
Supplementary Material for “Evolution of a male mating preference for a dual-utility trait used in female intrasexual competition in genetically monogamous populations”

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I. Analysis of the model and generation of the main text figures

A. Derivation of the model

Allele frequencies and variables

x_1 = freq of T1P1 genotype

x_2 = freq of T1P2 genotype

x_3 = freq of T2P1 genotype

x_4 = freq of T2P2 genotype

s = viability cost of carrying the trait allele for females

α = viability cost of carrying the trait allele for males

μ = rate at which T2 mutates to T1 in each generation

c = cost of the preference allele

ρ = strength of aversion that P2 males express to T1 females

f = fecundity benefit gained by males that mate with a T2 female (all matings with T2 females have fecundity = $1+f$, where $0 \leq f$, while matings with T1 females have fecundity = 1)

```
In[1]:= ClearAll[x1, x2, x3, x4, x1vf, x2vf, x3vf, x4vf, x1vm,
             x2vm, x3vm, x4vm,  $\rho$ , s, r, f,  $\mu$ , F, p2, t2, diseq,  $\alpha$ , c]
```

```
In[2]:=  $\alpha = s$ ;
```

If analyzing with no cost of preference, set $c = 0$ here:

```
In[3]:=  $c = 0$ ;
```

Mutation and viability selection

$x_i \mu$ = frequency of x_i individuals after mutation

$$\text{In[4]:= } \mathbf{x1\mu = x1 + \mu * x3}$$

$$\mathbf{x2\mu = x2 + \mu * x4}$$

$$\mathbf{x3\mu = x3 * (1 - \mu)}$$

$$\mathbf{x4\mu = x4 * (1 - \mu)}$$

$$\text{Out[4]= } x1 + x3 \mu$$

$$\text{Out[5]= } x2 + x4 \mu$$

$$\text{Out[6]= } x3 (1 - \mu)$$

$$\text{Out[7]= } x4 (1 - \mu)$$

viability selection:

x_{ivf} = frequency of x_i males after viability selection

x_{ivm} = frequency of x_i females after viability selection

```
In[8]:= x1vf =
  FullSimplify[x1 $\mu$  / (x1 $\mu$  + x2 $\mu$  * (1 - c) + x3 $\mu$  * (1 -  $\alpha$ ) + x4 $\mu$  * (1 -  $\alpha$ ) * (1 - c))]
x2vf = FullSimplify[(x2 $\mu$  * (1 - c)) /
  (x1 $\mu$  + x2 $\mu$  * (1 - c) + x3 $\mu$  * (1 -  $\alpha$ ) + x4 $\mu$  * (1 -  $\alpha$ ) * (1 - c))]
x3vf = FullSimplify[(x3 $\mu$  * (1 -  $\alpha$ )) /
  (x1 $\mu$  + x2 $\mu$  * (1 - c) + x3 $\mu$  * (1 -  $\alpha$ ) + x4 $\mu$  * (1 -  $\alpha$ ) * (1 - c))]
x4vf = FullSimplify[(x4 $\mu$  * (1 -  $\alpha$ ) * (1 - c)) /
  (x1 $\mu$  + x2 $\mu$  * (1 - c) + x3 $\mu$  * (1 -  $\alpha$ ) + x4 $\mu$  * (1 -  $\alpha$ ) * (1 - c))]

Out[8]= 
$$\frac{x_1 + x_3 \mu}{x_1 + x_2 + (x_3 + x_4) (1 + s (-1 + \mu))}$$


Out[9]= 
$$\frac{x_2 + x_4 \mu}{x_1 + x_2 + (x_3 + x_4) (1 + s (-1 + \mu))}$$


Out[10]= 
$$\frac{(-1 + s) x_3 (-1 + \mu)}{x_1 + x_2 + (x_3 + x_4) (1 + s (-1 + \mu))}$$


Out[11]= 
$$\frac{(-1 + s) x_4 (-1 + \mu)}{x_1 + x_2 + (x_3 + x_4) (1 + s (-1 + \mu))}$$

```

```
In[12]:= x1vm = FullSimplify[x1μ / (x1μ + x2μ + x3μ * (1 - s) + x4μ * (1 - s))]
x2vm = FullSimplify[x2μ / (x1μ + x2μ + x3μ * (1 - s) + x4μ * (1 - s))]
x3vm = FullSimplify[(x3μ * (1 - s)) / (x1μ + x2μ + x3μ * (1 - s) + x4μ * (1 - s))]
x4vm = FullSimplify[(x4μ * (1 - s)) / (x1μ + x2μ + x3μ * (1 - s) + x4μ * (1 - s))]
```

$$\text{Out[12]= } \frac{x1 + x3 \mu}{x1 + x2 + (x3 + x4) (1 + s (-1 + \mu))}$$

$$\text{Out[13]= } \frac{x2 + x4 \mu}{x1 + x2 + (x3 + x4) (1 + s (-1 + \mu))}$$

$$\text{Out[14]= } \frac{(-1 + s) x3 (-1 + \mu)}{x1 + x2 + (x3 + x4) (1 + s (-1 + \mu))}$$

$$\text{Out[15]= } \frac{(-1 + s) x4 (-1 + \mu)}{x1 + x2 + (x3 + x4) (1 + s (-1 + \mu))}$$

Mating

PiTjs = frequency of matings between Pi males and Tj females

For P1 males, the availability of T2 females for mating is affected by the frequency of P2 males

```
In[16]:= P2T1s = Simplify[(((x2vf + x4vf) / (x1vf + x2vf + x3vf + x4vf)) *
((x1vm + x2vm) / (x1vm + x2vm + x3vm + x4vm)) * (1 - ρ)];
P1T1s = Simplify[(((x1vf + x2vf) / (x1vf + x2vf + x3vf + x4vf)) - P2T1s];
P2T2s = Simplify[(((x2vf + x4vf) / (x1vf + x2vf + x3vf + x4vf)) - P2T1s];
P1T2s = Simplify[(((x1vf + x3vf) / (x1vf + x2vf + x3vf + x4vf)) - P1T1s];
```

Total frequency of P1 males: x1+x3

of P2 males: x2+x4

of T1 females: x1+x2

of T2 females: x_3+x_4

Within each mating type, the relative frequency of genotypes:

male P2T2: $x_4/(x_2+x_4)$

male P2T1: $x_2/(x_2+x_4)$

male PIT2: $x_3/(x_1+x_3)$

male PIT1: $x_1/(x_1+x_3)$

female P2T2: $x_4/(x_3+x_4)$

female PIT2: $x_3/(x_3+x_4)$

female P2T1: $x_2/(x_1+x_2)$

female PIT1: $x_1/(x_1+x_2)$

m_{ij} = frequency of x_i male genotype mating with x_j female genotype

Frequency of x_4 male genotype mating with x_3 female genotype =
 m_{43} = relative freq of the male genotype among P2 males * relative
 freq of the female genotype among T2 females * relative freq of a
 P2T2 mating

```

In[20]:= m11 = FullSimplify[(x1vf / (x1vf + x3vf)) * (x1vm / (x1vm + x2vm)) * P1T1s];
m12 = FullSimplify[(x1vf / (x1vf + x3vf)) * (x2vm / (x1vm + x2vm)) * P1T1s];
m13 = FullSimplify[(x1vf / (x1vf + x3vf)) * (x3vm / (x3vm + x4vm)) * P1T2s];
m14 = FullSimplify[(x1vf / (x1vf + x3vf)) * (x4vm / (x3vm + x4vm)) * P1T2s];
m21 = FullSimplify[(x2vf / (x2vf + x4vf)) * (x1vm / (x1vm + x2vm)) * P2T1s];
m22 = FullSimplify[(x2vf / (x2vf + x4vf)) * (x2vm / (x1vm + x2vm)) * P2T1s];
m23 = FullSimplify[(x2vf / (x2vf + x4vf)) * (x3vm / (x3vm + x4vm)) * P2T2s];
m24 = FullSimplify[(x2vf / (x2vf + x4vf)) * (x4vm / (x3vm + x4vm)) * P2T2s];
m31 = FullSimplify[(x3vf / (x1vf + x3vf)) * (x1vm / (x1vm + x2vm)) * P1T1s];
m32 = FullSimplify[(x3vf / (x1vf + x3vf)) * (x2vm / (x1vm + x2vm)) * P1T1s];
m33 = FullSimplify[(x3vf / (x1vf + x3vf)) * (x3vm / (x3vm + x4vm)) * P1T2s];
m34 = FullSimplify[(x3vf / (x1vf + x3vf)) * (x4vm / (x3vm + x4vm)) * P1T2s];
m41 = FullSimplify[(x4vf / (x2vf + x4vf)) * (x1vm / (x1vm + x2vm)) * P2T1s];
m42 = FullSimplify[(x4vf / (x2vf + x4vf)) * (x2vm / (x1vm + x2vm)) * P2T1s];
m43 = FullSimplify[(x4vf / (x2vf + x4vf)) * (x3vm / (x3vm + x4vm)) * P2T2s];
m44 = FullSimplify[(x4vf / (x2vf + x4vf)) * (x4vm / (x3vm + x4vm)) * P2T2s];

