

17 **S1 Text. The dependence of F_{st} on MAF**

18 Wu, *et al.* reported the PD among human genes using the HapMap data according
19 to F_{st} ¹. They called the genes of PD if they contained at least one SNP with an F_{st} more
20 than or equal to 0.6. However, it is not feasible that the F_{st} from rare variants exceed
21 0.6 and this method could not detect the population differentiation in rare variants. S2
22 figure shows the F_{st} in our WES data according to weighted average of MAF from each
23 ancestry groups, and MAF from total population. The red line describes the maximum
24 F_{st} when the MAF is given. The maximum F_{st} for MAF is defined when all the minor
25 allele only exist in the ancestry group with the smallest sample size. If we assume there
26 is no genotyping error, then we know how many loci are genotyped for each ancestry
27 groups. In case of our data, the max F_{st} was identified when American Hispanics (1938)
28 only have minor allele and other ancestry groups such as African Americans (2025),
29 East Asians (2164), South Asians (2199), and Europeans (4518) do not have minor
30 allele at all. The maximum F_{st} for the rare or less common variants ($MAF < 0.05$) is
31 less than 0.36 and especially the maximum of F_{st} for the rare variants ($MAF < 0.01$) is
32 less than 0.073. Therefore, for these variants, very high divergence ($0.25 < F_{st}$) or high
33 divergence ($0.15 < F_{st} < 0.25$) in the wright's criteria need to be modified to find PD in
34 rare variants.

35 The upper and lower bound of F_{st} when the MAF is given are already reported
36 ² and here we also showed that the maximum of F_{st} is bounded according to MAF when
37 MAF is small enough through the equation. We used the initial definition of F_{st} which
38 is developed by Wright for the simplification of proof. Since Wright developed this
39 measure, many estimators has been proposed to estimate the F_{st} correctly under various
40 situations. However, we only choose the Wright's F_{st} for our proof because other
41 estimates are originated to estimate this parameter. Since Wright's F_{st} assume the ideal

42 situation with infinite allele and balanced sample sizes, this ideal condition would be
 43 different from the real world. However, we can assume the ideal condition theoretically,
 44 and our proof will be able to be extended to other estimators.

45 n_i denotes minor allele count of ancestry group i ; N denotes the total genotyped
 46 allele counts; k denotes the total minor allele counts in population; m denotes the
 47 number of ancestry groups. Under Hardy Weinberg Equilibrium,

48 $H_s = \text{mean expected heterozygosity within random mating subpopulations} = 2 \overline{\frac{n_i}{N} \left(1 - \frac{n_i}{N}\right)}$

49 $H_T = \text{expected heterozygosity in random mating total population} = 2 \frac{k}{N} \left(1 - \frac{k}{N}\right)$

50 Wright's $F_{st} = \frac{H_T - H_s}{H_T}$

51 $= 1 - \frac{H_T - H_s}{H_T}$

52 $= 1 - \frac{\frac{2}{m} \sum_{i=1}^m \left(\frac{n_i}{N/m} \left(1 - \frac{n_i}{N/m}\right) \right)}{2 \left(\frac{k}{N} \right) \left(1 - \frac{k}{N}\right)}$

53 $= 1 - \frac{\sum_{i=1}^m m \left(n_i \left(\frac{N}{m} - n_i \right) \right)}{k(N - k)}$

54 When k is less than N/m , $\exists n_i = k$ such that $i = 1 \cdots m$ and the minimum of nominator is

55 $m \left(k \left(\frac{N}{m} - k \right) \right)$. Since $\sum_{i=1}^m n_i = k$, the minimum of nominator can be easily proved by

56 Jensen's inequality³. Therefore, when k is less than N/m , the maximum of above

57 equation is

$$\begin{aligned}
58 \quad &= 1 - \frac{m \left(\frac{N}{m} - k \right)}{N - k} \\
&= \frac{(m-1)k}{N - k} \\
59 \quad &= \frac{(1-m)(N-k) + (m-1)N}{N - k} \\
&= (1-m) + \frac{(m-1)N}{N - k}
\end{aligned}$$

60 As k is increasing; the denominator is decreasing; the maximum of F_{st} is increasing.

61 Therefore, when we focus on the variants with small MAF, then k is less than N/m and

62 the maximum of F_{st} is bounded according to their MAF

63

64 **S2 Text. The permutation to confirm the distribution of PDRC**

65 Considering small p-values from real data analysis, the distribution of PDRC
66 may be claimed not to follow chi-square distribution. As an attempt to answer this issue,
67 we permuted the ancestral allele information of each 48 VIP genes for 100 thousand
68 times and calculated the PDRC statistics with three different weighting schemes. All
69 the variants are used for this permutation regardless of their MAFs. In Supplementary
70 S3 Fig, from several permuted data sets, the PDRC statistics without weight did not
71 follow the chi-square distribution, but the PDRC statistics seem to be controlled by
72 using weights as inverse of MAF, or inverse of MAF^2 (S4 Fig, and S5 Fig).

