132)	
BE(0.05)	0.355 (0.244)	0.526 (0.251)	0.896 (0.086)	
BE(0.01)	0.287 (0.202)	0.376 (0.233)	0.962 (0.043)	
ZINB-ORACLE	0.046 (0.037)	1 (0)	1 (0)	

Table 4: Simulation results with example 2, $\rho = 0.8, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.377 (0.238)	0.925 (0.194)	0.831 (0.076)	0.857 (0.127)
NB-MCP	0.623 (0.505)	0.747 (0.291)	0.945 (0.048)	0.849 (0.133)
NB-SCAD	0.6 (0.52)	0.737 (0.309)	0.944 (0.057)	0.846 (0.116)
ZINB-LASSO	0.162 (0.138)	**0.93** (0.203)	0.906 (0.073)	2.499 (0.516)
ZINB-MCP	0.119 (0.159)	0.763 (0.3)	0.965 (0.043)	2.326 (0.605)
ZINB-SCAD	**0.086** (0.115)	0.806 (0.295)	**0.971** (0.042)	**2.252** (0.475)
BE(0.1573)	0.625 (0.286)	0.849 (0.231)	0.746 (0.142)	2.813 (0.786)
BE(0.05)	0.345 (0.332)	0.812 (0.254)	0.896 (0.098)	2.55 (0.606)
BE(0.01)	0.134 (0.183)	0.785 (0.27)	0.964 (0.048)	2.34 (0.537)
ZINB-ORACLE	0.023 (0.021)	1 (0)	1 (0)	2.317 (0.456)
		Zero component		
ZINB-LASSO	**0.116** (0.105)	0.36 (0.223)	0.947 (0.06)	
ZINB-MCP	0.203 (0.195)	0.53 (0.241)	0.862 (0.095)	
ZINB-SCAD	0.152 (0.145)	0.312 (0.197)	**0.967** (0.044)	
BE(0.1573)	0.554 (0.307)	**0.565** (0.244)	0.735 (0.14)	
BE(0.05)	0.254 (0.217)	0.419 (0.242)	0.907 (0.084)	
BE(0.01)	0.163 (0.156)	0.255 (0.205)	0.966 (0.047)	
ZINB-ORACLE	0.034 (0.038)	1 (0)	1 (0)	

Table 5: Simulation results with example 3, $\rho = 0.8, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.715 (0.463)	0.821 (0.272)	0.816 (0.076)	0.511 (0.058)
NB-MCP	1.193 (0.779)	0.6 (0.323)	0.914 (0.061)	0.521 (0.052)
NB-SCAD	1.126 (0.732)	0.621 (0.324)	0.899 (0.069)	0.514 (0.051)
ZINB-LASSO	**0.135** (0.111)	**0.884** (0.212)	0.888 (0.073)	2.314 (0.491)
ZINB-MCP	0.202 (0.234)	0.684 (0.31)	**0.96** (0.04)	2.229 (0.605)
ZINB-SCAD	0.188 (0.227)	0.716 (0.306)	0.95 (0.055)	**2.191** (0.642)
BE(0.1573)	0.597 (0.233)	0.816 (0.263)	0.739 (0.129)	2.729 (0.866)
BE(0.05)	0.363 (0.251)	0.747 (0.308)	0.888 (0.09)	2.473 (0.738)
BE(0.01)	0.223 (0.295)	0.689 (0.328)	0.954 (0.049)	2.227 (0.636)
ZINB-ORACLE	0.023 (0.021)	1 (0)	1 (0)	2.138 (0.532)
		Zero component		
ZINB-LASSO	**0.176** (0.118)	0.403 (0.231)	0.943 (0.06)	
ZINB-MCP	0.282 (0.219)	0.495 (0.255)	0.938 (0.065)	
ZINB-SCAD	0.295 (0.164)	0.363 (0.199)	0.957 (0.049)	
BE(0.1573)	0.575 (0.254)	**0.611** (0.252)	0.757 (0.131)	
BE(0.05)	0.407 (0.276)	0.471 (0.227)	0.885 (0.095)	
BE(0.01)	0.283 (0.195)	0.316 (0.216)	**0.961** (0.045)	
ZINB-ORACLE	0.046 (0.035)	1 (0)	1 (0)	

Table 6: Simulation results with example 4, $\rho = 0.8, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.992 (0.372)	0.164 (0.196)	0.938 (0.073)	0.591 (0.076)
NB-MCP	1.091 (0.453)	0.31 (0.201)	0.93 (0.055)	0.649 (0.093)
NB-SCAD	1.127 (0.486)	0.364 (0.241)	0.91 (0.078)	0.661 (0.098)
ZINB-LASSO	0.737 (0.338)	0.307 (0.298)	0.917 (0.087)	1.839 (0.497)
ZINB-MCP	0.691 (0.252)	0.398 (0.268)	**0.939** (0.057)	**2.076** (0.488)
ZINB-SCAD	0.654 (0.276)	0.488 (0.284)	0.917 (0.071)	2.101 (0.576)
BE(0.1573)	0.74 (0.263)	**0.726** (0.225)	0.757 (0.133)	2.452 (0.614)
BE(0.05)	**0.62** (0.334)	0.607 (0.246)	0.88 (0.107)	2.299 (0.426)
BE(0.01)	0.719 (0.368)	0.467 (0.225)	0.937 (0.063)	2.11 (0.395)
ZINB-ORACLE	0.111 (0.063)	1 (0)	1 (0)	2.24 (0.441)
		Zero component		
ZINB-LASSO	**0.207** (0.145)	0.176 (0.181)	**0.951** (0.061)	
ZINB-MCP	0.352 (0.256)	0.445 (0.208)	0.764 (0.148)	
ZINB-SCAD	0.313 (0.169)	0.252 (0.167)	0.918 (0.069)	
BE(0.1573)	0.576 (0.261)	**0.463** (0.188)	0.738 (0.126)	
BE(0.05)	0.413 (0.229)	0.325 (0.188)	0.864 (0.099)	
BE(0.01)	0.323 (0.26)	0.204 (0.171)	0.943 (0.06)	
ZINB-ORACLE	0.074 (0.059)	1 (0)	1 (0)	

Table 7: Simulation results with example 5, $\rho = 0.8, \theta = 2, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.746 (0.49)	0.974 (0.133)	0.837 (0.087)	0.743 (0.093)
NB-MCP	0.491 (0.308)	0.928 (0.177)	0.943 (0.054)	0.741 (0.09)
NB-SCAD	0.589 (0.536)	0.928 (0.177)	0.921 (0.067)	0.738 (0.097)
ZINB-LASSO	0.305 (0.197)	**0.995** (0.051)	0.92 (0.069)	2.136 (0.358)
ZINB-MCP	0.103 (0.104)	0.979 (0.1)	**0.99** (0.026)	2.033 (0.371)
ZINB-SCAD	**0.082** (0.091)	0.964 (0.13)	0.983 (0.037)	**2.015** (0.4)
BE(0.1573)	0.622 (0.224)	0.985 (0.087)	0.769 (0.119)	2.335 (0.482)
BE(0.05)	0.217 (0.227)	0.974 (0.111)	0.927 (0.079)	2.102 (0.414)
BE(0.01)	0.082 (0.083)	0.969 (0.121)	0.985 (0.033)	2.044 (0.382)
ZINB-ORACLE	0.059 (0.051)	1 (0)	1 (0)	2.034 (0.361)
		Zero component		
ZINB-LASSO	**0.426** (0.41)	0.523 (0.28)	0.934 (0.068)	
ZINB-MCP	0.482 (0.324)	0.673 (0.218)	0.881 (0.1)	
ZINB-SCAD	0.498 (0.408)	0.459 (0.233)	0.965 (0.043)	
BE(0.1573)	0.65 (0.341)	**0.711** (0.211)	0.782 (0.117)	
BE(0.05)	0.518 (0.351)	0.549 (0.221)	0.913 (0.093)	
BE(0.01)	0.52 (0.381)	0.394 (0.21)	**0.971** (0.041)	
ZINB-ORACLE	0.098 (0.079)	1 (0)	1 (0)	