```

Check: should add up to relative frequency of each male genotype, and total to 1:

```

In[36]:= m1j = FullSimplify[m11 + m12 + m13 + m14];
m2j = FullSimplify[m21 + m22 + m23 + m24];
m3j = FullSimplify[m31 + m32 + m33 + m34];
m4j = FullSimplify[m41 + m42 + m43 + m44];
FullSimplify[m1j + m2j + m3j + m4j];

In[41]:= mi1 = FullSimplify[m11 + m21 + m31 + m41];
mi2 = FullSimplify[m12 + m22 + m32 + m42];
mi3 = FullSimplify[m13 + m23 + m33 + m43];
mi4 = FullSimplify[m14 + m24 + m34 + m44];
FullSimplify[mi1 + mi2 + mi3 + mi4];

```

Fecundity selection

All matings with T2 females have fecundity = $1+f$, where $0 \leq f$,
while matings with T1 females have fecundity = 1

```
In[46]:= m11f = FullSimplify[m11 * 1];
m12f = FullSimplify[m12 * 1];
m13f = FullSimplify[m13 * (1 + f)];
m14f = FullSimplify[m14 * (1 + f)];
m21f = FullSimplify[m21 * 1];
m22f = FullSimplify[m22 * 1];
m23f = FullSimplify[m23 * (1 + f)];
m24f = FullSimplify[m24 * (1 + f)];
m31f = FullSimplify[m31 * 1];
m32f = FullSimplify[m32 * 1];
m33f = FullSimplify[m33 * (1 + f)];
m34f = FullSimplify[m34 * (1 + f)];
m41f = FullSimplify[m41 * 1];
m42f = FullSimplify[m42 * 1];
m43f = FullSimplify[m43 * (1 + f)];
m44f = FullSimplify[m44 * (1 + f)];
```

Normalize by f_b , which is the mean fecundity across the population

```
In[62]:= fb = FullSimplify[1 * (m11 + m12 + m21 + m22 + m31 + m32 + m41 + m42) +
(1 + f) * (m13 + m14 + m23 + m24 + m33 + m34 + m43 + m44)];
```


Normalized matings are thus:

```
In[63]:= no11 = FullSimplify[m11f / fb];
no12 = FullSimplify[m12f / fb];
no13 = FullSimplify[m13f / fb];
no14 = FullSimplify[m14f / fb];
no21 = FullSimplify[m21f / fb];
no22 = FullSimplify[m22f / fb];
no23 = FullSimplify[m23f / fb];
no24 = FullSimplify[m24f / fb];
no31 = FullSimplify[m31f / fb];
no32 = FullSimplify[m32f / fb];
no33 = FullSimplify[m33f / fb];
no34 = FullSimplify[m34f / fb];
no41 = FullSimplify[m41f / fb];
no42 = FullSimplify[m42f / fb];
no43 = FullSimplify[m43f / fb];
no44 = FullSimplify[m44f / fb];
```

Table format

```
In[79]:= ClearAll[F]
```

```
In[80]:= F = Table[0, {i, 4}, {j, 4}]
```

```
Out[80]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
In[81]:= F[[1, 1]] = no11;  
F[[1, 2]] = no12;  
F[[1, 3]] = no13;  
F[[1, 4]] = no14;  
F[[2, 1]] = no21;  
F[[2, 2]] = no22;  
F[[2, 3]] = no23;  
F[[2, 4]] = no24;  
F[[3, 1]] = no31;  
F[[3, 2]] = no32;  
F[[3, 3]] = no33;  
F[[3, 4]] = no34;  
F[[4, 1]] = no41;  
F[[4, 2]] = no42;  
F[[4, 3]] = no43;  
F[[4, 4]] = no44;
```

```
In[97]:= F;
```

```
In[98]:= FullSimplify[F[[1, 1]] + F[[1, 2]] + F[[1, 3]] + F[[1, 4]] +  
F[[2, 1]] + F[[2, 2]] + F[[2, 3]] + F[[2, 4]] + F[[3, 1]] + F[[3, 2]] +  
F[[3, 3]] + F[[3, 4]] + F[[4, 1]] + F[[4, 2]] + F[[4, 3]] + F[[4, 4]]]
```

```
Out[98]= 1
```

Recursions

Recursions in terms of genotypes:

```
In[99]:= ClearAll[r]
```

$$\begin{aligned} \text{In[100]:= } \mathbf{xt1}[1] &= \mathbf{F}[[1, 1]] + (1/2) \mathbf{F}[[1, 2]] + (1/2) \mathbf{F}[[1, 3]] + \\ & (1/2) (1-r) \mathbf{F}[[1, 4]] + (1/2) \mathbf{F}[[2, 1]] + (1/2) r \mathbf{F}[[2, 3]] + \\ & (1/2) \mathbf{F}[[3, 1]] + (1/2) r \mathbf{F}[[3, 2]] + (1/2) (1-r) \mathbf{F}[[4, 1]]; \end{aligned}$$

$$\begin{aligned} \text{In[101]:= } \mathbf{xt1}[2] &= (1/2) \mathbf{F}[[1,2]] + (1/2) r \mathbf{F}[[1,4]] + \\ & (1/2) \mathbf{F}[[2,1]] + \mathbf{F}[[2,2]] + (1/2) (1-r) \mathbf{F}[[2,3]] + \\ & (1/2) \mathbf{F}[[2,4]] + (1/2) (1-r) \mathbf{F}[[3,2]] + \\ & (1/2) r \mathbf{F}[[4,1]] + (1/2) \mathbf{F}[[4,2]]; \end{aligned}$$

$$\begin{aligned} \text{In[102]:= } \mathbf{xt1}[3] &= (1/2) \mathbf{F}[[1,3]] + (1/2) r \mathbf{F}[[1,4]] + \\ & (1/2) (1-r) \mathbf{F}[[2,3]] + (1/2) \mathbf{F}[[3,1]] + \\ & (1/2) (1-r) \mathbf{F}[[3,2]] + \mathbf{F}[[3,3]] + (1/2) \mathbf{F}[[3,4]] + \\ & (1/2) r \mathbf{F}[[4,1]] + (1/2) \mathbf{F}[[4,3]]; \end{aligned}$$

$$\begin{aligned} \text{In[103]:= } \mathbf{xt1}[4] &= \\ & (1/2) (1-r) \mathbf{F}[[1,4]] + (1/2) r \mathbf{F}[[2,3]] + \\ & (1/2) \mathbf{F}[[2,4]] + (1/2) r \mathbf{F}[[3,2]] + (1/2) \mathbf{F}[[3,4]] + \\ & (1/2) (1-r) \mathbf{F}[[4,1]] + (1/2) \mathbf{F}[[4,2]] + (1/2) \mathbf{F}[[4,3]] + \mathbf{F}[[4,4]]; \end{aligned}$$

In terms of allele frequencies:

$$\begin{aligned} \text{In[105]:= } \mathbf{x1} &= (1 - t2) * (1 - p2) + \mathbf{diseq}; \\ \mathbf{x2} &= (1 - t2) * p2 - \mathbf{diseq}; \\ \mathbf{x3} &= t2 * (1 - p2) - \mathbf{diseq}; \\ \mathbf{x4} &= t2 * p2 + \mathbf{diseq}; \end{aligned}$$

Frequencies in generation $t+1$: p_{2t1} , t_{2t1} , and $diseqt1$

If $c=0$, use `FullSimplify` for p_{2t1} , `Simplify` for t_{2t1} , and neither for $diseqt1$.

```
In[109]:= p2t1 = FullSimplify[xt1[2] + xt1[4]]
```

```
Out[109]= (p2 (1 + s t2 (-1 + μ)) (2 + 2 s t2 (-1 + μ) + f (ρ + t2 (-1 + μ) (-2 + 2 s + ρ))) +
  diseq (-1 + μ)
  (2 s (1 + s t2 (-1 + μ)) + f (-1 + s (1 + ρ + t2 (-1 + μ) (-2 + 2 s + ρ)))) /
  (2 (1 + s t2 (-1 + μ)) (1 + (f (-1 + s) + s) t2 (-1 + μ)))
```

```
In[110]:= t2t1 = Simplify[xt1[3] + xt1[4]]
```

```
Out[110]= ((-1 + s) (-1 + μ) ((-1 + p2) s (2 s + f (-1 + 2 s)) t2^3 (-1 + μ)^2 +
  t2^2 (-1 + μ) (2 s (-2 + 2 p2 + diseq s (-1 + μ)) +
  f (1 + p2 (-1 + 3 s) + 2 diseq s^2 (-1 + μ) + s (-3 + diseq - diseq μ))) +
  t2 (2 (-1 + p2 + diseq s (-1 + μ)) + f (-1 + p2 + diseq (-1 + μ) (s - ρ))) -
  diseq f ρ) /
  (2 (1 + s t2 (-1 + μ)) (1 + f (-1 + s) t2 (-1 + μ) + s t2 (-1 + μ))
  (-1 + s t2 + p2 (1 + s t2 (-1 + μ)) + diseq s (-1 + μ) - s t2 μ))
```

```
In[111]:= diseqt1 = xt1[1] xt1[4] - xt1[2] xt1[3];
```

B. Analysis of the model

The influence of the parameters on the preference and trait frequencies

How does p2 change with the parameters?

