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ADVANCED SCIENCE
Open Access

Supporting Information

for *Adv. Sci.*, DOI: 10.1002/advs.201700098

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DOI: 10.1002/advs.201700098
Article type: Full Paper

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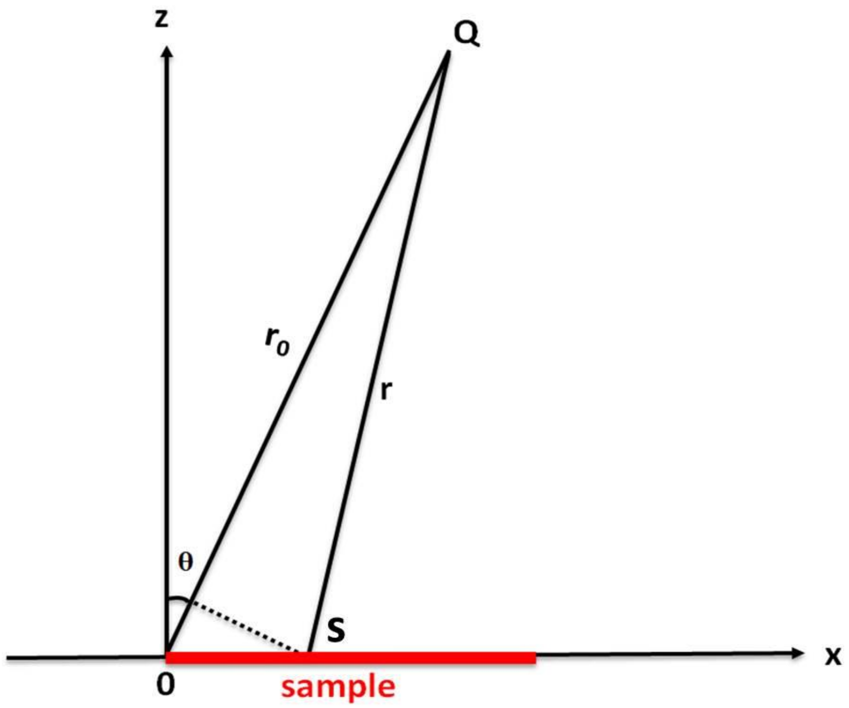
Supporting Text

Supporting Figs. S1

1. Far-field energy of 1-bit frequency coding metamaterials

We use the reflected electric field on the metamaterial $E(x)$ to calculate the far-field energy, as illustrated in Supporting **Figure S1**. Then the reflected energy by the metamaterial sample in the far distance is obtained as

$$E(r) = \int E(x) e^{j\omega t} \times e^{-j\frac{2\pi}{\lambda}r} dx \qquad (16)$$



Supporting Figure S1. Schematic to calculate far-field energy of 1-bit frequency coding metamaterial.

The distance r between points Q and S can be rewritten as

$$r \approx r_0 - x\sin\theta \quad (17)$$

since r is much larger than x. Here, r_0 is the distance of observation point Q to the origin point, and θ is the angle between the observation point to the z axis, as shown in Supporting Figure S1. Hence $E(r)$ is expressed as

$$E(r) = \int E(x) \times e^{j\omega t} \times e^{-j\frac{2\pi}{\lambda}r} dx = e^{j\omega t} \times e^{-j\frac{2\pi}{\lambda}r_0} \int (E(x) \times e^{j2\pi\frac{\sin\theta}{\lambda}x}) dx \quad (18)$$

The phase distribution of 1-bit frequency coding metamaterial is 0, φ, 0, φ, 0, φ … with N periods. Thus $E(x)$ is expressed as:

$$E(x) = \sum_{n=0}^{2N-1} E_0 \times \Pi_D[x - nD] \times e^{j\frac{1}{2}[1-(-1)^n]\varphi} \quad (19)$$

where E_0 is the magnitude of $E(x)$, D is half length of a period, N is the number of periods, and

$$\Pi_D(x) = \begin{cases} 1 & -D/2 \le x \le D/2 \\ 0 & others \end{cases} \quad (20)$$

Then we denote

$$A = e^{j\omega t} \times e^{-j\frac{2\pi}{\lambda}r_0}$$
$$P = -2\pi \cdot \sin\theta / \lambda \qquad (21)$$

