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Supplementary Materials for

Entropy-limited topological protection of skyrmions

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section 1. Image processing for the skyrmion decay measurements

Within this section the analysis of the LTEM-images for the skyrmion decay measurements is described. The image processing approach can be divided into the preprocessing and the extraction of physical quantities from the preprocessed images. For the two cases of the skyrmion decay to the helical phase and the decay to the conical phase two completely different image processing approaches were developed. Before describing these methods in detail first the preprocessing steps, which both methods have in common are explained.

Preprocessing

The raw LTEM-images not only contain information about the magnetic texture of the sample, but also nonmagnetic information for example due to crystal contrasts. The first step is hence to subtract the nonmagnetic background from the images. Since the nonmagnetic contrast is mostly present on much larger length scales than the helix pitch length or the size of a skyrmion, it can be removed from the images by subtracting a Gaussian filtered version of the image, where the sigma of the Gausian kernel has to be chosen much larger than the skyrmion size. The signal-to-noise ratio of these images can be further enhanced by applying a Fourier-filter using a suitable bandpass-like filter mask to keep only objects with the lateral dimensions of a skyrmion.

Analysis of the skyrmion decay to the conical phase

In the case of the decay to the conical phase the skyrmions are counted in each frame of the video by an automated detection approach, which is illustrated in fig. S2. The algorithm searches for local maxima of the intensity in each of the images. Each local maximum is considered as a candidate for a skyrmion and is further tested by computing a “local threshold” by comparing the image intensity at the possible skyrmion position with the intensity in a larger area around this position. If the skyrmion candidate exceeds a certain threshold value it will be counted as a skyrmion.

Analysis of the decay to the helical phase - skyrmion number method

The decay of the skyrmions to the helical phase is investigated by two different methods. One of them relies on counting the skyrmion number versus time as the skyrmions merge into the stripes of the helical phase. The other approach is based on tracing the intensity of the two peaks of the helical phase in reciprocal space versus time and is described within the next subsection.

In contrast to the previous subsection the skyrmion counting approach cannot be based on finding local maxima of the intensity, since not only the skyrmions but also the stripes of the helical phase would contribute local maxima. Hence a different skyrmion detection approach was developed and is illustrated in fig. S3. It relies on finding lines of equal intensity within the video frames by an edge detection algorithm using the Laplacian-of-Gaussian (LoG)-method. After processing with the edge detection skyrmions can be identified as circular contours with a certain enclosed area. The skyrmions are detected by identifying the connected components in the inverted contour image and subjecting each connected component to certain test criteria. The first criterium is that the area enclosed within the contour lies in a certain range, the second criterium is that the eccentricity of the contour has to be low enough (skyrmions are circular) and the final criterium is that the local threshold of the skyrmion compared to its environment is large enough.

Analysis of the decay to the helical phase - order method

The second method for the evaluation of the transition to the helical phase relies on an analysis of the video frames in reciprocal space. In the skyrmion phase the Fourier-transformed image shows six spots. During the transition intensity is transferred from the six spots into the two spots of the helical phase. By defining a ring-shaped mask and comparing the intensity (averaged absolute value of the FFT) of the helical spots with the intensity of the complement of these spots with respect to the ring-shaped mask, while normalizing to the overall intensity, an order parameter can be defined as follows

$$A_{FFT} = \frac{I_{angle} - I_{compl}}{I_{angle} + I_{compl}} \qquad (1)$$

This order parameter can be evaluated for each frame in the movies. Since during the transition to the helical phase intensity from the six spots of the skyrmion lattice is transferred into the two spots of the helical phase, the order parameter A_{FFT} will increase with time as shown in S4. Note that the decay time for skyrmion order is shorter than the decay time for the reduction of the number of skyrmions.