I. How does f influence preference frequency?

In[112]= $\delta p2f = \text{FullSimplify}[D[p2t1, f]]$

Out[112]=
$$p2 \rho + \frac{(-1 + \mu) (\text{diseq}(-1 + s) + (p2 t2 + s (\text{diseq} + p2 t2) (1 + t2 (-1 + \mu))) \rho)}{(2 (1 + (f (-1 + s) + s) t2 (-1 + \mu))^2)}$$

In[113]= $\text{FullSimplify}[\text{Reduce}[\delta p2f > 0 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p2 < t2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]]$

Out[113]=
$$0 \leq \mu < 1 \ \&\& \ t2 < 1 \ \&\& \ f > 0 \ \&\& \ 0 < p2 < t2 \ \&\& \left(\left(\text{diseq} \geq 0 \ \&\& \ \rho > 0 \ \&\& \ \left(\left(s > 0 \ \&\& \ \rho < 1 \ \&\& \ s \leq \frac{1}{2 + t2 (-1 + \mu)} \right) \ \&\& \ \left(s < 1 \ \&\& \ \frac{1}{2 + t2 (-1 + \mu)} < s \ \&\& \ \rho \leq \frac{1 - s}{s - s t2 + s t2 \mu} \right) \right) \right) \ \&\& \ \left(0 \leq \text{diseq} < - \left((p2 (1 + t2 (-1 + \mu)) (1 + s t2 (-1 + \mu)) \rho) / \frac{((-1 + \mu) (-1 + s + s (1 + t2 (-1 + \mu)) \rho))}{2 + t2 (-1 + \mu)} < s < 1 \ \&\& \ \frac{1 - s}{s - s t2 + s t2 \mu} < \rho < 1 \right) \right)$$

Is the condition $\frac{1-s}{s-s t_2+s t_2 \mu}$ possible (i.e., can ρ be greater than or less than this quantity)?

```
In[137]= FindMinimum[ {  $\frac{1-s}{s-s t_2+s t_2 \mu}$ , 0 < s < 1 && 0 <  $\mu$  < 1 && 0 < t2 < 1 },
  { {s, 0.001}, { $\mu$ , 0.001}, {t2, 0.001} } ]
```

FindMinimum::eit :

The algorithm does not converge to the tolerance of $4.806217383937354 \times 10^{-6}$ in 500 iterations.

The best estimated solution, with feasibility residual, KKT residual, or complementary residual of $\{9.2226 \times 10^{-13}, 0.000128012, 2.93169 \times 10^{-13}\}$, is returned. >>

```
Out[137]= {0.102265, {s  $\rightarrow$  0.912543,  $\mu$   $\rightarrow$  0.814941, t2  $\rightarrow$  0.339617}}
```

```
In[136]= FindMaximum[ {  $\frac{1-s}{s-s t_2+s t_2 \mu}$ , 0 < s < 1 && 0 <  $\mu$  < 1 && 0 < t2 < 1 },
  { {s, 0.001}, { $\mu$ , 0.001}, {t2, 0.001} } ]
```

FindMaximum::eit :

The algorithm does not converge to the tolerance of $4.806217383937354 \times 10^{-6}$ in 500 iterations.

The best estimated solution, with feasibility residual, KKT residual, or complementary residual of $\{1.3691 \times 10^{-9}, 1.22497 \times 10^{17}, 3.08682 \times 10^{-10}\}$, is returned. >>

```
Out[136]= {3.50369  $\times$  1010, {s  $\rightarrow$  2.85738  $\times$  10-11,  $\mu$   $\rightarrow$  0.00113752, t2  $\rightarrow$  0.00113866}}
```

Yes, this condition is relevant: ρ can be greater than the minimum and less than the maximum.

Is the condition

$$\text{diseq} < - \left(\frac{p2 (1 + t2 (-1 + \mu)) (1 + s t2 (-1 + \mu)) \rho}{((-1 + \mu) (-1 + s + s (1 + t2 (-1 + \mu)) \rho))} \right)$$

possible (can diseq be less than this quantity)?

```
In[139]= FindMinimum[{-((p2 (1 + t2 (-1 + μ)) (1 + s t2 (-1 + μ)) ρ) /
    ((-1 + μ) (-1 + s + s (1 + t2 (-1 + μ)) ρ))),
    0 < s < 1 && 0 < μ < 1 && 0 < t2 < 1 && 0 < p2 < 1 && 0 < ρ < 1},
    {{s, 0.001}, {μ, 0.001}, {t2, 0.001}, {p2, 0.001}, {ρ, 0.001}}]
```

FindMinimum::eit :

The algorithm does not converge to the tolerance of $4.806217383937354 \times 10^{-6}$ in 500 iterations.

The best estimated solution, with feasibility residual, KKT residual, or complementary residual of $\{1.23564 \times 10^{-8}, 3.44271 \times 10^{16}, 3.83499 \times 10^{-9}\}$, is returned. >>

```
Out[139]= {-1.16442 × 108, {s → 0.595045, μ → 0.370394,
    t2 → 0.0841672, p2 → 0.632266, ρ → 0.718625}}
```

```
In[140]= FindMaximum[{-((p2 (1 + t2 (-1 + μ)) (1 + s t2 (-1 + μ)) ρ) /
    ((-1 + μ) (-1 + s + s (1 + t2 (-1 + μ)) ρ))),
    0 < s < 1 && 0 < μ < 1 && 0 < t2 < 1 && 0 < p2 < 1 && 0 < ρ < 1},
    {{s, 0.001}, {μ, 0.001}, {t2, 0.001}, {p2, 0.001}, {ρ, 0.001}}]
```

```
Out[140]= {-3.97948 × 10-11, {s → 0.408574, μ → 0.381039,
    t2 → 0.293184, p2 → 0.0364918, ρ → 5.26764 × 10-10}}
```

$$\text{In[223]= } \frac{\left(p^2 (1 + t^2 (-1 + \mu)) (1 + s t^2 (-1 + \mu)) \rho \right) / \left((-1 + \mu) (-1 + s + s (1 + t^2 (-1 + \mu)) \rho) \right)}{t^2} \rightarrow 0.7 / .$$

$$p^2 \rightarrow 0.8 / . \mu \rightarrow 0.001 / . s \rightarrow 0.2 / . \rho \rightarrow 0.3$$

Out[223]= 0.045007

Yes, this condition is relevant.

The analysis above shows that there are three relevant conditions.

Preference frequency increases with f when:

a) $s \leq \frac{1}{2+t^2(-1+\mu)}$, OR

b) $\frac{1}{2+t^2(-1+\mu)} < s$ and $\rho \leq \frac{1-s}{s-s t^2+s t^2 \mu}$, OR

c) $0 \leq \text{diseq} <$

$$- \left((p^2 (1 + t^2 (-1 + \mu)) (1 + s t^2 (-1 + \mu)) \rho) / \left((-1 + \mu) (-1 + s + s (1 + t^2 (-1 + \mu)) \rho) \right) \right)$$

and $\frac{1}{2+t^2(-1+\mu)} < s < 1$ and $\frac{1-s}{s-s t^2+s t^2 \mu} < \rho < 1$

For (a), the whole fraction increases as t^2 increases and μ decreases. Thus, the condition is more likely to be met when t^2 is large and μ is small.

For (b), the ρ condition increases as s decreases, t^2 increases, and μ decreases.

For (c), the conditions for s is the same as for (b), but the condition for ρ is opposite.

One of $s \leq \frac{1}{2+t^2(-1+\mu)}$ and $\frac{1}{2+t^2(-1+\mu)} < s$ is always true.

In (b), preference frequency will always increase with f as long as

$$\rho \leq \frac{1-s}{s-s t_2+s t_2 \mu}. \text{ (c) requires } \frac{1-s}{s-s t_2+s t_2 \mu} < \rho.$$

ρ cannot be larger than 1, so when $\frac{1-s}{s-s t_2+s t_2 \mu}$ is > 1 , the conditions for case (b) will always be met, and the conditions for case (c) will never be met.

```
In[190]= FullSimplify[Reduce[ $\frac{1-s}{s-s t_2+s t_2 \mu} > 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ 0 < \mu < 1$ ]]
```

```
Out[190]=  $0 < \mu < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ 0 < s < \frac{1}{2+t_2(-1+\mu)}$ 
```

```
In[191]= FindMinimum[ $\left\{ \frac{1}{2+t_2(-1+\mu)}, 0 < \mu < 1 \ \&\& \ 0 < t_2 < 1 \right\},$   

 $\{\{\mu, 0.001\}, \{t_2, 0.001\}\}$ ]
```

```
Out[191]=  $\{0.5, \{\mu \rightarrow 0.50229, t_2 \rightarrow 6.09675 \times 10^{-7}\}\}$ 
```

```
In[192]= FindMaximum[ $\left\{ \frac{1}{2+t_2(-1+\mu)}, 0 < \mu < 1 \ \&\& \ 0 < t_2 < 1 \right\},$   

 $\{\{\mu, 0.001\}, \{t_2, 0.001\}\}$ ]
```

```
Out[192]=  $\{1., \{\mu \rightarrow 5.96859 \times 10^{-9}, t_2 \rightarrow 1.\}\}$ 
```

The condition for ρ in (b) is always true, and always false in (c), for $s < 0.5$. Because $s > 0.5$ seems very high, case (c) is unlikely to occur.