Therefore, E(r) can be obtained as

$$E(r) = A \cdot \int (E(x) \times e^{-jPx})dx$$
$$= \mathrm{A} \cdot \int \{\sum_{n=0}^{2N-1} E_0 \times \prod_D [x - nD] \times e^{j\frac{1}{2}[1-(-1)^n]\varphi} \times e^{-jPx}\}dx$$
$$= N \cdot D \cdot A \cdot E_0 \times \sin c(\frac{PD}{2}) \times (1 + e^{j\varphi}) \qquad (22)$$

When θ =0, we have P=-$2\pi \times \sin\theta/\lambda$=0, and the corresponding far filed energy at angle θ=0 is proportional to

$$|E(r)|^2 = N^2 D^2 A^2 E_0^2 \times (1 + e^{j\varphi})^2 \qquad (23)$$

where $N^2D^2A^2E^2$ is unchanged for a given sample. Hence the expression shows that the far-field energy at the angle θ=0 is proportional to $E^2|1+e^{j\varphi}|^2$ and φ is calculated from Equations (3), (5) and (6) in the main text as:

$$\varphi = 7\pi/9 - [7\pi/9 + (-\pi/4.5) \times (f - f_0)] = 2\pi/9 \times (f - f_0) \qquad (22)$$

which increases from 0 to π as the frequency increases from 6 to 10.5GHz. As a consequence, the far-field energy at angle θ=0 should approach zero when the frequency is close to f_1=10.5GHz because $E^2|1+e^{j\varphi}|^2$ is approaching zero when the frequency is close to f_1=10.5GHz.

2. Non-periodic frequency sweeping metamaterials

The phase responses for 2-bit frequency coding units are

$$\varphi(f)^{'00-00'} \approx \frac{7\pi}{9}$$

$$\varphi(f)^{'00-01'} \approx \frac{7\pi}{9} - \frac{\pi}{9}(f - f_0)$$

$$\varphi(f)^{'00-10'} \approx \frac{7\pi}{9} - \frac{2\pi}{9}(f - f_0)$$

$$\varphi(f)^{'00-11'} \approx \frac{7\pi}{9} - \frac{3\pi}{9}(f - f_0) \qquad (23)$$

Once the phase expressions are obtained, the phase differences of adjacent unit cells are calculated as

$$\varphi(f)^{'00-01'}-\varphi(f)^{'00-00'}\approx[\frac{7\pi}{9}-\frac{\pi}{9}(f-f_0)]-\frac{7\pi}{9}=-\frac{\pi}{9}(f-f_0)$$

$$\varphi(f)^{'00-10'}-\varphi(f)^{'00-01'}\approx[\frac{7\pi}{9}-\frac{2\pi}{7}(f-f_0)]-[\frac{7\pi}{9}-\frac{\pi}{9}(f-f_0)]=-\frac{\pi}{9}(f-f_0)$$

$$\varphi(f)^{'00-11'}-\varphi(f)^{'00-10'}\approx[\frac{7\pi}{9}-\frac{3\pi}{7}(f-f_0)]-[\frac{7\pi}{9}-\frac{2\pi}{9}(f-f_0)]=-\frac{\pi}{9}(f-f_0) \tag{24}$$

Note that phase differences between adjacent unit cells are the same and linearly proportional to the change of frequency f - f_0 in the operational band. From the generalized Snell's law, the deflected angle θ of the anomalous reflection beam is given by

$$\theta_r=\sin^{-1}[\frac{\lambda}{2\pi}\bullet\frac{d\varphi}{dx}] \tag{25}$$

Since $\frac{d\varphi}{dx}\approx\frac{\varphi^{'00-01'}-\varphi^{'00-00'}}{L}$ for approximation, in which L is the length of the super unit cell, and λ=$10^{-9}c/f$ (c=3×10^8m/s is the light speed in free space and the unit of f is GHz), the deflected angel θ_r is obtained as

$$\begin{aligned}\theta_r&=\sin^{-1}[\frac{\lambda}{2\pi}\bullet\frac{d\varphi}{dx}]\\&\approx\sin^{-1}\{\frac{10^{-9}\times c}{2\pi f}\bullet[\frac{-\frac{\pi}{9}(f-f_0)}{0.006\times3}]\}\\&=-\sin^{-1}[\frac{25}{27}(1-\frac{f_0}{f})]\end{aligned} \tag{26}$$

Hence the reflection angle $|\theta_r|$ monotonically increases in the operational frequency band.