Table 8: Simulation results with example 1, $\rho = 0.4, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	6.364 (2.677)	0.801 (0.364)	0.863 (0.113)	0.406 (0.099)
NB-MCP	7.538 (3.424)	0.847 (0.273)	0.871 (0.093)	0.418 (0.076)
NB-SCAD	7.792 (3.789)	0.837 (0.311)	0.844 (0.109)	0.421 (0.079)
ZINB-LASSO	0.287 (0.239)	**0.995** (0.051)	0.94 (0.073)	**12.919** (8.289)
ZINB-MCP	0.122 (0.103)	0.985 (0.087)	**0.985** (0.032)	13.615 (9.438)
ZINB-SCAD	**0.099** (0.099)	0.974 (0.111)	0.978 (0.057)	13.011 (9.103)
BE(0.1573)	0.625 (0.224)	0.995 (0.051)	0.771 (0.119)	31.598 (33.099)
BE(0.05)	0.279 (0.251)	0.995 (0.051)	0.916 (0.093)	19.925 (16.695)
BE(0.01)	0.113 (0.098)	0.979 (0.1)	0.983 (0.033)	14.294 (9.898)
ZINB-ORACLE	0.074 (0.063)	1 (0)	1 (0)	13.054 (8.789)
		Zero component		
ZINB-LASSO	0.582 (0.387)	0.735 (0.278)	0.959 (0.055)	
ZINB-MCP	**0.364** (0.265)	0.829 (0.182)	0.959 (0.05)	
ZINB-SCAD	0.465 (0.353)	0.645 (0.276)	0.985 (0.031)	
BE(0.1573)	0.52 (0.228)	**0.93** (0.139)	0.832 (0.105)	
BE(0.05)	0.387 (0.282)	0.84 (0.192)	0.941 (0.072)	
BE(0.01)	0.505 (0.338)	0.642 (0.257)	**0.986** (0.036)	
ZINB-ORACLE	0.145 (0.1)	1 (0)	1 (0)	

Table 9: Simulation results with example 2, $\rho = 0.4, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.795 (0.88)	**1** (0)	0.878 (0.101)	1.487 (0.282)
NB-MCP	1.891 (0.911)	0.99 (0.102)	0.947 (0.077)	1.515 (0.285)
NB-SCAD	1.801 (0.795)	1 (0)	0.93 (0.074)	1.545 (0.28)
ZINB-LASSO	0.381 (0.277)	1 (0)	0.949 (0.062)	11.482 (4.748)
ZINB-MCP	0.117 (0.102)	1 (0)	**0.994** (0.025)	11.691 (5.305)
ZINB-SCAD	0.105 (0.082)	1 (0)	0.99 (0.034)	**11.42** (5.335)
BE(0.1573)	0.605 (0.192)	1 (0)	0.787 (0.105)	16.323 (10.053)
BE(0.05)	0.276 (0.224)	1 (0)	0.931 (0.074)	13.242 (6.467)
BE(0.01)	**0.099** (0.085)	1 (0)	0.99 (0.027)	11.925 (5.358)
ZINB-ORACLE	0.077 (0.065)	1 (0)	1 (0)	11.818 (5.531)
		Zero component		
ZINB-LASSO	**0.389** (0.347)	0.698 (0.283)	0.946 (0.064)	
ZINB-MCP	0.475 (0.24)	**0.875** (0.17)	0.807 (0.13)	
ZINB-SCAD	0.408 (0.305)	0.589 (0.269)	0.972 (0.049)	
BE(0.1573)	0.543 (0.296)	0.852 (0.172)	0.794 (0.132)	
BE(0.05)	0.398 (0.241)	0.701 (0.213)	0.935 (0.08)	
BE(0.01)	0.423 (0.223)	0.479 (0.228)	**0.983** (0.036)	
ZINB-ORACLE	0.095 (0.09)	1 (0)	1 (0)	

Table 10: Simulation results with example 3, $\rho = 0.4, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	4.288 (1.752)	0.954 (0.162)	0.864 (0.1)	0.705 (0.145)
NB-MCP	4.884 (2.218)	0.954 (0.162)	0.888 (0.088)	0.755 (0.133)
NB-SCAD	4.744 (2.012)	0.964 (0.129)	0.864 (0.094)	0.737 (0.131)
ZINB-LASSO	0.324 (0.234)	**1** (0)	0.947 (0.063)	**10.964** (5.363)
ZINB-MCP	**0.123** (0.097)	0.995 (0.051)	**0.987** (0.034)	11.952 (6.244)
ZINB-SCAD	0.128 (0.094)	0.995 (0.051)	0.984 (0.043)	11.854 (6.199)
BE(0.1573)	0.598 (0.243)	1 (0)	0.79 (0.114)	18.2 (14.11)
BE(0.05)	0.324 (0.242)	1 (0)	0.922 (0.077)	14.51 (9.911)
BE(0.01)	0.146 (0.13)	1 (0)	0.981 (0.037)	12.687 (7.113)
ZINB-ORACLE	0.096 (0.07)	1 (0)	1 (0)	11.649 (5.979)
		Zero component		
ZINB-LASSO	0.472 (0.355)	0.717 (0.273)	0.946 (0.064)	
ZINB-MCP	0.403 (0.311)	0.829 (0.189)	0.938 (0.075)	
ZINB-SCAD	0.45 (0.34)	0.699 (0.227)	0.974 (0.043)	
BE(0.1573)	0.598 (0.212)	**0.908** (0.145)	0.817 (0.107)	
BE(0.05)	**0.388** (0.305)	0.832 (0.182)	0.945 (0.055)	
BE(0.01)	0.496 (0.355)	0.599 (0.235)	**0.985** (0.03)	
ZINB-ORACLE	0.12 (0.09)	1 (0)	1 (0)	

Table 11: Simulation results with example 4, $\rho = 0.4, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	4.587 (1.907)	0.845 (0.233)	0.822 (0.14)	1.066 (0.223)
NB-MCP	4.733 (1.843)	0.845 (0.163)	0.899 (0.085)	1.133 (0.227)
NB-SCAD	4.769 (1.742)	0.896 (0.158)	0.86 (0.102)	1.142 (0.208)
ZINB-LASSO	0.989 (0.466)	0.996 (0.025)	0.851 (0.105)	**10.32** (3.445)
ZINB-MCP	0.385 (0.226)	0.994 (0.03)	0.972 (0.052)	11.28 (3.848)
ZINB-SCAD	0.327 (0.246)	0.996 (0.025)	0.95 (0.071)	11.69 (4.307)
BE(0.1573)	0.683 (0.194)	**1** (0)	0.806 (0.119)	14.084 (5.458)
BE(0.05)	0.448 (0.228)	0.998 (0.018)	0.94 (0.065)	12.947 (4.933)
BE(0.01)	**0.296** (0.216)	0.994 (0.03)	**0.982** (0.044)	11.912 (4.478)
ZINB-ORACLE	0.214 (0.123)	1 (0)	1 (0)	11.758 (4.208)
		Zero component		
ZINB-LASSO	0.647 (0.459)	0.576 (0.259)	0.939 (0.091)	
ZINB-MCP	0.691 (0.24)	**0.79** (0.178)	0.828 (0.135)	
ZINB-SCAD	0.714 (0.37)	0.545 (0.249)	0.962 (0.061)	
BE(0.1573)	0.761 (0.255)	0.778 (0.153)	0.827 (0.11)	
BE(0.05)	**0.643** (0.262)	0.653 (0.188)	0.932 (0.073)	
BE(0.01)	0.777 (0.326)	0.402 (0.223)	**0.982** (0.044)	
ZINB-ORACLE	0.208 (0.118)	1 (0)	1 (0)	

Table 12: Simulation results with example 5, $\rho = 0.4, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	4.19 (2.444)	**1** (0)	0.905 (0.08)	1.167 (0.159)
NB-MCP	1.71 (1.287)	1 (0)	0.957 (0.067)	1.209 (0.162)
NB-SCAD	1.725 (1.236)	1 (0)	0.923 (0.087)	1.201 (0.176)
ZINB-LASSO	1.069 (0.806)	1 (0)	0.97 (0.04)	**9.883** (2.888)
ZINB-MCP	0.264 (0.286)	1 (0)	**0.996** (0.014)	10.445 (2.793)
ZINB-SCAD	**0.218** (0.191)	1 (0)	0.989 (0.025)	10.539 (2.854)
BE(0.1573)	0.62 (0.202)	1 (0)	0.812 (0.106)	12.278 (3.562)
BE(0.05)	0.39 (0.209)	1 (0)	0.939 (0.066)	11.074 (3.428)
BE(0.01)	0.248 (0.197)	1 (0)	0.988 (0.026)	10.466 (2.843)
ZINB-ORACLE	0.18 (0.166)	1 (0)	1 (0)	10.419 (2.7)
		Zero component		
ZINB-LASSO	1.261 (1.048)	0.773 (0.233)	0.952 (0.064)	
ZINB-MCP	0.623 (0.313)	0.921 (0.127)	0.828 (0.12)	
ZINB-SCAD	0.77 (0.615)	0.712 (0.242)	0.98 (0.04)	
BE(0.1573)	0.683 (0.245)	**0.923** (0.145)	0.811 (0.1)	
BE(0.05)	**0.603** (0.416)	0.847 (0.195)	0.939 (0.056)	
BE(0.01)	1.022 (0.896)	0.638 (0.241)	**0.987** (0.033)	
ZINB-ORACLE	0.252 (0.175)	1 (0)	1 (0)	