2. How does ρ influence preference frequency?

```
In[114]=  $\delta p_2 \rho = \text{FullSimplify}[D[p_2 t_1, \rho]]$ 
```

```
Out[114]=  $(f(p_2(1+s t_2(-1+\mu)) + \text{diseq}s(-1+\mu))(1+t_2(-1+\mu))) /$   

 $(2(1+s t_2(-1+\mu))(1+(f(-1+s)+s)t_2(-1+\mu)))$ 
```

```
In[115]= FullSimplify[Reduce[ $\delta p^2 \rho > 0 \ \&\& \ f > 0 \ \&\&$ 
```

```
0 <  $\rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p^2 < t^2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]]$ 
```

```
Out[115]=  $\rho > 0 \ \&\& \ \mu \geq 0 \ \&\& \ s > 0 \ \&\& \ p^2 > 0 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ f > 0 \ \&\& \ \rho < 1 \ \&\& \ \mu < 1 \ \&\&$ 
```

```
 $t^2 < 1 \ \&\& \ s < 1 \ \&\& \ p^2 < t^2 \ \&\& \ s (\text{diseq} + p^2 t^2) < p^2 + s (\text{diseq} + p^2 t^2) \ \mu$ 
```

Preference frequency increases with ρ when the following condition holds: $s \cdot (\text{diseq} + p^2 \cdot t^2) < p^2 + s \cdot (\text{diseq} + p^2 \cdot t^2) \cdot \mu$.

In terms of s , this condition simplifies to $s < p^2 / ((\text{diseq} + p^2 \cdot t^2) \cdot (1 - \mu))$.

Note that $\text{diseq} + p^2 \cdot t^2$ is equal to x^4 , the frequency of P2T2 genotypes. This frequency must be less than or equal to p^2 . Since $1 - \mu$ must be less than or equal to 1, this means that p^2 is always equal to or greater than $((\text{diseq} + p^2 \cdot t^2) \cdot (1 - \mu))$. This means that $p^2 / ((\text{diseq} + p^2 \cdot t^2) \cdot (1 - \mu))$ will always be greater than or equal to 1, and thus will always be larger than s . This means that the condition always holds true, and thus that $p^2 t$ will always increase with ρ .

3. How does s influence preference frequency?

```
In[116]=  $\delta p^2 s = \text{FullSimplify}[D[p^2 t^1, s]]$ 
```

```
Out[116]=  $\left( (-1 + \mu) \left( -f (1 + f) p^2 t^2 (1 + t^2 (-1 + \mu)) (1 + s t^2 (-1 + \mu))^2 \rho + \right. \right.$   

 $\text{diseq} \left( 2 (1 + s t^2 (-1 + \mu))^2 - f (1 + s t^2 (-1 + \mu)) (-1 - \rho + \right.$   

 $t^2 (-1 + \mu) (2 - 3 s + (-1 + s - s t^2 + s t^2 \mu) \rho) \left. \right) + f^2 t^2 (-1 + \mu)$   

 $\left. \left. (-\rho + t^2 (-1 + \mu) (1 - \rho + s (-2 + s + s (-1 + t^2 - t^2 \mu) \rho)) \right) \right) \right) /$   

 $\left( 2 (1 + s t^2 (-1 + \mu))^2 (1 + (f (-1 + s) + s) t^2 (-1 + \mu))^2 \right)$ 
```

When does preference frequency decrease with s?

In[170]= **FullSimplify**[Reduce[$\delta p^2 s < 0 \ \&\& \ f > 0 \ \&\&$

$0 < \rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p^2 < t^2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0$]]

Out[170]= $0 \leq \mu < 1 \ \&\& \ t^2 < 1 \ \&\& \ 0 < s < 1 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ 0 < p^2 < t^2 \ \&\&$

$\text{diseq} > - \left(\left(f (1+f) p^2 t^2 (1+t^2 (-1+\mu)) (1+s t^2 (-1+\mu))^2 \rho \right) / \right.$
 $\left. \left(-2 (1+s t^2 (-1+\mu))^2 + f (1+s t^2 (-1+\mu)) \right. \right.$
 $\left. \left. (-1-\rho+t^2 (-1+\mu) (2-\rho+s (-3+\rho-t^2 \rho+t^2 \mu \rho))) + f^2 t^2 \right. \right.$
 $\left. \left. (-1+\mu) (\rho+t^2 (-1+\mu) (-1+\rho+s (2+s (-1+\rho-t^2 \rho+t^2 \mu \rho)))) \right) \right)$

In[171]= **diseqthreshold** =

FullSimplify[$- \left(\left(f (1+f) p^2 t^2 (1+t^2 (-1+\mu)) (1+s t^2 (-1+\mu))^2 \rho \right) / \right.$
 $\left. \left(-2 (1+s t^2 (-1+\mu))^2 + f (1+s t^2 (-1+\mu)) \right. \right.$
 $\left. \left. (-1-\rho+t^2 (-1+\mu) (2-\rho+s (-3+\rho-t^2 \rho+t^2 \mu \rho))) + f^2 t^2 (-1+\mu) \right. \right.$
 $\left. \left. (\rho+t^2 (-1+\mu) (-1+\rho+s (2+s (-1+\rho-t^2 \rho+t^2 \mu \rho)))) \right) \right)];$

In[172]= **FindMinimum**[{**diseqthreshold**,

$0 < f < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < \mu < 1 \ \&\& \ 0 < t^2 < 1 \ \&\& \ 0 < p^2 < 1 \ \&\& \ 0 < \rho < 1$ },
 {{**s**, 0.001}, { **μ** , 0.001}, {**t2**, 0.001},
 {**p2**, 0.001}, { **ρ** , 0.001}, {**f**, 0.001}}

Out[172]= { 8.98246×10^{-15} , {**s** \rightarrow 0.208707, **μ** \rightarrow 0.205343, **t2** \rightarrow 0.0836925,
p2 \rightarrow 0.0348955, **ρ** \rightarrow 0.0388065, **f** \rightarrow 1.69806×10^{-10} }}

In[173]= **FindMaximum**[{**diseqthreshold**,

$0 < f < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < \mu < 1 \ \&\& \ 0 < t^2 < 1 \ \&\& \ 0 < p^2 < 1 \ \&\& \ 0 < \rho < 1$ },
 {{**s**, 0.001}, { **μ** , 0.001}, {**t2**, 0.001},
 {**p2**, 0.001}, { **ρ** , 0.001}, {**f**, 0.001}}

Out[173]= {0.499999, {**s** \rightarrow 0.431386, **μ** \rightarrow 1., **t2** \rightarrow 1., **p2** \rightarrow 1., **ρ** \rightarrow 0.999999, **f** \rightarrow 1.}}

As linkage disequilibrium increases, it becomes more likely that

preference frequency will decrease with s . This makes sense because greater linkage disequilibrium between the preference and the trait means that s will have a larger indirect effect on the preference frequency. Note that linkage disequilibrium cannot exceed 0.25.

When does preference frequency increase with s ?

```
In[174]= FullSimplify[Reduce[ $\delta p^2 s > 0 \ \&\& \ f > 0 \ \&\&$ 
```

```
0 <  $\rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p^2 < t^2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]]$ 
```

```
Out[174]= 0 ≤  $\mu < 1 \ \&\& \ t^2 < 1 \ \&\& \ 0 < s < 1 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ 0 < p^2 < t^2 \ \&\&$ 
```

```
0 ≤  $\text{diseq} < - \left( \left( f (1 + f) p^2 t^2 (1 + t^2 (-1 + \mu)) (1 + s t^2 (-1 + \mu))^2 \rho \right) / \right.$   

 $\left. \left( -2 (1 + s t^2 (-1 + \mu))^2 + f (1 + s t^2 (-1 + \mu)) \right. \right.$   

 $\left. \left. (-1 - \rho + t^2 (-1 + \mu) (2 - \rho + s (-3 + \rho - t^2 \rho + t^2 \mu \rho))) + f^2 t^2 \right. \right.$   

 $\left. \left. (-1 + \mu) (\rho + t^2 (-1 + \mu) (-1 + \rho + s (2 + s (-1 + \rho - t^2 \rho + t^2 \mu \rho)))) \right) \right)$ 
```

```
In[175]= diseqthreshold2 =
```

```
FullSimplify[-  $\left( \left( f (1 + f) p^2 t^2 (1 + t^2 (-1 + \mu)) (1 + s t^2 (-1 + \mu))^2 \rho \right) / \right.$   

 $\left. \left( -2 (1 + s t^2 (-1 + \mu))^2 + f (1 + s t^2 (-1 + \mu)) \right. \right.$   

 $\left. \left. (-1 - \rho + t^2 (-1 + \mu) (2 - \rho + s (-3 + \rho - t^2 \rho + t^2 \mu \rho))) + f^2 t^2 (-1 + \mu) \right. \right.$   

 $\left. \left. (\rho + t^2 (-1 + \mu) (-1 + \rho + s (2 + s (-1 + \rho - t^2 \rho + t^2 \mu \rho)))) \right) \right) \right];$ 
```

```
In[176]= Reduce[diseqthreshold == diseqthreshold2]
```

```
Out[176]= True
```

Preference frequency increases with s when linkage disequilibrium between the preference and the trait is greater than the same condition.

How does t_2 change with the parameters?

1. How does f influence trait frequency?

In[118]:= $\delta t_2 f = \text{FullSimplify}[D[t_2 t_1, f]]$

Out[118]=
$$\frac{((-1 + s) (1 + t_2 (-1 + \mu)) (-1 + \mu) (t_2 (-1 + p_2 + \text{diseq } s (-1 + \mu)) + (-1 + p_2) s t_2^2 (-1 + \mu) - \text{diseq } \rho))}{(2 (-1 + p_2 (1 + s t_2 (-1 + \mu)) + s (\text{diseq} - t_2) (-1 + \mu)) (1 + (f (-1 + s) + s) t_2 (-1 + \mu))^2)}$$

In[119]:= $\text{Reduce}[\delta t_2 f > 0 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p_2 < t_2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]$

Out[119]= $0 \leq \mu < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p_2 < t_2 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ f > 0$

Trait frequency increases with f for all realistic ranges of the parameters and variables.