section 2. Evaluation of intermediate states

The evaluation of the intermediate state intensity levels which were observed during the skyrmion decay into the conical phase was done as follows. First, the temporal average intensity of the spatially static skyrmion, intermediate state and conical phase is calculated. Second, a linescan through the maximum intensity of the respective spin object is taken and fitted by a Gaussian function, which gives the skyrmion and intermediate state intensity I_{Sk} and I_i. The intensity level of the conical background I_{con} is calculated by averaging within an area of the former skyrmion size. The levels shown in Fig. 3 (F) of the main text are then calculated by $\frac{I_L}{I_0} = \frac{I_i - I_{con}}{I_{Sk} - I_{con}}$. The temporal linescan in Fig. 3 (G) of the main text is obtained by averaging the intensity in a small circular area of approximately half the skyrmion diameter around the skyrmion core position that is tracked in time.

section 3. Scaling analysis of activation energies

As the size of skyrmions is much larger than lattice constants, one can efficiently describe their proporties in the continuum limit by a field theory. Such a description will, however, break down in the center of a Bloch point where the spin configuration is singular. Within a non-linear sigma model the energy of magnetic textures is computed from

$$E = \int d^3r \, A(\boldsymbol{\nabla}\hat{n})^2 + D\hat{n} \cdot \boldsymbol{\nabla} \times \hat{n} - \mu_0 H M \hat{n}_3 - \frac{\mu_0 M}{2} \boldsymbol{H_D} \hat{n} \qquad (2)$$

where $\boldsymbol{H}_D(r) = -\frac{M}{4\pi}\boldsymbol{\nabla}\int d^3r' \frac{r-r'}{|r-r'|^3}\hat{n}(r')$ is the demagnetization field from dipole-dipole interactions. Skyrmion formation is governed by the length scale $2\frac{A}{D}$. Therefore, we rewrite the energy by expressing all distances in this length scale. In the resulting dimensionless units, we label coordinates by $\mathbf{x}$ with $\boldsymbol{r} = \frac{2A}{D}\boldsymbol{x}$ and the wavelength of the helical state is 2π. The thickness d_s of the sample can be expressed by the dimensionless $d = \frac{d_s D}{2A}$. The energy functional then takes the form

$$E = \frac{4A^2}{D}\int d^3x \frac{1}{2}(\boldsymbol{\nabla}\hat{n})^2 + \hat{n}\cdot\boldsymbol{\nabla}\times\hat{n} - h\hat{n}_3 - \frac{\tau_D}{2}\hat{n}\cdot\boldsymbol{h}_D \qquad (3)$$

with the dimensionless demagnetization field $h_D(x) = -\frac{1}{4\pi}\boldsymbol{\nabla}\int d^3x' \frac{x-x'}{|x-x'|^3}\hat{n}(x')$, controlled by two dimensionless parameters h and τ_D, where $h = \frac{2A\mu_0 M}{D^2}H$ is the dimensionless magnetic field measured in units such that the transition from the conical to ferromagnetic state takes place at $h = 1$ and $\tau_D = \frac{2A\mu_0 M^2}{D^2}$ is a dimensionless measure for the strength of the dipole-dipole interaction which typically takes values of order one [with, e.g., $\tau_D = 0.65$ for $Fe_{0.8}Co_{0.2}Si$ (*44*)]. The activation energy ΔE for the decay of the skyrmion is determined by a high-energy state containing a pair of Bloch points [23, 25]. The continuum description breaks down at the core of these singular spin configurations. This regime contributes the core energy E_C which depends on microscopic details. In a metal, for example, the details of the electronic band-structure will enter. Nevertheless, one can estimate that this energy is of the order of the exchange coupling J, which can be estimated from the spin-stiffness A using $E_c \sim J \sim Aa$, where a is the relevant microscopic length scale (e.g. distance of magnetic ions).

These arguments show that the activation energy can be written in the form

$$\Delta E = \frac{4A^2}{D}f(h,\tau_D,d) + O(Aa) = \frac{JN_s}{2\pi}f(h,\tau_D,d) + O(J) \quad (4)$$