Table 13: Simulation results with example 1, $\rho = 0.8, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	2.706 (1.52)	0.732 (0.307)	0.788 (0.087)	0.447 (0.082)
NB-MCP	3.767 (2.045)	0.521 (0.367)	0.895 (0.059)	0.471 (0.078)
NB-SCAD	3.727 (2.151)	0.546 (0.376)	0.873 (0.067)	0.468 (0.074)
ZINB-LASSO	0.14 (0.113)	**0.979** (0.123)	0.88 (0.084)	**11.472** (6.25)
ZINB-MCP	**0.06** (0.077)	0.881 (0.248)	**0.973** (0.042)	12.759 (7.341)
ZINB-SCAD	0.092 (0.12)	0.835 (0.277)	0.961 (0.054)	11.907 (6.31)
BE(0.1573)	0.63 (0.306)	0.918 (0.2)	0.724 (0.137)	26.248 (23.51)
BE(0.05)	0.292 (0.273)	0.892 (0.219)	0.885 (0.092)	17.096 (12.592)
BE(0.01)	0.084 (0.113)	0.876 (0.229)	0.968 (0.047)	13.335 (7.747)
ZINB-ORACLE	0.025 (0.026)	1 (0)	1 (0)	11.684 (7.011)
		Zero component		
ZINB-LASSO	**0.233** (0.134)	0.505 (0.245)	0.93 (0.066)	
ZINB-MCP	0.288 (0.202)	0.485 (0.27)	0.952 (0.055)	
ZINB-SCAD	0.316 (0.172)	0.41 (0.25)	**0.965** (0.04)	
BE(0.1573)	0.621 (0.253)	**0.67** (0.238)	0.776 (0.113)	
BE(0.05)	0.49 (0.258)	0.58 (0.238)	0.893 (0.086)	
BE(0.01)	0.374 (0.256)	0.436 (0.248)	0.96 (0.056)	
ZINB-ORACLE	0.063 (0.047)	1 (0)	1 (0)	

Table 14: Simulation results with example 2, $\rho = 0.8, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.748 (0.4)	0.98 (0.098)	0.802 (0.077)	1.757 (0.398)
NB-MCP	1.008 (0.69)	0.86 (0.247)	0.938 (0.057)	1.767 (0.394)
NB-SCAD	1.048 (0.749)	0.85 (0.23)	0.937 (0.065)	1.745 (0.397)
ZINB-LASSO	0.177 (0.116)	**0.995** (0.05)	0.901 (0.073)	10.812 (4.071)
ZINB-MCP	**0.04** (0.039)	0.97 (0.119)	0.992 (0.02)	11.169 (4.962)
ZINB-SCAD	0.042 (0.038)	0.975 (0.11)	**0.994** (0.019)	**10.556** (4.192)
BE(0.1573)	0.559 (0.244)	0.985 (0.086)	0.763 (0.123)	14.539 (6.493)
BE(0.05)	0.243 (0.291)	0.965 (0.128)	0.912 (0.077)	11.994 (4.838)
BE(0.01)	0.044 (0.043)	0.96 (0.136)	0.98 (0.039)	11.472 (4.848)
ZINB-ORACLE	0.029 (0.026)	1 (0)	1 (0)	10.761 (4.405)
		Zero component		
ZINB-LASSO	**0.176** (0.101)	0.37 (0.247)	0.924 (0.067)	
ZINB-MCP	0.214 (0.143)	0.51 (0.241)	0.909 (0.09)	
ZINB-SCAD	0.229 (0.154)	0.342 (0.221)	**0.965** (0.052)	
BE(0.1573)	0.591 (0.222)	**0.568** (0.221)	0.741 (0.133)	
BE(0.05)	0.336 (0.206)	0.428 (0.211)	0.885 (0.077)	
BE(0.01)	0.258 (0.155)	0.322 (0.202)	0.956 (0.055)	
ZINB-ORACLE	0.041 (0.037)	1 (0)	1 (0)	

Table 15: Simulation results with example 3, $\rho = 0.8, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.702 (0.816)	0.914 (0.19)	0.777 (0.096)	0.841 (0.194)
NB-MCP	2.476 (1.298)	0.687 (0.3)	0.907 (0.057)	0.865 (0.194)
NB-SCAD	2.517 (1.597)	0.682 (0.315)	0.908 (0.067)	0.865 (0.185)
ZINB-LASSO	0.167 (0.125)	**0.99** (0.071)	0.895 (0.069)	11.377 (4.945)
ZINB-MCP	0.041 (0.047)	0.914 (0.215)	0.978 (0.039)	11.309 (5.182)
ZINB-SCAD	**0.038** (0.041)	0.899 (0.226)	**0.979** (0.038)	**10.779** (5.198)
BE(0.1573)	0.579 (0.291)	0.949 (0.151)	0.751 (0.136)	17.445 (11.205)
BE(0.05)	0.308 (0.269)	0.949 (0.151)	0.903 (0.086)	14.059 (7.691)
BE(0.01)	0.061 (0.077)	0.939 (0.164)	0.965 (0.056)	12.628 (5.458)
ZINB-ORACLE	0.025 (0.022)	1 (0)	1 (0)	11.13 (5.059)
		Zero component		
ZINB-LASSO	**0.208** (0.125)	0.462 (0.248)	0.926 (0.056)	
ZINB-MCP	0.318 (0.229)	0.482 (0.258)	0.948 (0.055)	
ZINB-SCAD	0.3 (0.211)	0.386 (0.235)	**0.964** (0.047)	
BE(0.1573)	0.653 (0.179)	**0.646** (0.247)	0.751 (0.123)	
BE(0.05)	0.437 (0.194)	0.523 (0.232)	0.887 (0.08)	
BE(0.01)	0.354 (0.212)	0.417 (0.226)	0.949 (0.052)	
ZINB-ORACLE	0.057 (0.049)	1 (0)	1 (0)	

Table 16: Simulation results with example 4, $\rho = 0.8, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	2.637 (1.407)	0.316 (0.329)	0.911 (0.103)	0.947 (0.202)
NB-MCP	2.684 (1.218)	0.495 (0.232)	0.903 (0.069)	1.098 (0.192)
NB-SCAD	3.108 (1.61)	0.517 (0.257)	0.883 (0.086)	1.096 (0.182)
ZINB-LASSO	1.277 (0.937)	0.773 (0.297)	0.839 (0.115)	8.839 (2.521)
ZINB-MCP	0.5 (0.406)	0.867 (0.158)	0.945 (0.064)	11.54 (3.183)
ZINB-SCAD	0.646 (0.571)	0.816 (0.196)	0.94 (0.068)	**10.626** (3.185)
BE(0.1573)	0.698 (0.184)	**0.947** (0.078)	0.784 (0.131)	13.824 (5.319)
BE(0.05)	0.506 (0.31)	0.923 (0.101)	0.895 (0.095)	12.391 (4.014)
BE(0.01)	**0.484** (0.435)	0.862 (0.151)	**0.959** (0.058)	11.416 (3.308)
ZINB-ORACLE	0.132 (0.099)	1 (0)	1 (0)	11.236 (2.911)
		Zero component		
ZINB-LASSO	**0.269** (0.156)	0.333 (0.202)	0.91 (0.078)	
ZINB-MCP	0.492 (0.239)	0.512 (0.236)	0.755 (0.136)	
ZINB-SCAD	0.42 (0.215)	0.314 (0.177)	0.902 (0.085)	
BE(0.1573)	0.724 (0.216)	**0.582** (0.205)	0.755 (0.107)	
BE(0.05)	0.495 (0.277)	0.42 (0.189)	0.882 (0.075)	
BE(0.01)	0.419 (0.234)	0.254 (0.166)	**0.941** (0.057)	
ZINB-ORACLE	0.089 (0.052)	1 (0)	1 (0)	