2. How does ρ influence trait frequency?

In[120]:= $\delta t_2 \rho = \text{FullSimplify}[D[t_2 t_1, \rho]]$

Out[120]=
$$-((\text{diseq } f (-1 + s) (1 + t_2 (-1 + \mu)) (-1 + \mu)) / (2 (-1 + p_2 (1 + s t_2 (-1 + \mu)) + s (\text{diseq} - t_2) (-1 + \mu)) (1 + s t_2 (-1 + \mu)) (1 + (f (-1 + s) + s) t_2 (-1 + \mu))))$$

In[121]:= $\text{Reduce}[\delta t_2 \rho > 0 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < p_2 < t_2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]$

Out[121]= $0 < \rho < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ 0 < p_2 < t_2 \ \&\& \ 0 \leq \mu < 1 \ \&\& \ 0 < s < 1 \ \&\& \ \text{diseq} > 0 \ \&\& \ f > 0$

Trait frequency increases with ρ for all realistic ranges of the

parameters and variables.

3. How does s influence trait frequency?

In[123]= $\delta t_2 s = \text{FullSimplify}[D[t_2 t_1 / . \mu \rightarrow 0, s]]$

$$\begin{aligned} \text{Out[123]= } & - \left(\left((-1 + s) t_2 (1 - s t_2) (-1 + p_2 - \text{diseq } s + s t_2 - p_2 s t_2) \right. \right. \\ & \quad \left. \left(1 - (f (-1 + s) + s) t_2 \right) \left((-1 + p_2) t_2 (-4 + 4 s t_2 + f (-3 - t_2 + 4 s t_2)) \right) + \right. \\ & \quad \left. \text{diseq} (-2 + 4 s t_2 + f (-1 - t_2 + 4 s t_2)) \right) + \\ & \quad (1 + f) (-1 + s) t_2 (1 - s t_2) (-1 + p_2 - \text{diseq } s + s t_2 - p_2 s t_2) \\ & \quad (t_2 (1 - p_2 + \text{diseq } s + (-1 + p_2) s t_2) \\ & \quad (-2 + 2 s t_2 + f (-1 + (-1 + 2 s) t_2)) + \text{diseq } f (-1 + t_2) \rho) - \\ & \quad (-1 + s) (-\text{diseq} + t_2 - p_2 t_2) (1 - s t_2) (1 - (f (-1 + s) + s) t_2) \\ & \quad (t_2 (1 - p_2 + \text{diseq } s + (-1 + p_2) s t_2) \\ & \quad (-2 + 2 s t_2 + f (-1 + (-1 + 2 s) t_2)) + \text{diseq } f (-1 + t_2) \rho) + \\ & \quad (-1 + s) t_2 (-1 + p_2 - \text{diseq } s + s t_2 - p_2 s t_2) (1 - (f (-1 + s) + s) t_2) \\ & \quad (t_2 (1 - p_2 + \text{diseq } s + (-1 + p_2) s t_2) \\ & \quad (-2 + 2 s t_2 + f (-1 + (-1 + 2 s) t_2)) + \text{diseq } f (-1 + t_2) \rho) + \\ & \quad (1 - s t_2) (-1 + p_2 - \text{diseq } s + s t_2 - p_2 s t_2) (1 - (f (-1 + s) + s) t_2) \\ & \quad (t_2 (1 - p_2 + \text{diseq } s + (-1 + p_2) s t_2) \\ & \quad (-2 + 2 s t_2 + f (-1 + (-1 + 2 s) t_2)) + \text{diseq } f (-1 + t_2) \rho)) / \\ & \quad \left(2 (-1 + s t_2)^2 (-1 + f (-1 + s) t_2 + s t_2)^2 \right. \\ & \quad \left. (1 - p_2 + \text{diseq } s + (-1 + p_2) s t_2)^2 \right) \end{aligned}$$

In[124]= $\text{Reduce}[\delta t_2 s < 0 \ \&\& \ f > 0 \ \&\& \ 0 < \rho < 1 \ \&\& \$

$$0 < s < 1 \ \&\& \ 0 < p_2 < t_2 < 1 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 1 > \mu \geq 0]$$

$$\text{Out[124]= } 0 \leq \mu < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ f > 0 \ \&\& \ 0 < p_2 < t_2 \ \&\& \ \text{diseq} \geq 0 \ \&\& \ 0 < \rho < 1$$

Trait frequency decreases with s for all realistic ranges of the parameters and variables.

Evolution in the absence of the aversion ($\rho = 0$)

In[196]:= **p2ρ0 = FullSimplify[p2t1 /. ρ → 0]**

$$\text{Out[196]= } p2 + \frac{1}{2 t2} \text{diseq} \left(2 - \frac{1}{1 + (f (-1 + s) + s) t2 (-1 + \mu)} + \frac{1}{-1 + s t2 - s t2 \mu} \right)$$

In[197]:= **t2ρ0 = FullSimplify[t2t1 /. ρ → 0]**

$$\text{Out[197]= } \frac{((-1 + s) t2 (2 + f (1 + (-1 + 2 s) t2 (-1 + \mu)) + 2 s t2 (-1 + \mu)) (-1 + \mu))}{(2 (1 + s t2 (-1 + \mu)) (1 + (f (-1 + s) + s) t2 (-1 + \mu)))}$$

In[198]:= **diseqr0 = FullSimplify[diseqt1 /. ρ → 0]**

$$\text{Out[198]= } - \left(\left(\text{diseq} (-1 + s) (-1 + \mu) (4 (-1 + r) (1 + s t2 (-1 + \mu))^2 + 2 f (-1 + r) (1 + s t2 (-1 + \mu)) (1 + (-2 + 3 s) t2 (-1 + \mu)) + f^2 (-1 + s) t2 (-1 + \mu) (-1 + 2 r (1 + s t2 (-1 + \mu)) + t2 (-1 + 2 s + \mu - 2 s \mu))) \right) / (4 (1 + s t2 (-1 + \mu))^2 (1 + (f (-1 + s) + s) t2 (-1 + \mu))^2) \right)$$

I. Will p2 increase with s when $\rho=0$?

dp2ρ0s = FullSimplify[D[p2ρ0, s]]

$$\frac{1}{2} \text{diseq} \left(\frac{1}{(1 + s t2 (-1 + \mu))^2} + \frac{1 + f}{(1 + (f (-1 + s) + s) t2 (-1 + \mu))^2} \right) (-1 + \mu)$$

Reduce[dp2ρ0s > 0 && diseq ≥ 0 && 0 ≤ t2 ≤ 1 && 0 < f < 1 && 0 < s < 1 && 0 ≤ μ < 1]

False

p_2 will not increase with s when $\rho=0$.

2. Will p_2 increase with f when $\rho=0$?

```
dp2rho0f = FullSimplify[D[p2rho0, f]]
```

$$\frac{\text{diseq} (-1 + s) (-1 + \mu)}{2 (1 + (f (-1 + s) + s) t_2 (-1 + \mu))^2}$$

```
Reduce[dp2rho0f > 0 && diseq >= 0 && 0 <= t2 <= 1 && 0 < f < 1 && 0 < s < 1 && 0 <= mu < 1]
```

```
0 < s < 1 && 0 < f < 1 && 0 <= mu < 1 && 0 <= t2 <= 1 && diseq > 0
```

p_2 will only increase with f when $\rho=0$ if diseq is greater than 0.

3. If $\text{diseq} = 0$ when $\text{diseq} = 0$ and $\rho = 0$, we can conclude that linkage disequilibrium cannot build up between the preference and the trait in the absence of the aversion. Does $\text{diseq} = 0$ when $\text{diseq} = 0$ and $\rho = 0$?

```
In[199]:= diseqrho0 /. diseq -> 0
```

```
Out[199]= 0
```

Yes.

Conclusion: when $\rho=0$, p_2 will not increase with s under any conditions, and p_2 will only increase with f when $\text{diseq} > 0$. However, because our starting diseq in the simulations is 0, p_2 will never increase when f is increasing in the simulations, as long as $\rho=0$. This demonstrates both the importance of the aversion, and the

importance of linkage disequilibrium between the preference and the trait, for the evolution of the preference. Further, we have shown that linkage disequilibrium does not build up between the preference and the trait in the absence of the aversion.

Does sexual selection occur in the model?

If sexual selection occurs, aversion strength alone will influence p_2 and t_2 . Test this by setting s and f equal to zero, and determining whether aversion strength influences the allele frequencies.

```
In[193]:= FullSimplify[p2t1 /. s -> 0 /. f -> 0]
```

```
Out[193]= p2
```

```
In[194]:= FullSimplify[t2t1 /. s -> 0 /. f -> 0]
```

```
Out[194]= t2 - t2 μ
```

In the absence of s and f , neither p_2t_1 nor t_2t_1 is influenced by the aversion (ρ).

When will t_2 persist in the population with no preference?