73

74 **S3 Text. The variance of common odds ratio**

$$\begin{aligned}
\text{var}(\log(\hat{\theta}_{MH_i})) &= \frac{\sum_k w_k^2 (n_{i1k} + n_{52k})(n_{i1k} n_{52k}) / (n_{i,k} + n_{5,k})^2}{2 \left(\sum_k w_k (n_{i2k} \cdot n_{51k}) / (n_{i,k} + n_{5,k}) \right)^2} \\
&+ \frac{\sum_k w_k^2 \left((n_{i1k} + n_{52k})(n_{i2k} n_{51k}) + (n_{i2k} + n_{51k})(n_{i1k} n_{52k}) \right) / (n_{i,k} + n_{5,k})^2}{2 \left(\sum_k w_k (n_{i1k} \cdot n_{52k}) / (n_{i,k} + n_{5,k}) \right) \left(\sum_k w_k (n_{i2k} \cdot n_{51k}) / (n_{i,k} + n_{5,k}) \right)} \\
&+ \frac{\sum_k w_k^2 (n_{i2k} + n_{51k})(n_{i2k} n_{51k}) / (n_{i,k} + n_{5,k})^2}{2 \left(\sum_k w_k (n_{i2k} \cdot n_{51k}) / (n_{i,k} + n_{5,k}) \right)^2}
\end{aligned}$$

76

77 **S4 Text. The simulation under the assumed null distribution**

78 For the simulation to evaluate the type-1 error rate, we generated the data set
79 without PD. We designed the data with four ancestral groups with 500 individuals and
80 one ancestral group with 1000 individuals to assume a similar setting of sample sizes
81 like our WES dataset. We also specified the number of SNPs in genes and assumed the
82 distributions of MAF for a range of scenarios to investigate the potential effect of the
83 number of variants in a gene and the distribution of MAF. In Scenarios 1 to 5, the rare
84 or less common variants were generated; in Scenarios 6 to 10, the common variants
85 were generated; in Scenarios 11 to 15, the variants were generated with the same MAF
86 distribution as our WES data. The ancestral group information was randomly assigned
87 to follow the null hypothesis, and 10^5 genes are simulated under fifteen different
88 scenarios. For each setting of MAF distribution, 5 different numbers of SNP in a gene
89 are assumed as following, 5, 10, 20, 50, and 100. Let $\text{Gene}_{i,s}$ represent the i th Gene of
90 scenario s for $i \in \{1, 2, \dots, 99999, 100000\}$ and $\text{SNP}_{i,j,s}$ the j th SNP in $\text{Gene}_{i,s}$ for

91 $j \in \{1, 2, \dots, n_s\}$. The MAF of $SNP_{i,j,s}$, $p_{i,j,s}$, is sampled from $unif(0, 0.05)$ for
92 scenarios 1 to 5, and from $unif(0.05, 0.5)$ for scenarios 6 to 10. For scenarios 11 to 15,
93 $p_{i,j,s}$ is sampled from the MAF distribution of all 3,130,381 variants in our WES data
94 sets (S1 Fig). Also, it is known that the p -values from the null distribution follow the
95 uniform distribution and the distribution of simulated p -values can be investigated by
96 QQ-plot. Supplementary Figures S6 to S8 show that the p -value of the PDRC test from
97 the simulated data sets follows the uniform distribution when using three types of
98 weights, 1, inverse of MAF, and inverse of MAF^2 . According to these results, the type-
99 1 error rate of PDRC tests seems to be reasonably controlled regardless the MAF values
100 and the number of variants in a gene.

101

- 102 1 Wu, D. D. & Zhang, Y. P. Different level of population differentiation among human genes.
103 *Bmc Evol Biol* **11**, 16, doi:10.1186/1471-2148-11-16 (2011).
104 2 Jakobsson, M., Edge, M. D. & Rosenberg, N. A. The relationship between F_{ST} and the
105 frequency of the most frequent allele. *Genetics* **193**, 515-528,
106 doi:10.1534/genetics.112.144758 (2013).
107 3 Hölder, O. Ueber einen Mittelwertsatz. *Göttinger Nachr*, 38-47 (1889).