Table 17: Simulation results with example 5, $\rho = 0.8, \theta = 10, n = 300$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.639 (0.948)	0.995 (0.051)	0.793 (0.081)	1.382 (0.208)
NB-MCP	0.988 (0.783)	0.98 (0.099)	0.94 (0.065)	1.382 (0.222)
NB-SCAD	1.075 (0.753)	0.985 (0.087)	0.928 (0.079)	1.382 (0.221)
ZINB-LASSO	0.467 (0.368)	**1** (0)	0.936 (0.062)	**10.235** (2.539)
ZINB-MCP	0.102 (0.103)	1 (0)	0.996 (0.014)	10.737 (2.59)
ZINB-SCAD	**0.079** (0.078)	1 (0)	**0.997** (0.012)	10.842 (2.686)
BE(0.1573)	0.587 (0.199)	1 (0)	0.789 (0.108)	12.877 (4.067)
BE(0.05)	0.28 (0.23)	1 (0)	0.92 (0.073)	11.57 (3.467)
BE(0.01)	0.119 (0.122)	1 (0)	0.981 (0.037)	10.841 (2.876)
ZINB-ORACLE	0.071 (0.07)	1 (0)	1 (0)	10.821 (2.503)
		Zero component		
ZINB-LASSO	0.63 (0.452)	0.645 (0.278)	0.939 (0.059)	
ZINB-MCP	**0.44** (0.358)	0.745 (0.224)	0.932 (0.066)	
ZINB-SCAD	0.617 (0.376)	0.571 (0.243)	**0.974** (0.035)	
BE(0.1573)	0.707 (0.299)	**0.811** (0.18)	0.789 (0.117)	
BE(0.05)	0.625 (0.338)	0.686 (0.219)	0.914 (0.081)	
BE(0.01)	0.671 (0.351)	0.528 (0.235)	0.971 (0.045)	
ZINB-ORACLE	0.133 (0.081)	1 (0)	1 (0)	

Table 18: Simulation results with example 1, $\rho = 0.4, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.215 (0.135)	0.158 (0.292)	**0.982** (0.037)	0.242 (0.064)
NB-MCP	0.382 (0.327)	0.332 (0.344)	0.931 (0.093)	0.292 (0.109)
NB-SCAD	0.278 (0.225)	0.24 (0.339)	0.943 (0.109)	0.255 (0.088)
ZINB-LASSO	**0.063** (0.051)	0.337 (0.405)	0.953 (0.082)	**2.034** (1.436)
ZINB-MCP	0.111 (0.089)	0.418 (0.364)	0.933 (0.072)	3.396 (2.501)
ZINB-SCAD	0.151 (0.15)	0.541 (0.364)	0.884 (0.099)	4.468 (4.165)
BE(0.1573)	0.784 (0.201)	**0.735** (0.282)	0.428 (0.163)	2953523 (4378881)
BE(0.05)	0.563 (0.283)	0.562 (0.33)	0.672 (0.201)	258 (381)
BE(0.01)	0.185 (0.156)	0.406 (0.369)	0.903 (0.127)	4.334 (4.274)
ZINB-ORACLE	0.023 (0.025)	1 (0)	1 (0)	2.945 (1.949)
		Zero component		
ZINB-LASSO	**8e-05** (1e-04)	0.148 (0.179)	**0.985** (0.036)	
ZINB-MCP	0.00025 (0.00035)	0.508 (0.245)	0.779 (0.2)	
ZINB-SCAD	0.00017 (0.00023)	0.316 (0.22)	0.947 (0.079)	
BE(0.1573)	0.36144 (0.48413)	**0.551** (0.228)	0.618 (0.193)	
BE(0.05)	0.03615 (0.05347)	0.344 (0.268)	0.879 (0.12)	
BE(0.01)	0.01967 (0.02907)	0.078 (0.118)	0.984 (0.035)	
ZINB-ORACLE	6e-05 (8e-05)	1 (0)	1 (0)	

Table 19: Simulation results with example 2, $\rho = 0.4, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.341 (0.162)	0.486 (0.393)	**0.963** (0.07)	0.672 (0.183)
NB-MCP	0.396 (0.24)	0.646 (0.379)	0.924 (0.084)	0.856 (0.324)
NB-SCAD	0.374 (0.171)	0.583 (0.402)	0.914 (0.117)	0.779 (0.319)
ZINB-LASSO	**0.238** (0.142)	0.521 (0.406)	0.961 (0.062)	**2.909** (1.737)
ZINB-MCP	0.295 (0.199)	0.438 (0.402)	0.961 (0.057)	3.124 (1.758)
ZINB-SCAD	0.283 (0.249)	0.743 (0.336)	0.878 (0.113)	4.628 (2.921)
BE(0.1573)	0.867 (0.416)	**0.811** (0.264)	0.599 (0.192)	11.985 (11.849)
BE(0.05)	0.526 (0.438)	0.708 (0.346)	0.794 (0.175)	5.703 (4.136)
BE(0.01)	0.277 (0.296)	0.566 (0.367)	0.939 (0.113)	3.543 (2.508)
ZINB-ORACLE	0.056 (0.045)	1 (0)	1 (0)	3.224 (1.833)
		Zero component		
ZINB-LASSO	**0.012** (0.018)	0.194 (0.198)	0.97 (0.051)	
ZINB-MCP	0.139 (0.205)	**0.597** (0.267)	0.612 (0.273)	
ZINB-SCAD	0.045 (0.067)	0.292 (0.214)	0.902 (0.123)	
BE(0.1573)	0.393 (0.381)	0.467 (0.235)	0.613 (0.167)	
BE(0.05)	0.079 (0.117)	0.25 (0.235)	0.888 (0.114)	
BE(0.01)	0.041 (0.06)	0.066 (0.122)	**0.988** (0.03)	
ZINB-ORACLE	0.016 (0.023)	1 (0)	1 (0)	

Table 20: Simulation results with example 3, $\rho = 0.4, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.322 (0.161)	0.239 (0.363)	**0.976** (0.056)	0.389 (0.112)
NB-MCP	0.419 (0.304)	0.438 (0.407)	0.929 (0.089)	0.501 (0.171)
NB-SCAD	0.362 (0.187)	0.312 (0.403)	0.939 (0.102)	0.439 (0.141)
ZINB-LASSO	**0.131** (0.095)	0.455 (0.426)	0.943 (0.108)	**2.548** (1.673)
ZINB-MCP	0.201 (0.17)	0.449 (0.394)	0.932 (0.087)	3.258 (2.144)
ZINB-SCAD	0.226 (0.218)	0.642 (0.347)	0.891 (0.106)	4.287 (2.851)
BE(0.1573)	0.526 (0.381)	**0.815** (0.3)	0.586 (0.171)	14.994 (17.361)
BE(0.05)	0.339 (0.283)	0.712 (0.345)	0.775 (0.174)	8.39 (8.511)
BE(0.01)	0.132 (0.138)	0.581 (0.368)	0.947 (0.085)	3.307 (2.31)
ZINB-ORACLE	0.026 (0.022)	1 (0)	1 (0)	3.088 (1.742)
		Zero component		
ZINB-LASSO	**0.009** (0.014)	0.179 (0.185)	0.968 (0.046)	
ZINB-MCP	0.032 (0.047)	0.528 (0.269)	0.756 (0.223)	
ZINB-SCAD	0.017 (0.025)	0.27 (0.212)	0.927 (0.082)	
BE(0.1573)	0.113 (0.167)	**0.562** (0.264)	0.639 (0.177)	
BE(0.05)	0.024 (0.035)	0.281 (0.262)	0.895 (0.125)	
BE(0.01)	0.014 (0.02)	0.072 (0.121)	**0.995** (0.018)	
ZINB-ORACLE	0.004 (0.006)	1 (0)	1 (0)	

