Appendix A: Related Functions

Incomplete Gamma Function

$$\gamma\left(k, \frac{x}{\theta}\right) = \int_0^{\frac{x}{\theta}} t^{k-1} e^{-t}\, dt \tag{A.1}$$

Mean = kθ (A.2)

Var = $k\theta^2$ (A.3)

Gamma Function

$$\Gamma(k) = \int_0^{\infty} t^{k-1} e^{-t} dt \tag{A.4}$$

Normalized Incomplete Gamma Function

$$P\left(k, \frac{x}{\theta}\right) = \frac{\gamma\left(k, \frac{x}{\theta}\right)}{\Gamma(k)} \tag{A.5}$$

Gamma Distribution p.d.f.

$$p(x|k, \theta) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \tag{A.6}$$

Gamma Distribution c.d.f

$$c(x|k, \theta) = P\left(k, \frac{x}{\theta}\right) \tag{A.7}$$

Normal Distribution p.d.f

$$p(x|\mu, \sigma) = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{\sigma^2}} \tag{A.8}$$

Normal Distribution c.d.f.

$$c(x|\mu, \sigma) = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right) \tag{A.9}$$

Relationship of incomplete gamma function to error function

$$\frac{\Gamma\left(\frac{1}{2}\right)P\left(\frac{1}{2},x\right)}{\sqrt{\pi}} = \operatorname{erf}(x) \tag{A.10}$$

Appendix B: Sigmoidal curve using Normal C.D.F.

The normal p.d.f. is frequently used for values that can range over positive and negative values. In that case the sigmoidal function used in the GEM calculation is the normal c.d.f..

$$\mathrm{GEM} = \frac{\sum_i \left[2^{-(\mathrm{Priority}_i\ -1)} \cdot {}^1\!/_2 \left(1 + \mathrm{erf}\left(\frac{\mathrm{PlanValue}_i\ - \mathrm{ConstraintValue}_i}{q_i\ \cdot \mathrm{ConstraintValue}_i}\right)\right)\right]}{\sum_i 2^{-(\mathrm{Priority}_i - 1)}} \qquad \text{(B.1)}$$

If Upper 90% CI ≥ Constraint Value, q is selected for

$${}^1\!/_2 \left(1 + \mathrm{erf}\left(\frac{\text{Upper 90\% CI}_i - \text{Constraint Value}_i}{q_i \cdot \text{Constraint Value}_i}\right)\right) = 0.95 \qquad \text{(B.2)}$$

If historical values are well below constraint values (Upper 90% $\mathrm{CI}_i < \mathit{Constraint\ Value}_i$), q is set equal to 0.05 approximating a steep step function.