where f is a dimensionless scaling function which depends on the dimensionless parameters h, τ_D and d and is independent of the cutoff parameter a in $d = 3$. For the second equality, we identified $A = \frac{J}{2a}$ with the exchange coupling J of a classical Heisenberg model on a cubic lattice with lattice constant a and introduced $N_s = \frac{4\pi A}{Da}$, the number of spins per pitch of the helix. Importantly, for $Fe_{0.5}Co_{0.5}Si$, where the pitch of the helix is $\lambda \sim 90$ nm, the universal first term arising from the continuum model, is expected to exceed the non-universal second term by a large factor of order 10^2 for temperatures $T \ll T_c$ where the above analysis applies. Previous theoretical results computed ΔE for $N_s \sim 10$ and $\tau_D = 0$ [23, 25]. Assuming that the numerical results were dominated by the universal contribution, we can estimate $f(0,0,31) \approx 3.6$ [9] and $f(0.8,0,12) \approx f(0.8,0,18) \approx 13$ [23] for the decay into the helical and conical state, respectively. Using $J \sim k_B T_c$ and $N_s \approx 300$ for an order-of-magnitude estimate, we obtain $\frac{4A^2}{D} \sim \frac{JN_S}{2\pi} \sim 150$ meV. Using this value, we conclude that the predicted activation energies are about an order of magnitude larger than the measured ones.

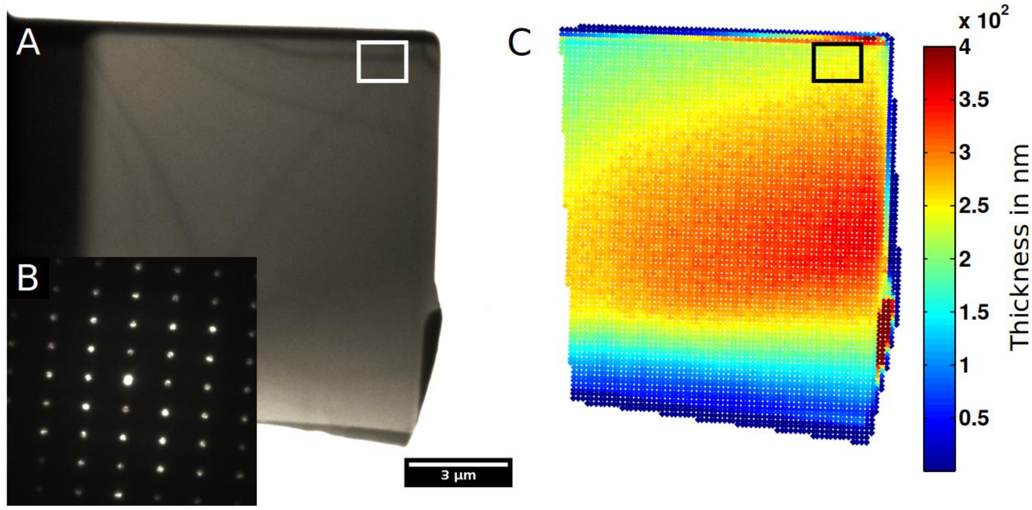

fig. S1. Thickness determination of the FIB lamella. (**A**) shows the FIB prepared sample with a dimension 10×10 µm of A diffraction pattern in (**B**) confirms the thinning of the specimen in [100] direction. The thickness extracted from HAADF-STEM in (**C**) gives a thickness of 241 ± 8 nm for the measurement position.

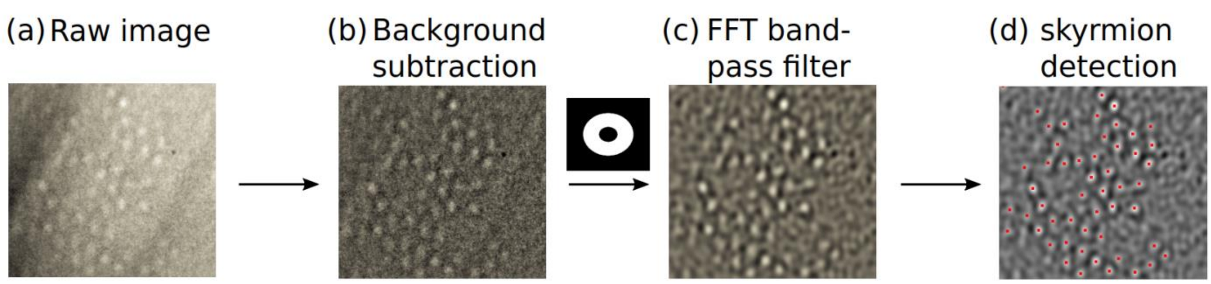

fig. S2. Illustration of the evaluation of the skyrmion decay to the conical phase. (**a**) shows raw data, (**b**) the image after background subtraction and (**c**) a bandpass Fourier-filtered version, which is used for the local maximum detection in step (**d**) resulting in the skyrmion positions.