108

109 **Legends to Supplementary Figures**

110

111 **S1 Fig.** Histogram of $\log_{10}(MAF)$ from our WES data sets

112

113 **S2 Fig.** The maximum of F_{st} is bounded according to the MAF from total population

114 The red line represents theoretical maximum of F_{st} .

115

116 **S3 Fig.** QQ-plot results of PDRC without weight from the ancestral allele information
117 permutation of VIP gene datasets

118

119 **S4 Fig.** QQ-plot results of PDRC with the weight as inverse of MAF from the ancestral
120 allele information permutation of VIP gene datasets

121

122 **S5 Fig.** QQ-plot results of PDRC with the weight as inverse of MAF^2 from the ancestral
123 allele information permutation of VIP gene datasets

124

125 **S6 Fig.** QQ-plot results from the simulation under null hypothesis 1

126 MAF of $SNP_{i,j,s}$, $p_{i,j,s}$, is sampled from $unif(0, 0.05)$ for scenarios 1 to 5

127 **1:** Number of SNPs in a Gene is 5. **2:** Number of SNPs in a Gene is 10. **3:** Number of
128 SNPs in a Gene is 20. **4:** Number of SNPs in a Gene is 50. **5:** Number of SNPs in a
129 Gene is 100. **A:** No weight, **B:** Weight is $1/MAF$, **C:** Weight is $1/MAF^2$

130

131 **S7 Fig.** QQ-plot results from the simulation under null hypothesis 2

132 MAF of $SNP_{i,j,s}$, $p_{i,j,s}$, is sampled from $unif(0.05, 0.5)$ for scenarios 6 to 10

133 **6:** Number of SNPs in a Gene is 5. **7:** Number of SNPs in a Gene is 10. **8:** Number of
134 SNPs in a Gene is 20. **9:** Number of SNPs in a Gene is 50. **10:** Number of SNPs in a
135 Gene is 100. **A:** No weight, **B:** Weight is $1/MAF$, **C:** Weight is $1/MAF^2$

136

137 **S8 Fig.** QQ-plot results from the simulation under null hypothesis 3

138 MAF of $SNP_{i,j,s}$, $p_{i,j,s}$, is sampled from the real MAF distribution of our WES data for

139 scenarios 11 to 15

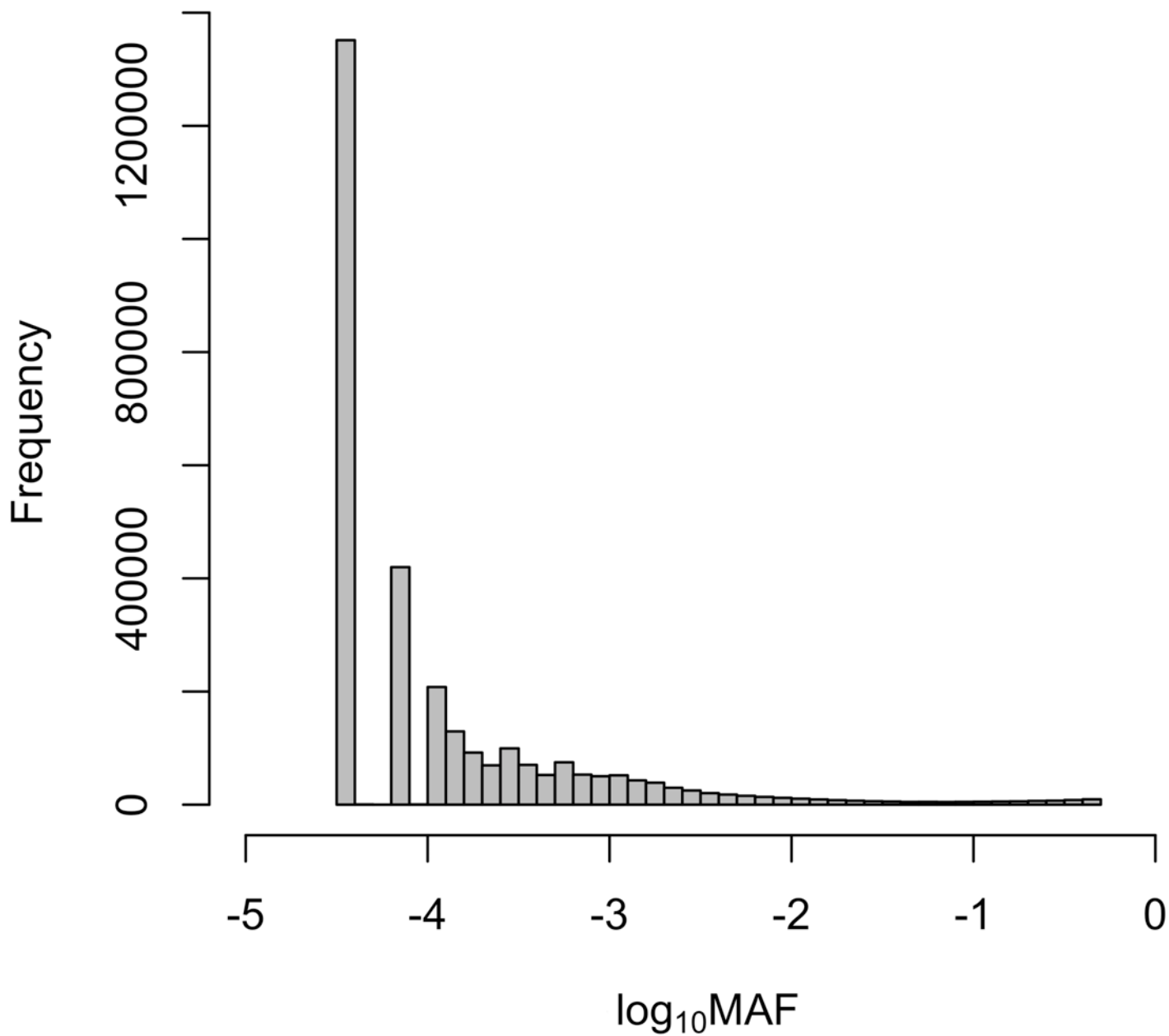
140 **11:** Number of SNPs in a Gene is 5. **12:** Number of SNPs in a Gene is 10. **13:** Number