Table 21: Simulation results with example 4, $\rho = 0.4, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.751 (0.432)	0.177 (0.19)	**0.967** (0.059)	0.507 (0.112)
NB-MCP	0.966 (0.568)	0.336 (0.284)	0.932 (0.088)	0.565 (0.155)
NB-SCAD	0.885 (0.616)	0.271 (0.264)	0.937 (0.1)	0.547 (0.171)
ZINB-LASSO	**0.606** (0.277)	0.246 (0.316)	0.952 (0.095)	**1.917** (0.823)
ZINB-MCP	0.662 (0.352)	0.303 (0.284)	0.951 (0.063)	2.498 (1.139)
ZINB-SCAD	0.683 (0.405)	0.463 (0.285)	0.886 (0.103)	3.389 (1.866)
BE(0.1573)	0.824 (0.391)	**0.707** (0.243)	0.648 (0.174)	5.806 (2.842)
BE(0.05)	0.704 (0.408)	0.558 (0.3)	0.812 (0.14)	4.462 (2.095)
BE(0.01)	0.666 (0.497)	0.369 (0.303)	0.951 (0.074)	2.894 (1.301)
ZINB-ORACLE	0.161 (0.111)	1 (0)	1 (0)	3.263 (0.966)
		Zero component		
ZINB-LASSO	**0.036** (0.051)	0.168 (0.137)	0.962 (0.081)	
ZINB-MCP	0.205 (0.252)	**0.506** (0.238)	0.691 (0.238)	
ZINB-SCAD	0.095 (0.124)	0.257 (0.173)	0.92 (0.096)	
BE(0.1573)	0.337 (0.316)	0.485 (0.224)	0.653 (0.182)	
BE(0.05)	0.096 (0.137)	0.274 (0.224)	0.903 (0.107)	
BE(0.01)	0.041 (0.059)	0.069 (0.101)	**0.992** (0.022)	
ZINB-ORACLE	0.041 (0.055)	1 (0)	1 (0)	

Table 22: Simulation results with example 5, $\rho = 0.4, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.181 (0.788)	0.394 (0.387)	**0.977** (0.045)	0.595 (0.139)
NB-MCP	0.97 (0.753)	0.563 (0.387)	0.945 (0.076)	0.678 (0.202)
NB-SCAD	1.059 (0.85)	0.472 (0.413)	0.953 (0.072)	0.625 (0.178)
ZINB-LASSO	0.761 (0.602)	0.451 (0.407)	0.961 (0.07)	**2.289** (1.057)
ZINB-MCP	0.497 (0.397)	0.592 (0.381)	0.964 (0.056)	2.421 (1.056)
ZINB-SCAD	0.517 (0.483)	0.697 (0.363)	0.916 (0.097)	3.022 (1.432)
BE(0.1573)	0.696 (0.241)	**0.887** (0.21)	0.668 (0.119)	5.227 (2.702)
BE(0.05)	0.508 (0.305)	0.796 (0.288)	0.845 (0.11)	4.071 (1.819)
BE(0.01)	**0.466** (0.472)	0.613 (0.38)	0.962 (0.057)	2.644 (1.388)
ZINB-ORACLE	0.084 (0.077)	1 (0)	1 (0)	2.712 (1.01)
		Zero component		
ZINB-LASSO	**0.194** (0.237)	0.165 (0.215)	0.969 (0.046)	
ZINB-MCP	0.424 (0.469)	**0.57** (0.247)	0.691 (0.197)	
ZINB-SCAD	0.282 (0.279)	0.285 (0.244)	0.934 (0.078)	
BE(0.1573)	0.502 (0.384)	0.535 (0.275)	0.727 (0.14)	
BE(0.05)	0.293 (0.325)	0.289 (0.23)	0.924 (0.095)	
BE(0.01)	0.257 (0.277)	0.081 (0.151)	**0.987** (0.03)	
ZINB-ORACLE	0.087 (0.105)	1 (0)	1 (0)	

Table 23: Simulation results with example 1, $\rho = 0.8, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.114 (0.075)	0.215 (0.335)	0.921 (0.089)	0.3 (0.131)
NB-MCP	0.22 (0.134)	0.24 (0.28)	0.917 (0.061)	0.366 (0.112)
NB-SCAD	0.233 (0.147)	0.22 (0.304)	0.888 (0.084)	0.368 (0.117)
ZINB-LASSO	**0.033** (0.027)	0.37 (0.393)	0.913 (0.088)	**2.649** (1.727)
ZINB-MCP	0.047 (0.042)	0.295 (0.334)	**0.932** (0.076)	3.084 (2.1)
ZINB-SCAD	0.06 (0.059)	0.34 (0.347)	0.909 (0.077)	3.562 (2.713)
BE(0.1573)	0.205 (0.279)	**0.638** (0.298)	0.565 (0.178)	13.179 (15.833)
BE(0.05)	0.089 (0.107)	0.467 (0.322)	0.731 (0.174)	6.404 (6.105)
BE(0.01)	0.059 (0.053)	0.295 (0.282)	0.903 (0.127)	2.902 (1.611)
ZINB-ORACLE	0.007 (0.007)	1 (0)	1 (0)	2.565 (1.307)
		Zero component		
ZINB-LASSO	**6e-05** (8e-05)	0.14 (0.192)	0.969 (0.04)	
ZINB-MCP	5e-04 (0.00073)	0.358 (0.242)	0.877 (0.11)	
ZINB-SCAD	0.00033 (0.00048)	0.185 (0.183)	0.957 (0.055)	
BE(0.1573)	0.00479 (0.00708)	**0.482** (0.246)	0.609 (0.188)	
BE(0.05)	0.00035 (0.00051)	0.247 (0.206)	0.87 (0.145)	
BE(0.01)	8e-05 (0.00012)	0.084 (0.124)	**0.98** (0.037)	
ZINB-ORACLE	7e-05 (0.00011)	1 (0)	1 (0)	

Table 24: Simulation results with example 2, $\rho = 0.8, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.136 (0.085)	0.488 (0.377)	0.901 (0.094)	0.778 (0.281)
NB-MCP	0.237 (0.185)	0.494 (0.356)	0.92 (0.067)	0.927 (0.286)
NB-SCAD	0.252 (0.183)	0.518 (0.355)	0.902 (0.076)	0.959 (0.29)
ZINB-LASSO	**0.094** (0.068)	0.494 (0.404)	0.907 (0.087)	3.095 (1.915)
ZINB-MCP	0.12 (0.107)	0.348 (0.374)	**0.944** (0.071)	**2.828** (1.698)
ZINB-SCAD	0.139 (0.122)	0.451 (0.357)	0.907 (0.089)	3.967 (2.237)
BE(0.1573)	0.768 (0.401)	**0.628** (0.321)	0.592 (0.134)	11.055 (11.739)
BE(0.05)	0.395 (0.313)	0.472 (0.355)	0.757 (0.145)	5.886 (4.9)
BE(0.01)	0.193 (0.143)	0.299 (0.297)	0.91 (0.11)	3.399 (2.049)
ZINB-ORACLE	0.018 (0.019)	1 (0)	1 (0)	3.095 (1.461)
		Zero component		
ZINB-LASSO	**0.003** (0.005)	0.171 (0.204)	0.959 (0.048)	
ZINB-MCP	0.044 (0.065)	0.393 (0.302)	0.748 (0.216)	
ZINB-SCAD	0.011 (0.016)	0.216 (0.207)	0.927 (0.095)	
BE(0.1573)	0.168 (0.249)	**0.437** (0.254)	0.618 (0.195)	
BE(0.05)	0.007 (0.01)	0.235 (0.205)	0.865 (0.158)	
BE(0.01)	0.005 (0.007)	0.073 (0.11)	**0.978** (0.039)	
ZINB-ORACLE	0.002 (0.002)	1 (0)	1 (0)	