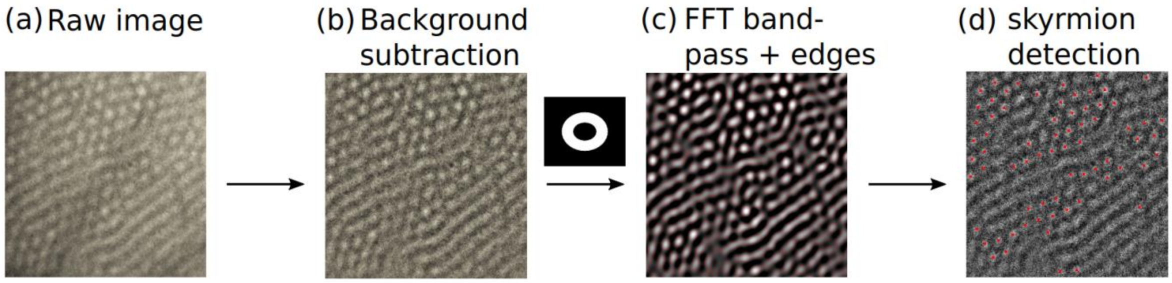

fig. S3. Illustration of the evaluation of the skyrmion decay to the helical phase. (**a**) shows raw data, (**b**) the image after background subtraction, (**c**) a bandpass Fourier-filtered version with contours obtained by the LoG-edge detection algorithm. In (**d**) the skyrmion positions obtained from the contour image are visualized by red squares.

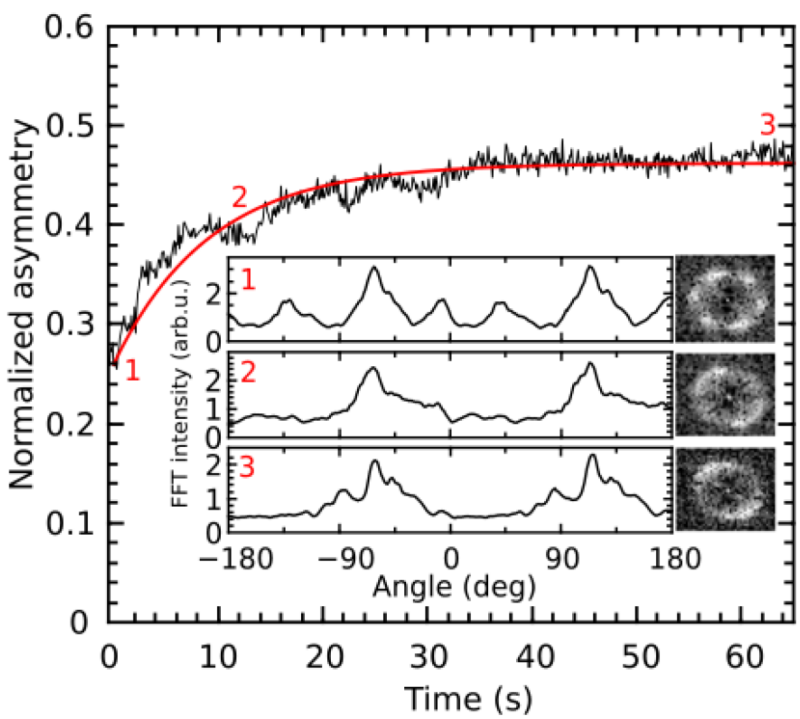

fig. S4. Reduction of the skyrmion order deduced from the Fourier transform of the real-space data. Reduction of skyrmion order deduced from the Fourier transform of the real space data after the magnetic field has been reduced to $B_{ext} = -2.6$ mT from an initial field value of $B_{ext} = 23$ mT. The insets show radial averages at distinct times over the FT data shown at the right. The measurement temperature is $T_m = 20.4$ K.