Calculated using $c = 0$

Goal: find the conditions under which Δt_2 is positive when $p_2=0$ (and thus $diseq=0$). This should tell us when we expect t_2 to persist in the absence of a preference.

```
In[114]:=  $\Delta t_2 = \text{FullSimplify}[t_2 t_1 - t_2]$ 
```

```
Out[114]= 
$$\frac{-t_2 + ((-1 + s)(-1 + \mu)(t_2(-1 + p_2(1 + s t_2(-1 + \mu))) + s(diseq - t_2)(-1 + \mu)) + (2 + f(1 + (-1 + 2s)t_2(-1 + \mu)) + 2s t_2(-1 + \mu)) + diseq f(-1 + t_2 - t_2 \mu) \rho))}{(2(-1 + p_2(1 + s t_2(-1 + \mu))) + s(diseq - t_2)(-1 + \mu))(1 + s t_2(-1 + \mu))(1 + (f(-1 + s) + s)t_2(-1 + \mu))}$$

```

```
In[124]:=  $\Delta t_{p_2=0} = \text{FullSimplify}[\Delta t_2 /. p_2 \rightarrow 0 /. diseq \rightarrow 0]$ 
```

```
Out[124]= 
$$\frac{1}{2} t_2 \frac{(-2 + ((-1 + s)(2 + f(1 + (-1 + 2s)t_2(-1 + \mu)) + 2s t_2(-1 + \mu))(-1 + \mu))}{((1 + s t_2(-1 + \mu))(1 + (f(-1 + s) + s)t_2(-1 + \mu)))}$$

```

Note that c does not appear in the expression for Δt_2 when $p_2=0$, indicating that further analyses apply equally to cases with and without a cost.

Under what conditions is Δt_2 positive (meaning that we can expect t_2 to persist in the population) when $p_2=0$?

```
In[118]:= FullSimplify[
  Reduce[ $\Delta t_2 > 0 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < t_2 < 1 \ \&\& \ 0 < f < 1 \ \&\& \ 0 < \mu < 1$ ]]
Out[118]=  $0 < \mu < \frac{1}{3} \ \&\& \ 0 < t_2 < \frac{-1 + 3\mu}{-1 + \mu^2} \ \&\& \ s > 0 \ \&\& \ 2 + \frac{3}{t_2 - t_2 \mu} >$ 
 $8 s + \sqrt{\left( (9 + 4(-1 + t_2) t_2) (1 + t_2 (-1 + \mu))^2 \right) / \left( (-1 + t_2)^2 t_2^2 (-1 + \mu)^2 \right)} +$ 
 $\frac{3\mu}{(-1 + t_2) (-1 + \mu)} \ \&\& \ - \left( (2 (1 + s t_2 (-1 + \mu)) (s (-1 + t_2) (-1 + \mu) + \mu)) / \right.$ 
 $\left. ((-1 + s) (-1 + \mu) (-1 + t_2 (1 + 2 s (-1 + t_2) (-1 + \mu) + \mu))) \right) < f < 1$ 
```

I. What is the condition for Δt_2 to be positive when $s=0.1$ and $\mu=0.01$, as in our simulations shown in Figures 1A, 2A, and 2B?

```
In[127]:=  $\Delta t_2$ 201 = FullSimplify[ $\Delta t_2$ 20 / . s -> 0.1 / .  $\mu$  -> 0.01]
Out[127]=  $\frac{1}{2} t_2 (-2 + (0.891 (2 + f - 0.198 t_2 + 0.792 f t_2)) /$ 
 $((1 - 0.099 t_2) (1 - 0.099 t_2 + 0.891 f t_2)))$ 
```

```
In[128]:= FullSimplify[Reduce[ $\Delta t_2$ 201 > 0 && 0 ≤ f ≤ 1 && 0 < t_2 ≤ 1]]
```

Reduce::ratnz :

Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result. >>

```
Out[128]=  $0 < t_2 < 0.951106 \ \&\& \ (1.2357 + (-1.24467 + 0.111111 t_2) t_2) /$ 
 $(5.05051 + t_2 (-6.10101 + 1. t_2)) < f \leq 1.$ 
```

```
In[137]:= fthreshold1 = (1.2357015724019094` +
  (-1.2446689113355782` + 0.11111111111111112` t2) t2) /
  (5.05050505050505` + t2 (-6.101010101010101` + 1.` t2));
```

Δt_2 is positive when $t_2 < 0.951106$ and $f >$

$\frac{1.2357 + (-1.24467 + 0.111111 t_2) t_2}{5.05051 + t_2 (-6.10101 + 1. t_2)}$. This means that, for the

conditions $s=0.1$ and $\mu=0.01$, when $p_2=0$, t_2 will increase as long as $t_2 < 0.951106$ and f meets the condition above. Plugging in the starting values we use in our simulations ($t_2=0.8$ in the simulations shown in the main text, and $t_2=0.1$ in the simulations shown in the supplementary figures) gives the threshold value of f above which t_2 will persist in the absence of p_2 .

```
In[138]:= fthreshold1 /. t2 -> 0.1
  fthreshold1 /. t2 -> 0.8
```

```
Out[138]= 0.249943
```

```
Out[139]= 0.38419
```

These values are displayed as dashed lines on the appropriate figures (1A, 2A, and 2B in the main text; supplementary figure 1), showing the value of f above which t_2 is expected to persist in the absence of a preference.

2. What is the condition for Δt_2 to be positive when $s=0.2$ and $\mu=0.01$, as in our simulations shown in Figure 1B?

In[133]:= $\Delta t_2 p_2 202 = \text{FullSimplify}[\Delta t_2 p_2 20 / . s \rightarrow 0.2 / . \mu \rightarrow 0.01]$

$$\text{Out[133]} = \frac{1}{2} t_2 (-2 + (0.792 (2 + f - 0.396 t_2 + 0.594 f t_2))) / ((1 - 0.198 t_2) (1 - 0.198 t_2 + 0.792 f t_2))$$

In[134]:= $\text{FullSimplify}[\text{Reduce}[\Delta t_2 p_2 202 > 0 \ \&\& \ 0 \leq f \leq 1 \ \&\& \ 0 < t_2 \leq 1]]$

Reduce::ratnz :

Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numerizing the result. >>

$$\text{Out[134]} = 0 < t_2 < 0.876377 \ \&\& \ \frac{1.3264 + (-1.52525 + 0.25 t_2) t_2}{2.52525 + t_2 (-3.55051 + 1. t_2)} < f \leq 1.$$

In[136]:= $f_{\text{threshold}2} =$

$$\frac{(1.326395265789205 + (-1.5252525252525253 + 0.25 t_2) t_2)}{(2.525252525252525 + t_2 (-3.55050505050506 + 1. t_2))};$$

Δt_2 is positive when $t_2 < 0.876377$ and $f >$

$\frac{1.3264 + (-1.52525 + 0.25 t_2) t_2}{2.52525 + t_2 (-3.55051 + 1. t_2)}$. This means that, for the conditions $s=0.2$ and $\mu=0.01$, when $p_2=0$, t_2 will increase as long as $t_2 < 0.876377$ and f meets the condition above. Plugging in the starting

values we use in our simulations ($t_2=0.8$ in the simulations shown in the main text, and $t_2=0.1$ in the simulations shown in the supplementary figures) gives the threshold value of f above which t_2 will persist in the absence of p_2 .

```
In[140]:= fthreshold2 /. t2 -> 0.1
```

```
fthreshold2 /. t2 -> 0.8
```

```
Out[140]= 0.539569
```

```
Out[141]= 0.819438
```

These values are displayed as dashed lines on the appropriate figures (1B in the main text, and supplementary figure 2), showing the value of f above which t_2 is expected to persist in the absence of a preference.

C. Simulations

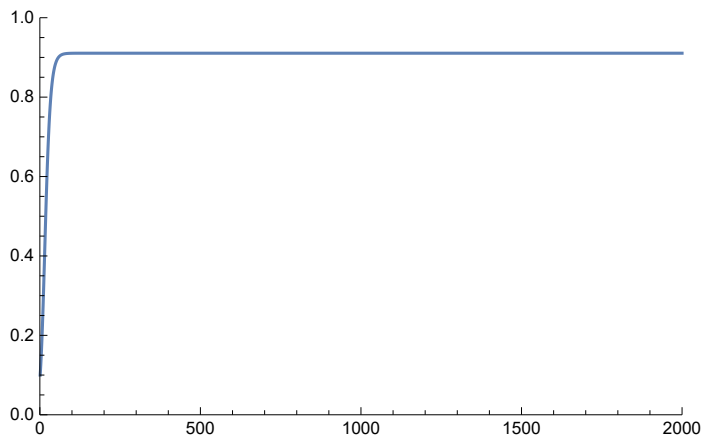
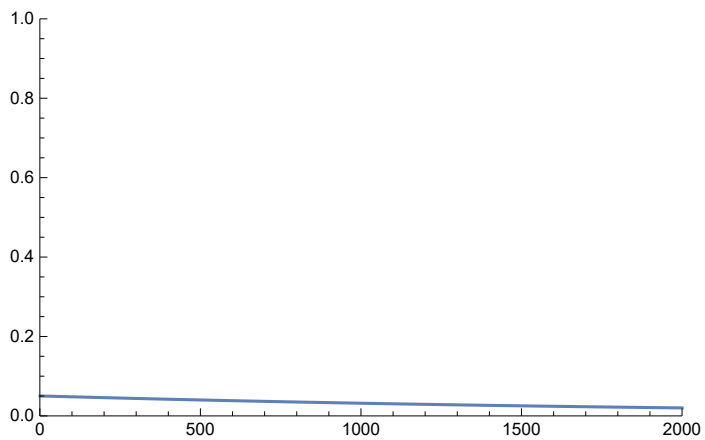
For simulations