141 of SNPs in a Gene is 20. **14:** Number of SNPs in a Gene is 50. **15:** Number of SNPs in

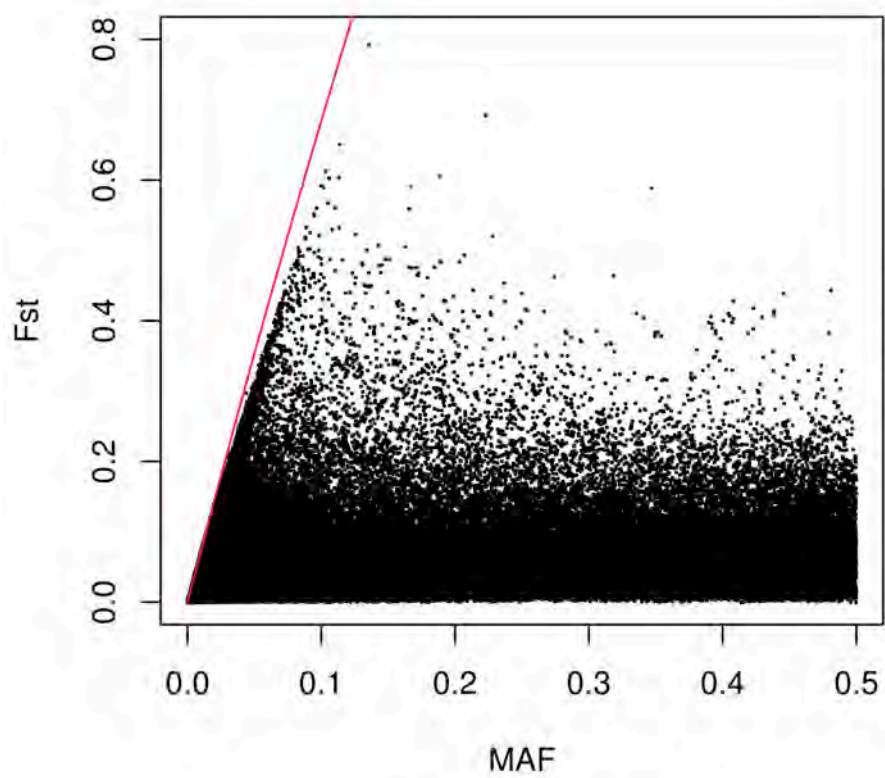
142 a Gene is 100. **A:** No weight, **B:** Weight is $1/MAF$, **C:** Weight is $1/MAF^2$