Table 25: Simulation results with example 3, $\rho = 0.8, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.139 (0.095)	0.34 (0.388)	0.909 (0.094)	0.472 (0.158)
NB-MCP	0.275 (0.215)	0.31 (0.347)	0.913 (0.065)	0.569 (0.162)
NB-SCAD	0.276 (0.241)	0.32 (0.352)	0.893 (0.084)	0.566 (0.179)
ZINB-LASSO	**0.05** (0.031)	0.405 (0.425)	0.912 (0.089)	**3.207** (2.163)
ZINB-MCP	0.089 (0.067)	0.26 (0.297)	**0.934** (0.063)	3.297 (2.422)
ZINB-SCAD	0.112 (0.104)	0.36 (0.318)	0.898 (0.079)	4.041 (2.734)
BE(0.1573)	0.708 (0.426)	**0.623** (0.342)	0.549 (0.151)	13.682 (16.157)
BE(0.05)	0.273 (0.282)	0.458 (0.373)	0.748 (0.157)	8.268 (8.239)
BE(0.01)	0.134 (0.124)	0.29 (0.336)	0.884 (0.119)	4.084 (3.066)
ZINB-ORACLE	0.015 (0.013)	1 (0)	1 (0)	2.893 (1.553)
		Zero component		
ZINB-LASSO	**0.002** (0.003)	0.162 (0.211)	0.956 (0.064)	
ZINB-MCP	0.01 (0.015)	0.338 (0.289)	0.844 (0.171)	
ZINB-SCAD	0.007 (0.01)	0.145 (0.16)	0.931 (0.095)	
BE(0.1573)	0.063 (0.094)	**0.458** (0.245)	0.591 (0.193)	
BE(0.05)	0.006 (0.009)	0.231 (0.203)	0.855 (0.148)	
BE(0.01)	0.003 (0.004)	0.076 (0.112)	**0.973** (0.041)	
ZINB-ORACLE	0.001 (0.002)	1 (0)	1 (0)	

Table 26: Simulation results with example 4, $\rho = 0.8, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.252 (0.13)	0.072 (0.092)	**0.957** (0.047)	0.587 (0.128)
NB-MCP	0.399 (0.266)	0.153 (0.167)	0.926 (0.059)	0.667 (0.153)
NB-SCAD	0.385 (0.253)	0.155 (0.18)	0.919 (0.095)	0.656 (0.166)
ZINB-LASSO	**0.214** (0.09)	0.102 (0.16)	0.95 (0.075)	**2.122** (0.985)
ZINB-MCP	0.245 (0.145)	0.12 (0.14)	0.929 (0.071)	2.639 (1.188)
ZINB-SCAD	0.308 (0.178)	0.181 (0.17)	0.902 (0.095)	2.761 (1.322)
BE(0.1573)	0.758 (0.277)	**0.537** (0.233)	0.595 (0.138)	6.067 (3.976)
BE(0.05)	0.498 (0.311)	0.354 (0.271)	0.775 (0.137)	4.581 (2.518)
BE(0.01)	0.297 (0.21)	0.183 (0.215)	0.916 (0.105)	2.775 (1.25)
ZINB-ORACLE	0.086 (0.071)	1 (0)	1 (0)	3.07 (1.109)
		Zero component		
ZINB-LASSO	**0.017** (0.023)	0.062 (0.106)	**0.969** (0.051)	
ZINB-MCP	0.14 (0.167)	0.366 (0.214)	0.707 (0.176)	
ZINB-SCAD	0.043 (0.049)	0.134 (0.142)	0.926 (0.071)	
BE(0.1573)	0.354 (0.36)	**0.44** (0.215)	0.653 (0.151)	
BE(0.05)	0.106 (0.144)	0.238 (0.159)	0.852 (0.122)	
BE(0.01)	0.031 (0.039)	0.06 (0.103)	0.965 (0.053)	
ZINB-ORACLE	0.018 (0.026)	1 (0)	1 (0)	

Table 27: Simulation results with example 5, $\rho = 0.8, \theta = 2, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.483 (0.306)	0.529 (0.439)	0.914 (0.082)	0.672 (0.186)
NB-MCP	0.583 (0.499)	0.547 (0.359)	0.924 (0.057)	0.741 (0.135)
NB-SCAD	0.552 (0.507)	0.576 (0.397)	0.907 (0.075)	0.737 (0.157)
ZINB-LASSO	**0.253** (0.21)	0.676 (0.421)	0.914 (0.073)	**2.647** (1.052)
ZINB-MCP	0.376 (0.272)	0.518 (0.366)	0.948 (0.063)	2.711 (1.166)
ZINB-SCAD	0.324 (0.324)	0.641 (0.35)	0.917 (0.087)	3.125 (1.132)
BE(0.1573)	0.68 (0.391)	**0.841** (0.247)	0.646 (0.153)	4.82 (1.94)
BE(0.05)	0.35 (0.307)	0.747 (0.305)	0.835 (0.119)	3.83 (1.611)
BE(0.01)	0.323 (0.33)	0.618 (0.359)	**0.949** (0.064)	3.046 (1.374)
ZINB-ORACLE	0.047 (0.047)	1 (0)	1 (0)	2.61 (0.771)
		Zero component		
ZINB-LASSO	**0.052** (0.067)	0.224 (0.211)	0.951 (0.059)	
ZINB-MCP	0.155 (0.176)	**0.479** (0.242)	0.796 (0.163)	
ZINB-SCAD	0.107 (0.121)	0.244 (0.222)	0.943 (0.07)	
BE(0.1573)	0.42 (0.388)	0.465 (0.256)	0.665 (0.181)	
BE(0.05)	0.134 (0.167)	0.25 (0.208)	0.899 (0.12)	
BE(0.01)	0.077 (0.093)	0.091 (0.144)	**0.981** (0.033)	
ZINB-ORACLE	0.029 (0.035)	1 (0)	1 (0)	

Table 28: Simulation results with example 1, $\rho = 0.4, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.429 (0.219)	0.247 (0.362)	**0.966** (0.065)	0.344 (0.104)
NB-MCP	0.573 (0.399)	0.371 (0.391)	0.924 (0.101)	0.443 (0.19)
NB-SCAD	0.516 (0.357)	0.34 (0.392)	0.923 (0.113)	0.386 (0.168)
ZINB-LASSO	**0.071** (0.06)	0.593 (0.429)	0.95 (0.067)	**12.587** (13.361)
ZINB-MCP	0.083 (0.088)	0.603 (0.414)	0.956 (0.054)	23.035 (28.778)
ZINB-SCAD	0.099 (0.092)	0.716 (0.353)	0.926 (0.078)	34.95 (42.79)
BE(0.1573)	0.694 (0.412)	**0.806** (0.275)	0.6 (0.201)	1589894 (2357156)
BE(0.05)	0.249 (0.192)	0.735 (0.353)	0.845 (0.152)	316624 (469402)
BE(0.01)	0.177 (0.193)	0.559 (0.404)	0.958 (0.068)	30.367 (34.407)
ZINB-ORACLE	0.015 (0.018)	1 (0)	1 (0)	20.829 (24.069)
		Zero component		
ZINB-LASSO	**5e-05** (5e-05)	0.253 (0.23)	0.978 (0.035)	
ZINB-MCP	0.00023 (0.00033)	0.564 (0.256)	0.809 (0.173)	
ZINB-SCAD	0.00011 (0.00015)	0.348 (0.241)	0.949 (0.065)	
BE(0.1573)	0.11708 (0.16425)	**0.604** (0.289)	0.66 (0.172)	
BE(0.05)	0.02478 (0.03663)	0.39 (0.262)	0.901 (0.123)	
BE(0.01)	0.01368 (0.02018)	0.176 (0.18)	**0.993** (0.02)	
ZINB-ORACLE	3e-05 (5e-05)	1 (0)	1 (0)	

Table 29: Simulation results with example 2, $\rho = 0.4, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.525 (0.285)	0.73 (0.354)	0.941 (0.079)	1.395 (0.594)
NB-MCP	0.601 (0.404)	0.792 (0.318)	0.941 (0.079)	1.588 (0.785)
NB-SCAD	0.616 (0.386)	0.758 (0.346)	0.918 (0.107)	1.546 (0.825)
ZINB-LASSO	0.2 (0.171)	0.798 (0.351)	0.94 (0.083)	**13.543** (12.434)
ZINB-MCP	0.211 (0.239)	0.77 (0.346)	**0.968** (0.057)	14.248 (13.997)
ZINB-SCAD	**0.175** (0.2)	**0.893** (0.219)	0.927 (0.085)	16.574 (16.775)
BE(0.1573)	0.792 (0.368)	0.887 (0.233)	0.646 (0.172)	557352 (826283)
BE(0.05)	0.272 (0.351)	0.853 (0.27)	0.87 (0.138)	16568 (24551)
BE(0.01)	0.186 (0.248)	0.745 (0.366)	0.966 (0.06)	17.989 (18.842)
ZINB-ORACLE	0.048 (0.045)	1 (0)	1 (0)	18.609 (16.618)
		Zero component		
ZINB-LASSO	**4e-05** (4e-05)	0.289 (0.264)	0.945 (0.073)	
ZINB-MCP	0.05664 (0.08392)	**0.66** (0.248)	0.563 (0.298)	
ZINB-SCAD	0.00218 (0.00318)	0.365 (0.261)	0.899 (0.119)	
BE(0.1573)	0.00587 (0.00866)	0.566 (0.269)	0.685 (0.171)	
BE(0.05)	0.00036 (0.00051)	0.284 (0.265)	0.907 (0.128)	
BE(0.01)	5e-05 (5e-05)	0.083 (0.147)	**0.994** (0.023)	
ZINB-ORACLE	5e-05 (6e-05)	1 (0)	1 (0)	

