

**Web-based Supplementary Materials for
Groupwise Envelope Models for Imaging Genetic Analysis**

Yeonhee Park^{1,*}, Zhihua Su^{2,} and Hongtu Zhu^{1,***}**

¹Department of Biostatistics, The University of Texas MD Anderson Cancer Center, Houston, TX 77030, U.S.A.

²Department of Statistics, University of Florida, Gainesville, FL 32611, U.S.A.

**email:* ypark3@mdanderson.org

***email:* zhihuasu@stat.ufl.edu

****email:* hzhu5@mdanderson.org

Web Appendix A: Simulation study with non-normal errors

In this simulation section, we investigate the finite-sample performance of the groupwise envelope model when the errors are not normally distributed. We generated the data under the same settings as in Section 4, but with three different non-normal error distributions, including t distribution with degrees of freedom 6, uniform distribution defined on the unit interval, and the chi-squared distribution with degrees of freedom 4. The results are presented in Web Figure 1. For comparison, we included the normal errors in the first row of the figure. From Web Figure 1, we notice that a moderate departure from normality does not cause a difference in the estimation standard deviations.

[Web Figure 1 about here.]

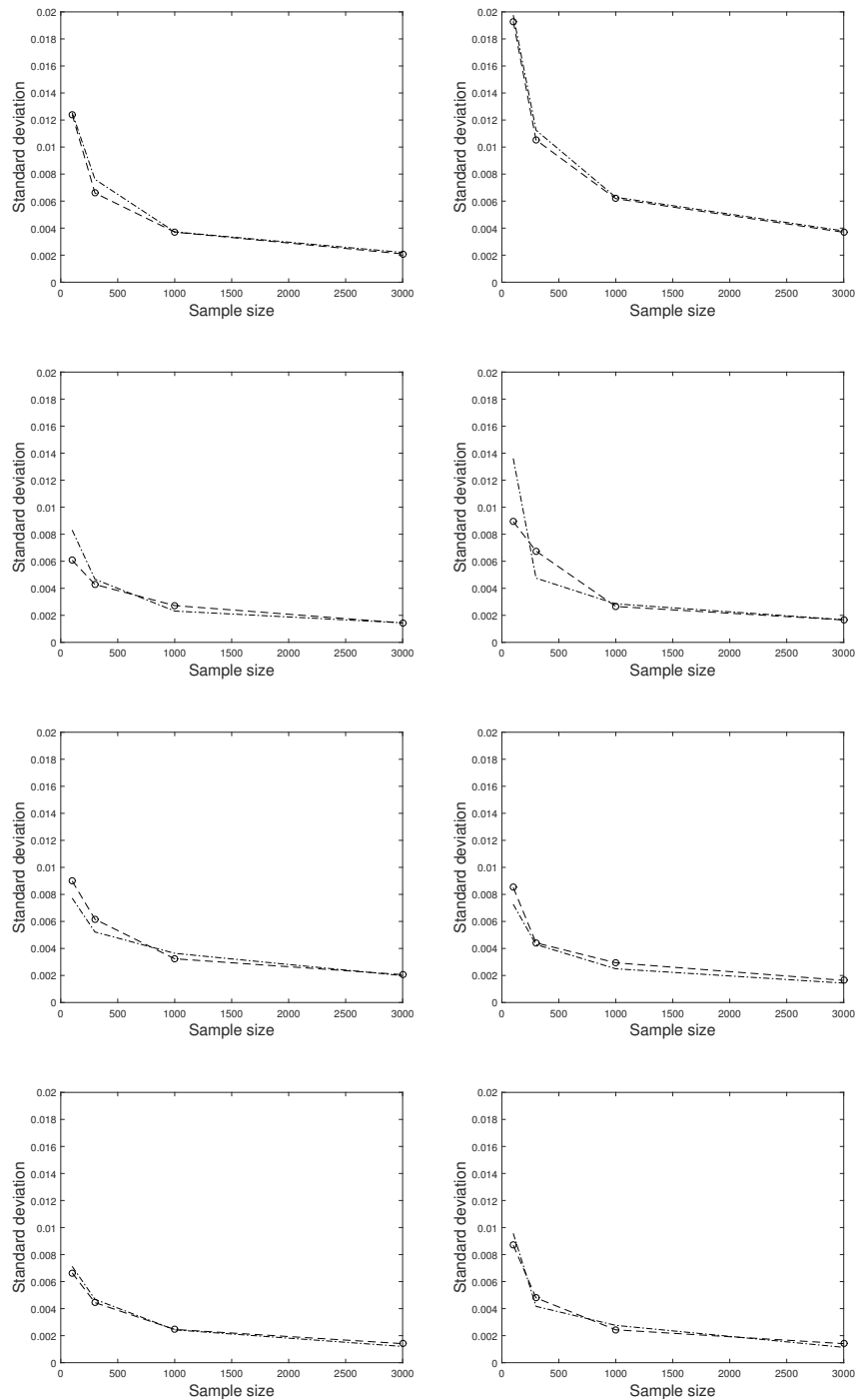
Web Appendix B: Dimension selection using BIC without normality assumption

This simulation study investigates the selection performance of BIC under non-normal errors. When the errors do not follow the normal distribution, we still use the normal likelihood as an objective function for getting the envelope estimators. As shown in Proposition 1, the groupwise envelope estimator is \sqrt{n} -consistent even when the errors do not follow a normal distribution. This suggests that BIC is still a reasonable criterion even if the normality assumption does not hold. To confirm this, we perform the following numerical experiment. We used the same setting as in Section 4, except that we generated the errors from the t distribution with degrees of freedom 6, uniform distribution defined on the unit interval and the chi-squared distribution with degrees of freedom 4. We generated 200 replications for each case and computed the proportion in 200 replications that BIC correctly selects the true dimension u . The result is summarized in Web

Table 1. Web Table 1 indicates that the finite-sample performance of BIC is very stable in this setting.

[Web Table 1 about here.]

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Web Figure1 Left panel: Standard deviations for a random picked element in $\beta_{(1)}$. Right panel: Standard deviations for a random picked element in $\beta_{(2)}$. The four rows corresponds to normal errors, t errors, uniform errors and chi-squared errors. The $-o-$ and $- \cdot -$ lines mark the actual and bootstrap standard deviations of the groupwise envelope model.

Web Table1

The proportion in 200 replications that BIC correctly selects the true dimension

Sample size	t	uniform	χ^2
100	0.9900	1	1
300	0.9900	1	1
1000	0.9950	1	1
3000	1	1	1