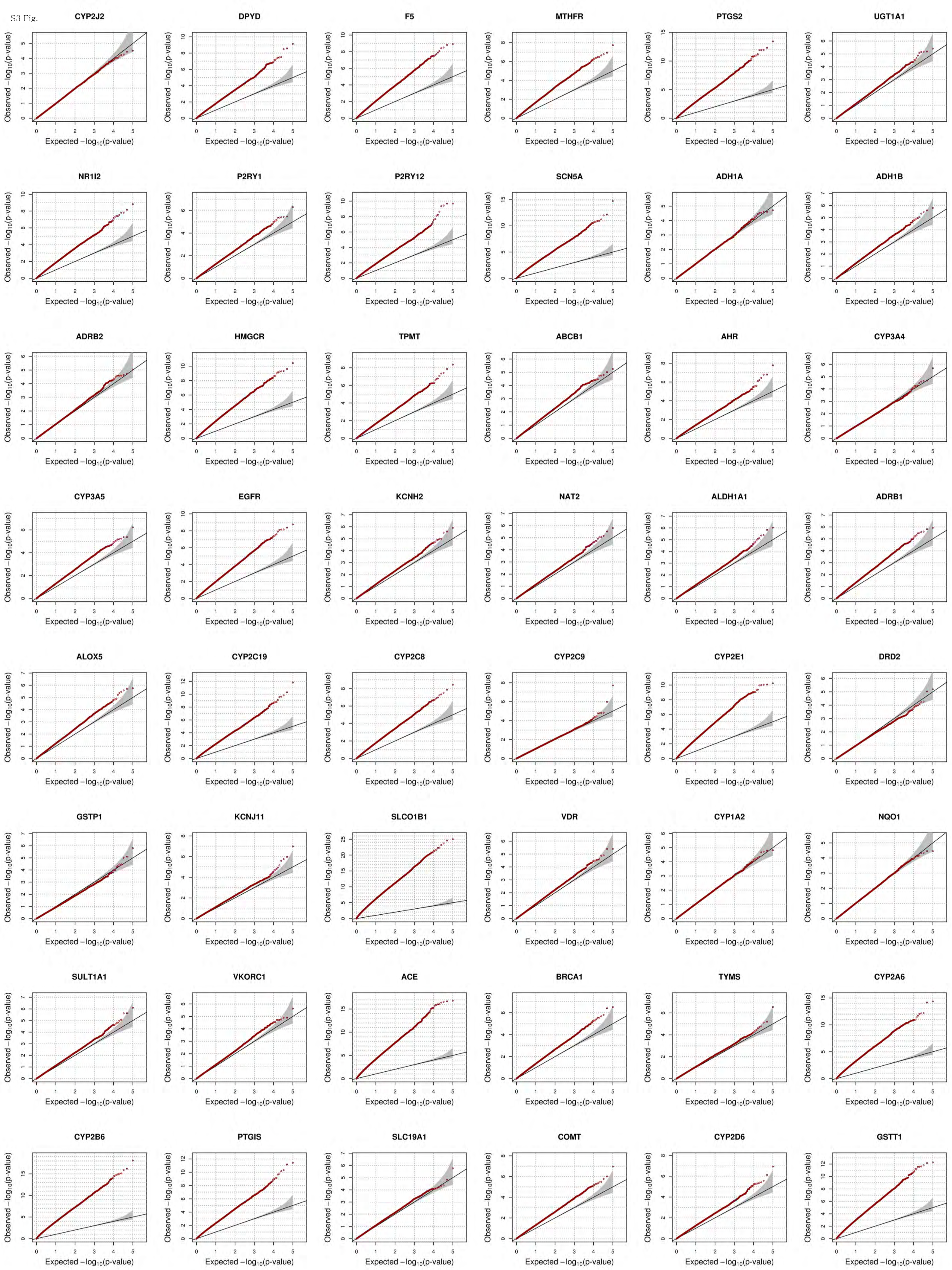
S1 Fig.