Table 30: Simulation results with example 3, $\rho = 0.4, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.551 (0.312)	0.438 (0.422)	0.944 (0.086)	0.607 (0.195)
NB-MCP	0.713 (0.527)	0.567 (0.379)	0.911 (0.094)	0.79 (0.311)
NB-SCAD	0.692 (0.51)	0.536 (0.403)	0.891 (0.125)	0.74 (0.325)
ZINB-LASSO	0.145 (0.128)	0.737 (0.383)	0.94 (0.077)	17.123 (17.926)
ZINB-MCP	0.131 (0.148)	0.665 (0.393)	**0.966** (0.058)	**15.876** (18.774)
ZINB-SCAD	0.143 (0.167)	0.835 (0.286)	0.914 (0.097)	28.343 (32.342)
BE(0.1573)	0.533 (0.397)	**0.888** (0.225)	0.632 (0.177)	795841 (1179852)
BE(0.05)	0.243 (0.255)	0.827 (0.291)	0.853 (0.127)	21268 (31529)
BE(0.01)	**0.096** (0.121)	0.733 (0.352)	0.966 (0.053)	24.578 (27.685)
ZINB-ORACLE	0.03 (0.022)	1 (0)	1 (0)	23.858 (24.497)
		Zero component		
ZINB-LASSO	**4e-05** (4e-05)	0.284 (0.249)	0.963 (0.05)	
ZINB-MCP	0.00026 (0.00037)	**0.631** (0.258)	0.771 (0.204)	
ZINB-SCAD	0.00012 (0.00016)	0.407 (0.248)	0.93 (0.086)	
BE(0.1573)	0.00466 (0.00686)	0.553 (0.266)	0.651 (0.182)	
BE(0.05)	0.00101 (0.00148)	0.337 (0.262)	0.891 (0.144)	
BE(0.01)	7e-05 (9e-05)	0.123 (0.171)	**0.991** (0.028)	
ZINB-ORACLE	3e-05 (5e-05)	1 (0)	1 (0)	

Table 31: Simulation results with example 4, $\rho = 0.4, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	1.677 (0.741)	0.208 (0.222)	**0.967** (0.068)	0.719 (0.181)
NB-MCP	1.721 (0.915)	0.418 (0.294)	0.929 (0.079)	0.966 (0.448)
NB-SCAD	1.829 (0.971)	0.374 (0.323)	0.925 (0.107)	0.854 (0.386)
ZINB-LASSO	0.955 (0.837)	0.474 (0.416)	0.916 (0.118)	5.382 (4.92)
ZINB-MCP	0.696 (0.481)	0.61 (0.354)	0.937 (0.088)	**10.523** (10.614)
ZINB-SCAD	0.579 (0.365)	0.79 (0.264)	0.877 (0.114)	14.193 (12.419)
BE(0.1573)	0.734 (0.272)	**0.896** (0.154)	0.698 (0.161)	75.515 (100)
BE(0.05)	**0.467** (0.333)	0.838 (0.194)	0.887 (0.11)	33.711 (37.303)
BE(0.01)	0.553 (0.408)	0.68 (0.279)	0.954 (0.076)	17.989 (17.228)
ZINB-ORACLE	0.144 (0.101)	1 (0)	1 (0)	16.072 (10.147)
		Zero component		
ZINB-LASSO	**0.052** (0.076)	0.19 (0.173)	0.963 (0.063)	
ZINB-MCP	0.264 (0.377)	**0.604** (0.233)	0.625 (0.248)	
ZINB-SCAD	0.121 (0.168)	0.251 (0.211)	0.91 (0.102)	
BE(0.1573)	0.437 (0.403)	0.518 (0.241)	0.719 (0.172)	
BE(0.05)	0.128 (0.179)	0.274 (0.243)	0.924 (0.089)	
BE(0.01)	0.092 (0.127)	0.07 (0.11)	**0.982** (0.041)	
ZINB-ORACLE	0.039 (0.058)	1 (0)	1 (0)	

Table 32: Simulation results with example 5, $\rho = 0.4, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	2.311 (1.797)	0.704 (0.386)	0.954 (0.068)	1.081 (0.405)
NB-MCP	1.498 (1.382)	0.796 (0.324)	0.938 (0.084)	1.284 (0.467)
NB-SCAD	1.937 (1.837)	0.753 (0.363)	0.928 (0.091)	1.134 (0.465)
ZINB-LASSO	0.481 (0.362)	0.969 (0.165)	0.916 (0.076)	**15.013** (9.881)
ZINB-MCP	0.321 (0.309)	0.963 (0.173)	**0.971** (0.047)	15.48 (10.393)
ZINB-SCAD	0.261 (0.22)	**1** (0)	0.922 (0.078)	16.044 (8.899)
BE(0.1573)	0.679 (0.334)	0.969 (0.1)	0.696 (0.155)	307 (443)
BE(0.05)	0.332 (0.27)	0.96 (0.11)	0.9 (0.092)	26.894 (24.716)
BE(0.01)	**0.213** (0.203)	0.915 (0.195)	0.963 (0.065)	17.511 (11.689)
ZINB-ORACLE	0.097 (0.106)	1 (0)	1 (0)	14.512 (8.938)
		Zero component		
ZINB-LASSO	**0.111** (0.162)	0.401 (0.226)	0.943 (0.075)	
ZINB-MCP	0.362 (0.436)	**0.704** (0.227)	0.633 (0.207)	
ZINB-SCAD	0.122 (0.18)	0.398 (0.219)	0.934 (0.075)	
BE(0.1573)	0.328 (0.459)	0.561 (0.217)	0.727 (0.181)	
BE(0.05)	0.179 (0.246)	0.312 (0.223)	0.933 (0.083)	
BE(0.01)	0.117 (0.173)	0.089 (0.143)	**0.991** (0.032)	
ZINB-ORACLE	0.032 (0.046)	1 (0)	1 (0)	

Table 33: Simulation results with example 1, $\rho = 0.8, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.212 (0.173)	0.3 (0.369)	0.893 (0.105)	0.44 (0.191)
NB-MCP	0.446 (0.361)	0.355 (0.357)	0.891 (0.078)	0.556 (0.173)
NB-SCAD	0.455 (0.374)	0.345 (0.367)	0.869 (0.09)	0.555 (0.189)
ZINB-LASSO	**0.036** (0.035)	**0.67** (0.364)	0.9 (0.078)	**16.309** (19.102)
ZINB-MCP	0.056 (0.064)	0.505 (0.366)	**0.942** (0.063)	27.176 (32.465)
ZINB-SCAD	0.057 (0.068)	0.56 (0.364)	0.93 (0.068)	27.307 (34.062)
BE(0.1573)	0.357 (0.402)	0.648 (0.32)	0.58 (0.211)	589869 (874529)
BE(0.05)	0.135 (0.158)	0.572 (0.337)	0.767 (0.225)	161266 (239082)
BE(0.01)	0.075 (0.082)	0.497 (0.347)	0.87 (0.227)	53.693 (70.808)
ZINB-ORACLE	0.006 (0.006)	1 (0)	1 (0)	20.124 (20.541)
		Zero component		
ZINB-LASSO	**3e-05** (4e-05)	0.208 (0.201)	**0.948** (0.045)	
ZINB-MCP	0.00019 (0.00027)	0.365 (0.237)	0.872 (0.098)	
ZINB-SCAD	8e-05 (0.00011)	0.225 (0.218)	0.944 (0.058)	
BE(0.1573)	0.00534 (0.0079)	**0.461** (0.286)	0.609 (0.228)	
BE(0.05)	0.00079 (0.00117)	0.276 (0.257)	0.813 (0.234)	
BE(0.01)	1e-04 (0.00015)	0.161 (0.242)	0.914 (0.227)	
ZINB-ORACLE	2e-05 (3e-05)	1 (0)	1 (0)	