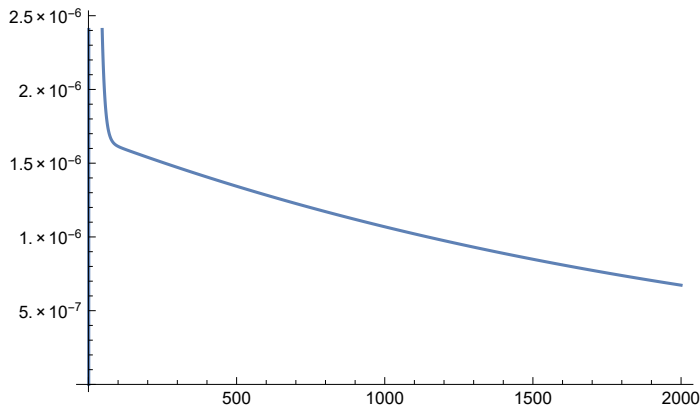
```
p2sims = p2t1;  
t2sims = t2t1;  
diseqsims = diseqt1;
```

Simulations

```
r = 0.5;
f = 0.6;
ρ = 0.001;
s = 0.1;
μ = 0.01;
p2 = 0.05;
t2 = 0.1;
diseq = 0;
j = 2000;

tabp2 = Table[0, {i, j}];
tabt2 = Table[0, {i, j}];
tabdiseq = Table[0, {i, j}];
Do[
  tabp2[[i]] = p2;
  tabt2[[i]] = t2;
  tabdiseq[[i]] = diseq;
  p2t1 = p2sims;
  t2t1 = t2sims;
  diseqt1 = diseqsims;
  p2 = p2t1;
  t2 = t2t1;
  diseq = diseqt1; If[p2 > t2, Break[]],
  {i, j}];
p2plot =
  ListPlot[tabp2, Joined → True, PlotRange → {{0, j}, {0.0, 1.0}}]
t2plot = ListPlot[tabt2, Joined → True,
  PlotRange → {{0, j}, {0.0, 1.0}}]
diseqplot = ListPlot[tabdiseq, Joined → True]
```





tabp2[[1900 ;; 2000]]

```
{0.0209519, 0.0209421, 0.0209323, 0.0209225, 0.0209127,
 0.020903, 0.0208932, 0.0208834, 0.0208737, 0.0208639, 0.0208542,
 0.0208444, 0.0208347, 0.020825, 0.0208152, 0.0208055, 0.0207958,
 0.0207861, 0.0207764, 0.0207667, 0.020757, 0.0207473, 0.0207376,
 0.0207279, 0.0207182, 0.0207085, 0.0206988, 0.0206892, 0.0206795,
 0.0206698, 0.0206602, 0.0206505, 0.0206409, 0.0206312, 0.0206216,
 0.020612, 0.0206023, 0.0205927, 0.0205831, 0.0205735, 0.0205638,
 0.0205542, 0.0205446, 0.020535, 0.0205254, 0.0205158, 0.0205063,
 0.0204967, 0.0204871, 0.0204775, 0.0204679, 0.0204584, 0.0204488,
 0.0204393, 0.0204297, 0.0204202, 0.0204106, 0.0204011, 0.0203915,
 0.020382, 0.0203725, 0.020363, 0.0203534, 0.0203439, 0.0203344,
 0.0203249, 0.0203154, 0.0203059, 0.0202964, 0.0202869, 0.0202775,
 0.020268, 0.0202585, 0.020249, 0.0202396, 0.0202301, 0.0202206,
 0.0202112, 0.0202017, 0.0201923, 0.0201829, 0.0201734, 0.020164,
 0.0201546, 0.0201451, 0.0201357, 0.0201263, 0.0201169, 0.0201075,
 0.0200981, 0.0200887, 0.0200793, 0.0200699, 0.0200605, 0.0200511,
 0.0200418, 0.0200324, 0.020023, 0.0200137, 0.0200043, 0.019995}
```

tabt2[[1 ;; 1000]]

```
{0.1, 0.116955, 0.136168, 0.157742, 0.181723, 0.208088, 0.236735,
 0.267478, 0.300046, 0.334093, 0.369216, 0.404972, 0.440905, 0.476572,
```


{1.01047 × 10⁻⁶, 1.01001 × 10⁻⁶, 1.00954 × 10⁻⁶, 1.00908 × 10⁻⁶, 1.00861 × 10⁻⁶,
 1.00815 × 10⁻⁶, 1.00768 × 10⁻⁶, 1.00722 × 10⁻⁶, 1.00675 × 10⁻⁶, 1.00629 × 10⁻⁶,
 1.00582 × 10⁻⁶, 1.00536 × 10⁻⁶, 1.00489 × 10⁻⁶, 1.00443 × 10⁻⁶,
 1.00397 × 10⁻⁶, 1.0035 × 10⁻⁶, 1.00304 × 10⁻⁶, 1.00258 × 10⁻⁶, 1.00212 × 10⁻⁶,
 1.00165 × 10⁻⁶, 1.00119 × 10⁻⁶, 1.00073 × 10⁻⁶, 1.00027 × 10⁻⁶,
 9.99805 × 10⁻⁷, 9.99344 × 10⁻⁷, 9.98883 × 10⁻⁷, 9.98422 × 10⁻⁷,
 9.97961 × 10⁻⁷, 9.97501 × 10⁻⁷, 9.97041 × 10⁻⁷, 9.96581 × 10⁻⁷,
 9.96121 × 10⁻⁷, 9.95661 × 10⁻⁷, 9.95202 × 10⁻⁷, 9.94742 × 10⁻⁷,
 9.94283 × 10⁻⁷, 9.93824 × 10⁻⁷, 9.93366 × 10⁻⁷, 9.92907 × 10⁻⁷,
 9.92449 × 10⁻⁷, 9.91991 × 10⁻⁷, 9.91533 × 10⁻⁷, 9.91076 × 10⁻⁷,
 9.90618 × 10⁻⁷, 9.90161 × 10⁻⁷, 9.89704 × 10⁻⁷, 9.89247 × 10⁻⁷,
 9.88791 × 10⁻⁷, 9.88335 × 10⁻⁷, 9.87878 × 10⁻⁷, 9.87422 × 10⁻⁷, 9.86967 × 10⁻⁷,
 9.86511 × 10⁻⁷, 9.86056 × 10⁻⁷, 9.85601 × 10⁻⁷, 9.85146 × 10⁻⁷,
 9.84691 × 10⁻⁷, 9.84237 × 10⁻⁷, 9.83782 × 10⁻⁷, 9.83328 × 10⁻⁷,
 9.82874 × 10⁻⁷, 9.82421 × 10⁻⁷, 9.81967 × 10⁻⁷, 9.81514 × 10⁻⁷,
 9.81061 × 10⁻⁷, 9.80608 × 10⁻⁷, 9.80155 × 10⁻⁷, 9.79703 × 10⁻⁷, 9.7925 × 10⁻⁷,
 9.78798 × 10⁻⁷, 9.78347 × 10⁻⁷, 9.77895 × 10⁻⁷, 9.77443 × 10⁻⁷,
 9.76992 × 10⁻⁷, 9.76541 × 10⁻⁷, 9.7609 × 10⁻⁷, 9.7564 × 10⁻⁷, 9.75189 × 10⁻⁷,
 9.74739 × 10⁻⁷, 9.74289 × 10⁻⁷, 9.73839 × 10⁻⁷, 9.7339 × 10⁻⁷, 9.7294 × 10⁻⁷,
 9.72491 × 10⁻⁷, 9.72042 × 10⁻⁷, 9.71593 × 10⁻⁷, 9.71145 × 10⁻⁷,
 9.70696 × 10⁻⁷, 9.70248 × 10⁻⁷, 9.698 × 10⁻⁷, 9.69352 × 10⁻⁷, 9.68905 × 10⁻⁷,
 9.68457 × 10⁻⁷, 9.6801 × 10⁻⁷, 9.67563 × 10⁻⁷, 9.67116 × 10⁻⁷, 9.6667 × 10⁻⁷,
 9.66223 × 10⁻⁷, 9.65777 × 10⁻⁷, 9.65331 × 10⁻⁷, 9.64886 × 10⁻⁷}

D. Figures

Figure 1A: effect of f and ρ on the sign of Δp_2 and Δt_2 when $\mu = 0.01$, $c=0$, and $s=0.1$

$\{\rho, f\}$

In the gray region, $\Delta p_2 > 0$ and $\Delta t_2 < 0$. In the white region, $\Delta p_2 > 0$ and $\Delta t_2 > 0$.