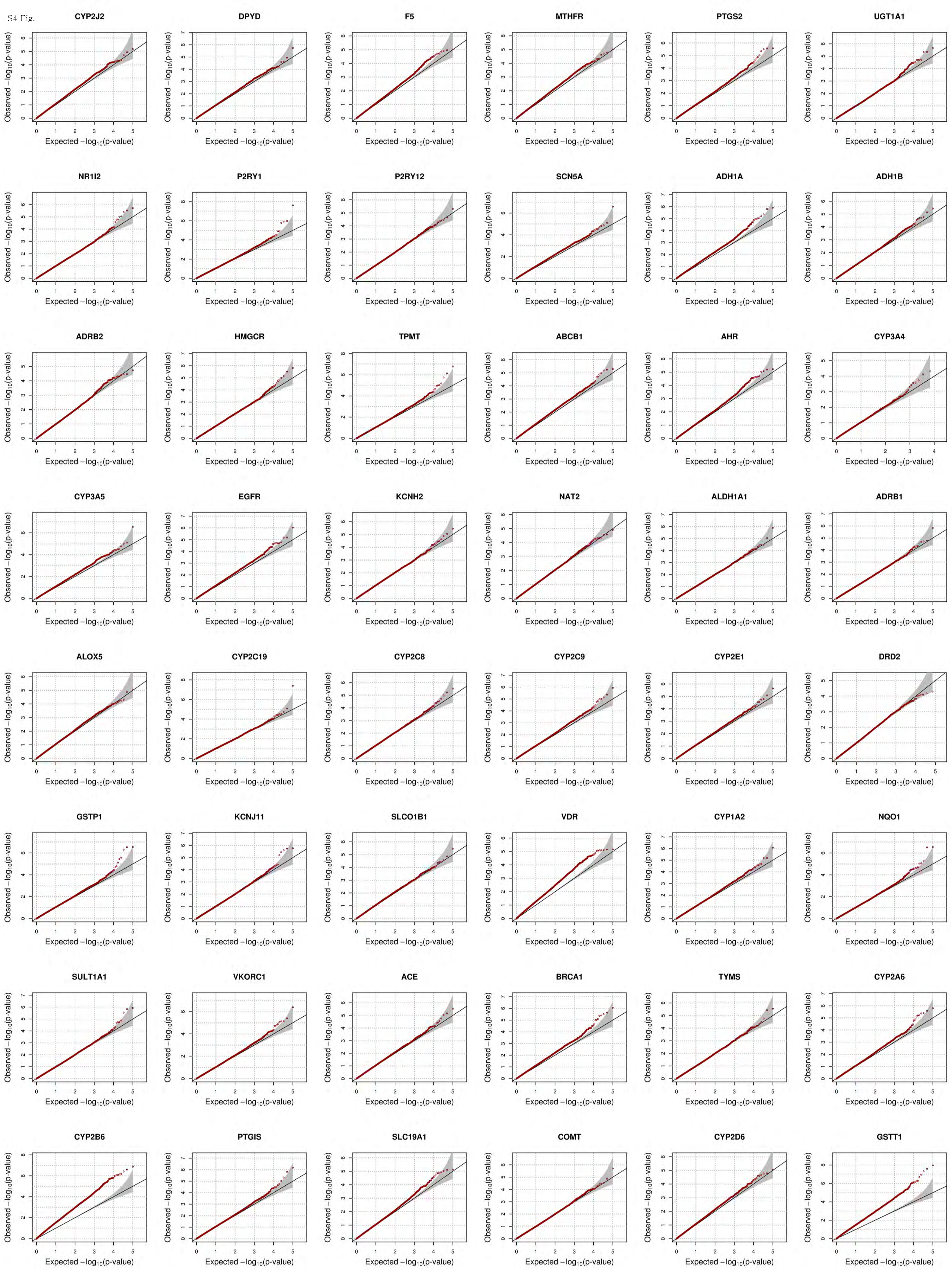
Histogram of $\log_{10}