Table 34: Simulation results with example 2, $\rho = 0.8, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.228 (0.141)	0.722 (0.361)	0.861 (0.09)	2.139 (1.018)
NB-MCP	0.424 (0.32)	0.557 (0.353)	0.915 (0.074)	2.249 (1.045)
NB-SCAD	0.391 (0.284)	0.562 (0.355)	0.897 (0.084)	2.254 (1.123)
ZINB-LASSO	**0.089** (0.083)	**0.851** (0.272)	0.888 (0.079)	17.243 (14.158)
ZINB-MCP	0.112 (0.131)	0.67 (0.345)	**0.955** (0.061)	**13.141** (12.641)
ZINB-SCAD	0.132 (0.166)	0.711 (0.321)	0.942 (0.058)	15.831 (13.283)
BE(0.1573)	0.757 (0.392)	0.696 (0.291)	0.613 (0.198)	129721 (192312)
BE(0.05)	0.455 (0.381)	0.597 (0.328)	0.797 (0.195)	105 (147)
BE(0.01)	0.309 (0.3)	0.538 (0.347)	0.89 (0.19)	19.645 (20.253)
ZINB-ORACLE	0.016 (0.017)	1 (0)	1 (0)	15.948 (14.846)
		Zero component		
ZINB-LASSO	**3e-05** (2e-05)	0.186 (0.205)	**0.944** (0.059)	
ZINB-MCP	0.0093 (0.01377)	**0.503** (0.278)	0.664 (0.266)	
ZINB-SCAD	0.00011 (0.00014)	0.198 (0.201)	0.934 (0.079)	
BE(0.1573)	0.00131 (0.00186)	0.427 (0.256)	0.626 (0.215)	
BE(0.05)	0.00015 (0.00021)	0.263 (0.236)	0.838 (0.203)	
BE(0.01)	4e-05 (5e-05)	0.118 (0.189)	0.937 (0.191)	
ZINB-ORACLE	3e-05 (4e-05)	1 (0)	1 (0)	

Table 35: Simulation results with example 3, $\rho = 0.8, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.242 (0.167)	0.495 (0.399)	0.874 (0.097)	0.864 (0.326)
NB-MCP	0.497 (0.331)	0.387 (0.335)	0.893 (0.077)	0.967 (0.276)
NB-SCAD	0.538 (0.342)	0.428 (0.361)	0.871 (0.083)	0.979 (0.313)
ZINB-LASSO	**0.057** (0.051)	**0.747** (0.324)	0.898 (0.085)	16.794 (15.28)
ZINB-MCP	0.096 (0.088)	0.577 (0.341)	**0.938** (0.07)	24.51 (25.774)
ZINB-SCAD	0.102 (0.11)	0.644 (0.353)	0.925 (0.084)	24.902 (25.673)
BE(0.1573)	0.364 (0.341)	0.66 (0.289)	0.65 (0.146)	19763 (29291)
BE(0.05)	0.246 (0.214)	0.54 (0.313)	0.832 (0.121)	37.979 (45.314)
BE(0.01)	0.147 (0.155)	0.434 (0.317)	0.924 (0.082)	**16.591** (14.889)
ZINB-ORACLE	0.009 (0.009)	1 (0)	1 (0)	18.359 (17.251)
		Zero component		
ZINB-LASSO	**4e-05** (6e-05)	0.273 (0.204)	0.938 (0.062)	
ZINB-MCP	3e-04 (0.00043)	0.374 (0.237)	0.869 (0.113)	
ZINB-SCAD	0.00014 (2e-04)	0.237 (0.205)	0.95 (0.053)	
BE(0.1573)	0.00283 (0.00416)	**0.443** (0.233)	0.649 (0.183)	
BE(0.05)	0.00056 (0.00082)	0.245 (0.2)	0.869 (0.124)	
BE(0.01)	0.00012 (0.00016)	0.114 (0.134)	**0.973** (0.041)	
ZINB-ORACLE	4e-05 (6e-05)	1 (0)	1 (0)	

Table 36: Simulation results with example 4, $\rho = 0.8, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.596 (0.362)	0.107 (0.141)	**0.952** (0.066)	0.903 (0.236)
NB-MCP	0.804 (0.572)	0.197 (0.182)	0.933 (0.07)	1.085 (0.276)
NB-SCAD	0.79 (0.604)	0.169 (0.184)	0.933 (0.078)	1.036 (0.323)
ZINB-LASSO	**0.468** (0.253)	0.222 (0.234)	0.91 (0.107)	6.018 (3.823)
ZINB-MCP	0.53 (0.262)	0.27 (0.228)	0.906 (0.102)	**9.616** (6.265)
ZINB-SCAD	0.562 (0.3)	0.349 (0.222)	0.879 (0.102)	11.539 (7.574)
BE(0.1573)	0.838 (0.367)	**0.657** (0.242)	0.65 (0.146)	74.347 (92.371)
BE(0.05)	0.607 (0.407)	0.525 (0.29)	0.825 (0.122)	22.75 (19.008)
BE(0.01)	0.556 (0.356)	0.41 (0.294)	0.913 (0.088)	13.614 (8.564)
ZINB-ORACLE	0.087 (0.067)	1 (0)	1 (0)	15.051 (9.463)
		Zero component		
ZINB-LASSO	**0.022** (0.029)	0.107 (0.132)	0.965 (0.05)	
ZINB-MCP	0.112 (0.136)	0.347 (0.228)	0.728 (0.178)	
ZINB-SCAD	0.05 (0.068)	0.169 (0.154)	0.909 (0.095)	
BE(0.1573)	0.284 (0.368)	**0.409** (0.241)	0.695 (0.168)	
BE(0.05)	0.104 (0.132)	0.21 (0.195)	0.879 (0.109)	
BE(0.01)	0.044 (0.055)	0.095 (0.135)	**0.975** (0.041)	
ZINB-ORACLE	0.022 (0.033)	1 (0)	1 (0)	

Table 37: Simulation results with example 5, $\rho = 0.8, \theta = 10, n = 100$. Median and robust standard deviations (in parentheses) of the MSE ratio between the penalized model and full model, and the estimated θ. Mean and standard deviations of sensitivity and specificity.

Method	MSE	Sensitivity	Specificity	$\hat{\theta}$
		NB component		
NB-LASSO	0.762 (0.644)	0.802 (0.312)	0.871 (0.099)	1.271 (0.521)
NB-MCP	0.758 (0.816)	0.74 (0.332)	0.924 (0.066)	1.396 (0.445)
NB-SCAD	0.771 (0.804)	0.766 (0.307)	0.895 (0.084)	1.425 (0.489)
ZINB-LASSO	0.208 (0.186)	**0.958** (0.173)	0.909 (0.078)	**14.113** (8.669)
ZINB-MCP	0.144 (0.147)	0.917 (0.214)	**0.966** (0.047)	15.297 (9.06)
ZINB-SCAD	0.122 (0.136)	0.938 (0.181)	0.954 (0.064)	15.253 (9.21)
BE(0.1573)	0.681 (0.232)	0.949 (0.173)	0.675 (0.148)	53.846 (62.814)
BE(0.05)	0.291 (0.308)	0.936 (0.187)	0.859 (0.128)	25.546 (23.342)
BE(0.01)	**0.105** (0.118)	0.91 (0.209)	0.965 (0.054)	14.568 (8.236)
ZINB-ORACLE	0.043 (0.043)	1 (0)	1 (0)	13.862 (6.641)
		Zero component		
ZINB-LASSO	**0.041** (0.06)	0.362 (0.243)	0.932 (0.062)	
ZINB-MCP	0.112 (0.142)	**0.583** (0.242)	0.773 (0.173)	
ZINB-SCAD	0.088 (0.108)	0.336 (0.256)	0.95 (0.057)	
BE(0.1573)	0.467 (0.288)	0.519 (0.241)	0.666 (0.183)	
BE(0.05)	0.143 (0.129)	0.298 (0.235)	0.901 (0.105)	
BE(0.01)	0.1 (0.098)	0.154 (0.186)	**0.977** (0.04)	
ZINB-ORACLE	0.026 (0.038)	1 (0)	1 (0)	