\text{MAF}$

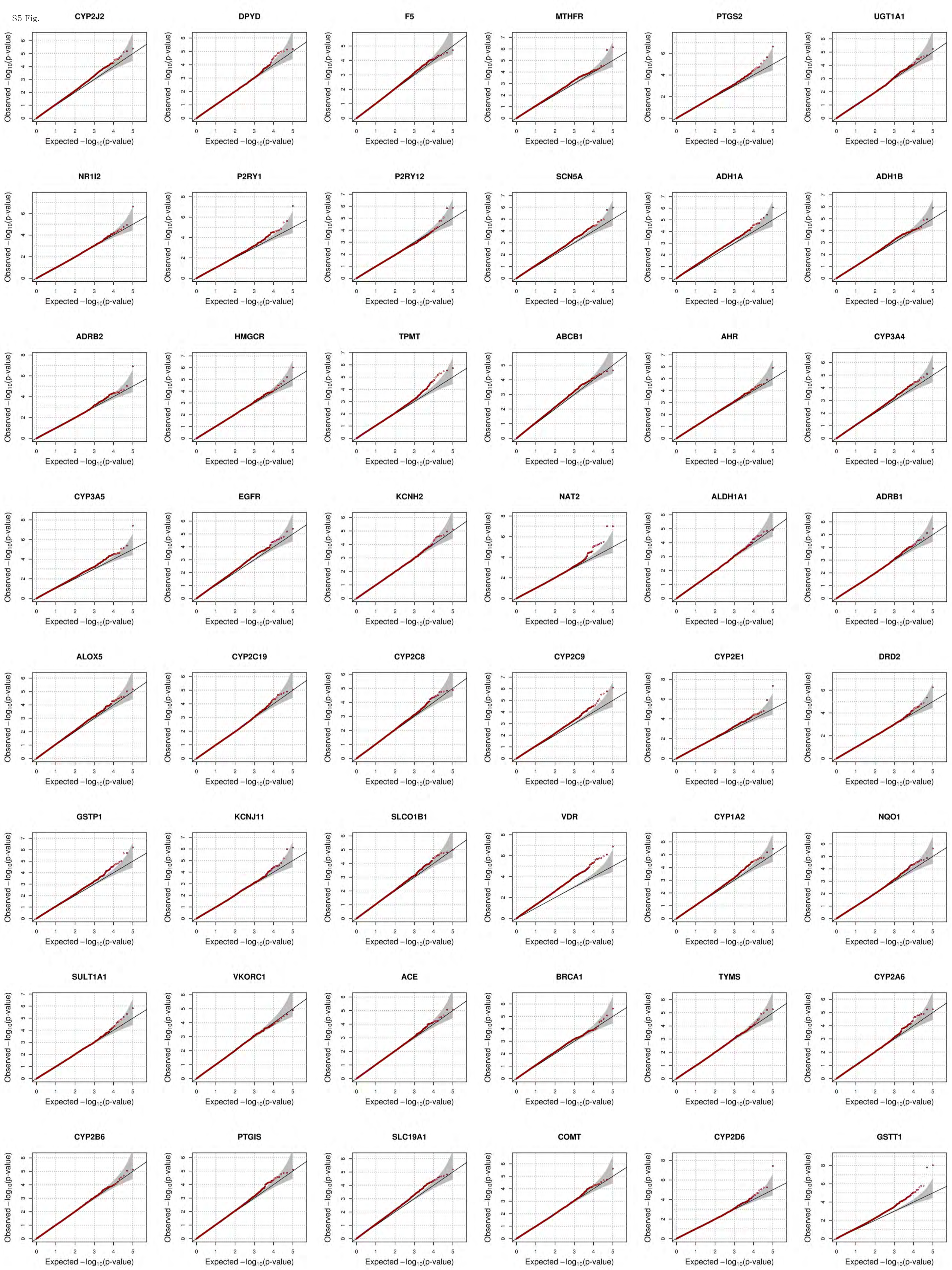


S2 Fig.

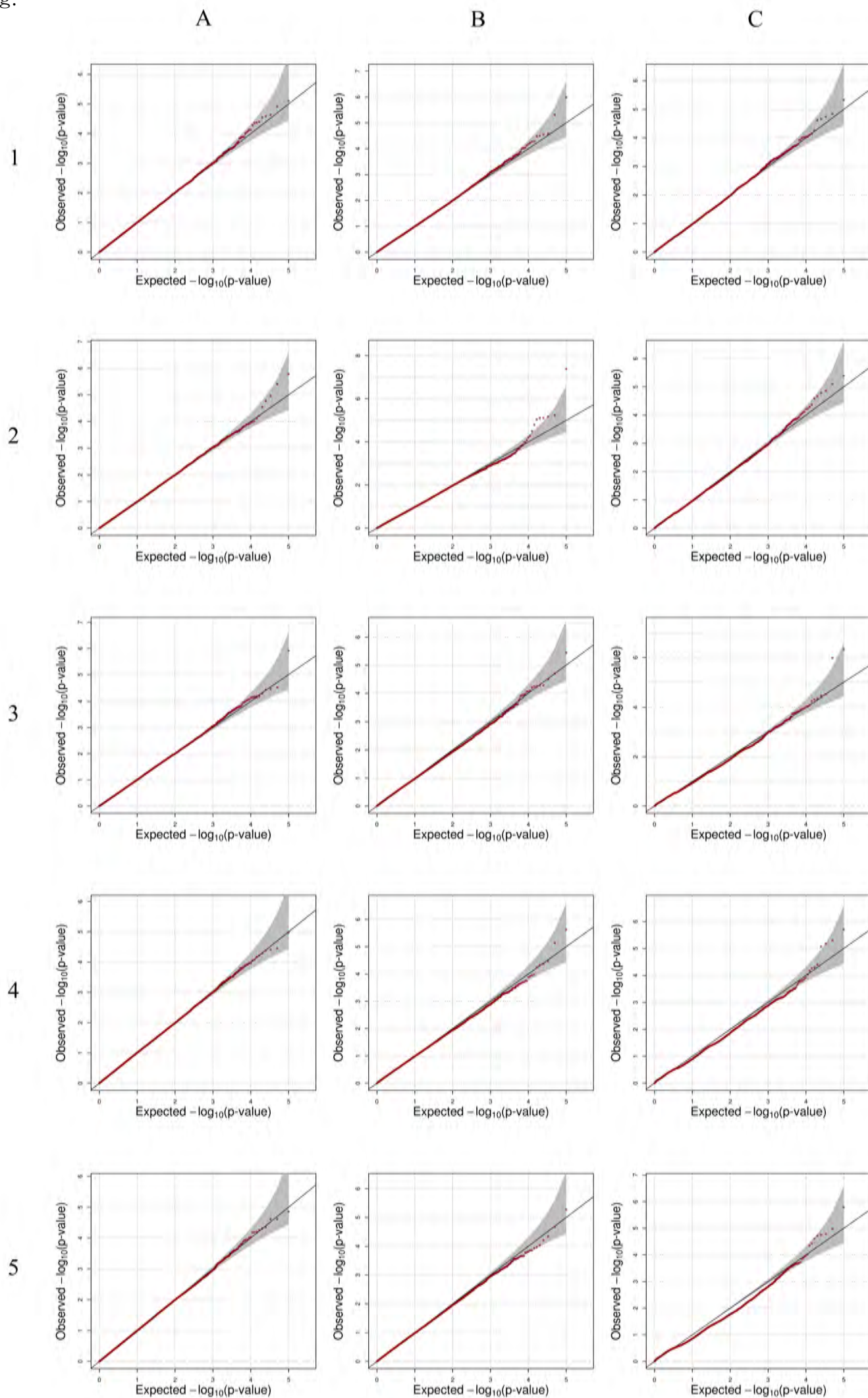




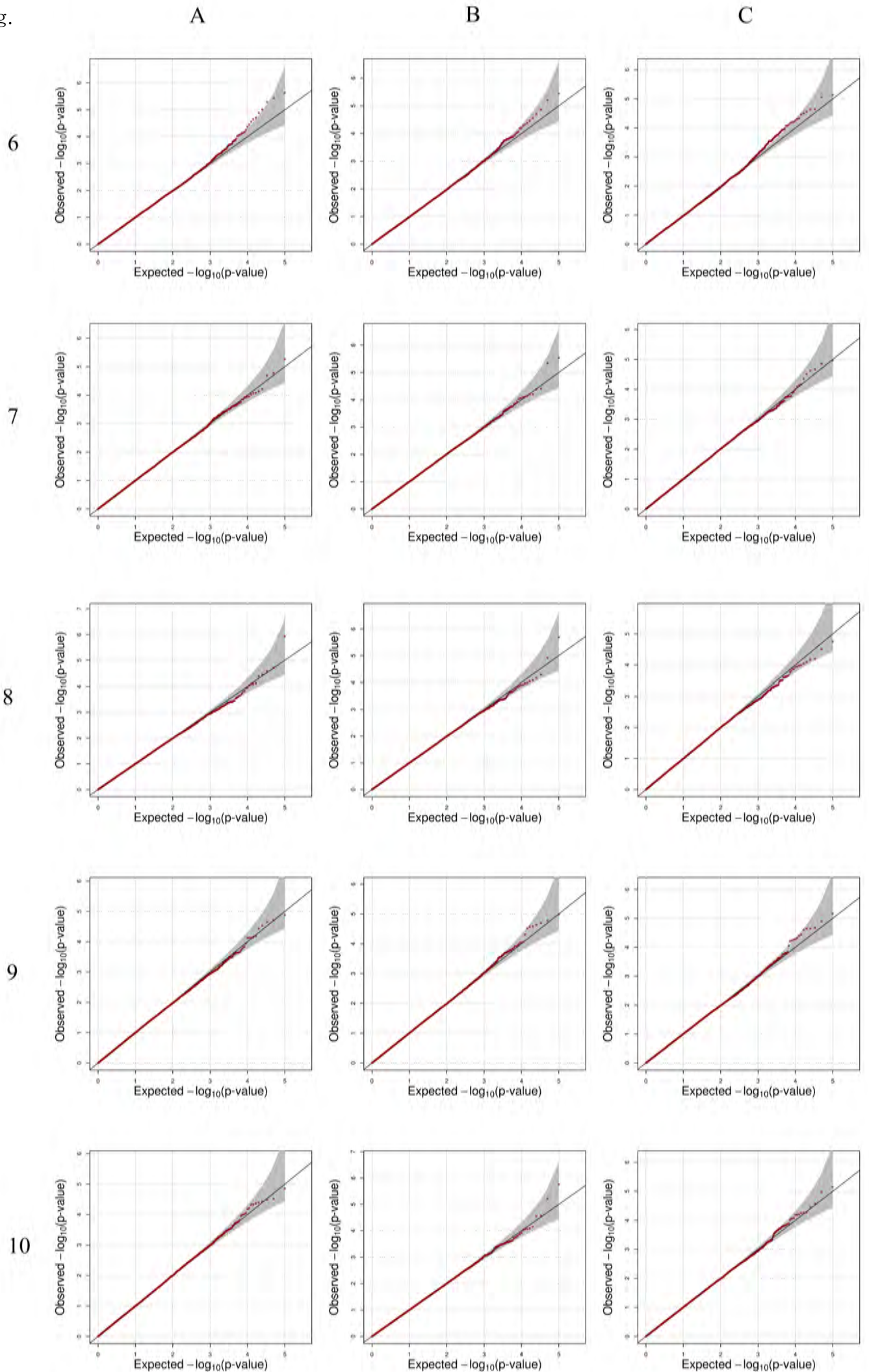




S6 Fig.



S7 Fig.



S8 Fig.