The dashed line indicates the value that f must exceed in order for Δt_2 to be positive in the absence of p_2 ($p_2=0$).

```
Style["c", Italic]
```

```
c
```

```
Figure1A = Show[
  ListPlot[{{0.001, 0.25}, {0.005, 0.25}, {0.01, 0.25}, {0.1, 0.27},
    {0.2, 0.27}, {0.3, 0.26}, {0.4, 0.25}, {0.5, 0.24},
    {0.6, 0.22}, {0.7, 0.21}}, {{0, 0.38419}, {1, 0.38419}}],
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
    "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {{GrayLevel[0.6]}, {Black, Thick, Dashed}},
  FrameStyle → Black, Filling → {1 → Axis},
  FillingStyle → GrayLevel[0.6]],
  Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 > 0$ ", {0.35, 0.5}]],
  Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 < 0$ ", {0.35, 0.1}]],
  Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```

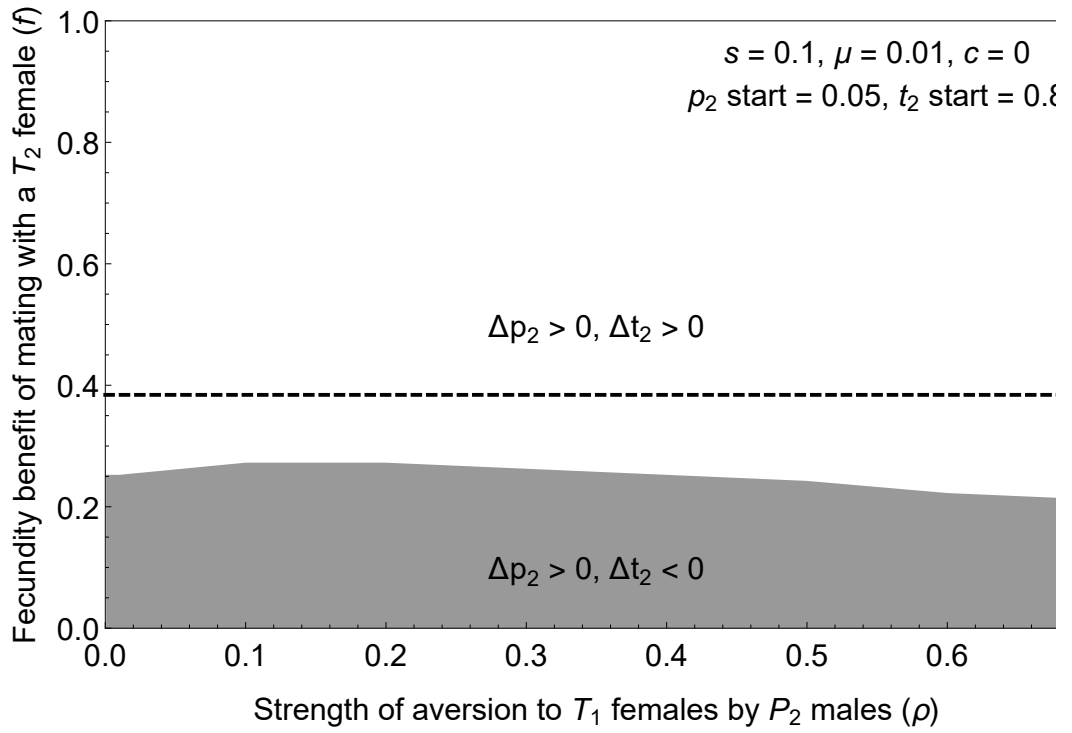



Figure 1B: effect of f and ρ on the sign of Δp_2 and Δt_2 when $\mu = 0.01$, $c=0$, and $s=0.2$

$\{\rho, f\}$

The dashed line indicates the value that f must exceed in order for Δt_2 to be positive in the absence of p_2 ($p_2=0$).

```
Show[ListPlot[{{0.001, 0.54}, {0.01, 0.56}, {0.1, 0.6}, {0.2, 0.6},
  {0.3, 0.58}, {0.4, 0.56}, {0.5, 0.52}, {0.6, 0.5}, {0.7, 0.47}},
  {{0.001, 0.14}, {0.01, 0.14}, {0.1, 0.14}, {0.2, 0.14},
  {0.3, 0.14}, {0.4, 0.14}, {0.5, 0.14}, {0.6, 0.14}, {0.7, 0.15}},
  {{0, 0.819438}, {1, 0.819438}}], Joined -> True,
PlotRange -> {{0.0, 0.7}, {0.0, 1.0}}, Frame -> True,
FrameTicks -> {{True, False}, {True, False}},
FrameLabel -> {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
ImageSize -> Large, BaseStyle -> {FontSize -> 16}, PlotStyle ->
  {{GrayLevel[0.6]}, {GrayLevel[0.3]}, {Black, Thick, Dashed}},
FrameStyle -> Black, Filling ->
  {{1 -> {{2}, GrayLevel[0.6]}}, {2 -> {Axis, GrayLevel[0.3]}}},
Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 > 0$ ", {0.35, 0.74}]],
Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 < 0$ ", {0.35, 0.25}]],
Graphics[Text[" $s = 0.2$ ,  $\mu = 0.01$ ,  $c = 0$ ", {0.55, 0.95}]],
Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]],
Graphics[Text[Style[" $\Delta p_2 < 0$ ,  $\Delta t_2 < 0$ ", White], {0.35, 0.07}]]]
```

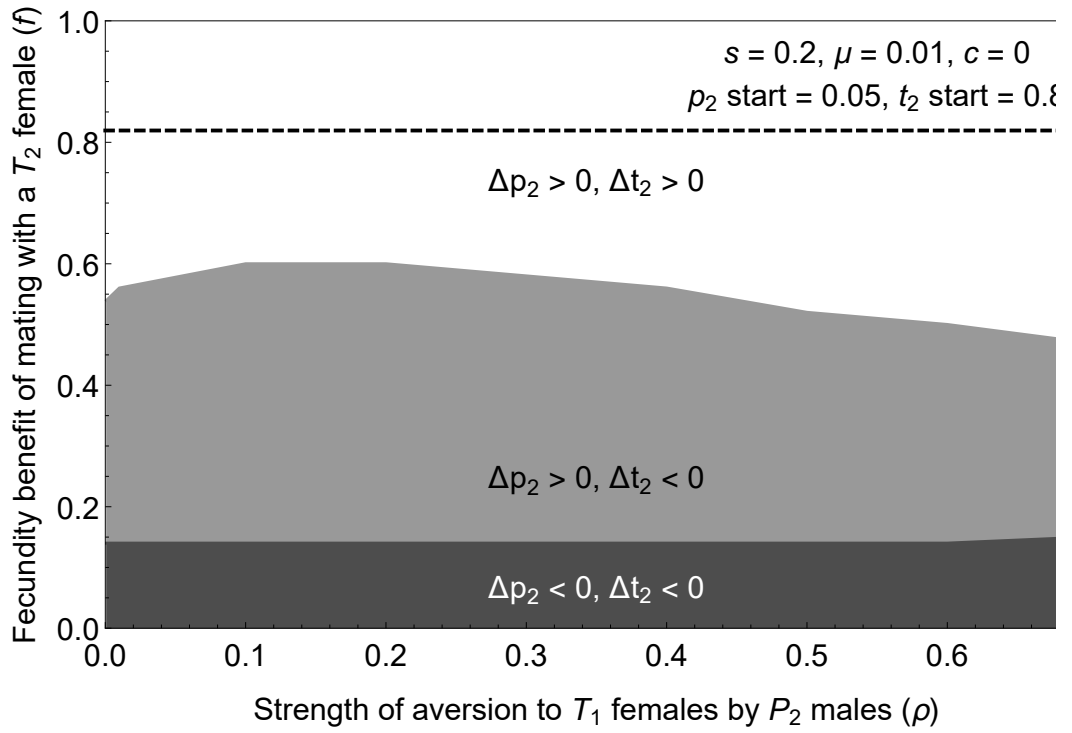
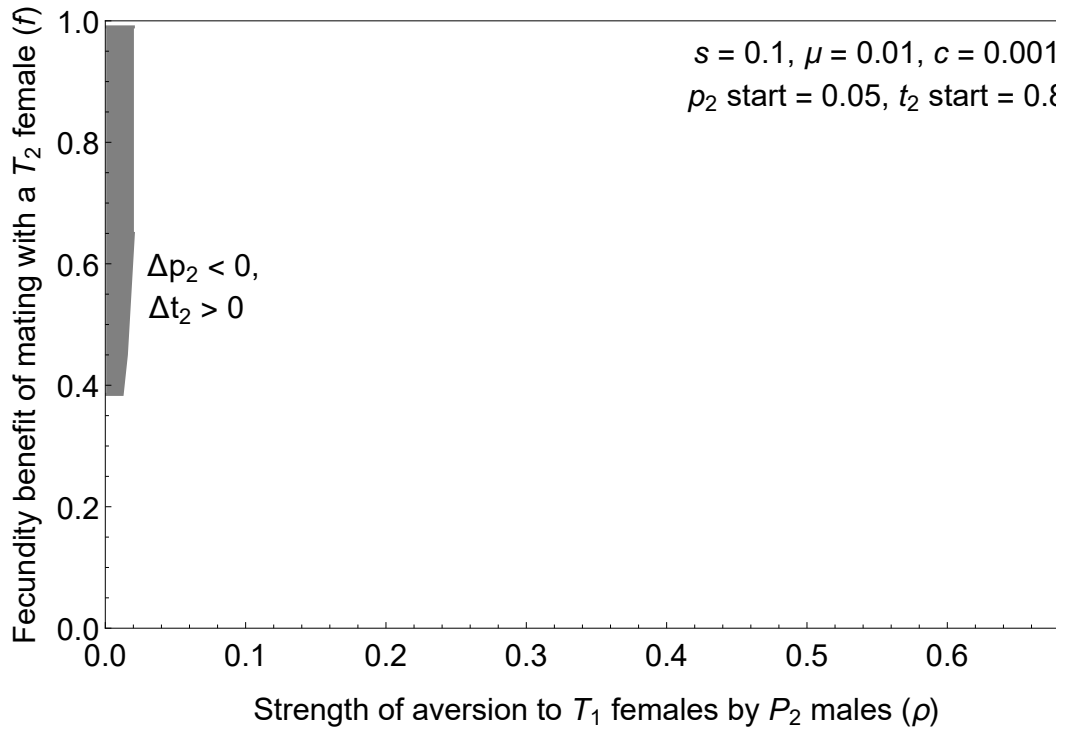


Figure 2A: effect of f and ρ on the sign of Δp_2 and Δt_2 when $\mu = 0.01$ and $c=0.001$

$\{\rho, f\}$

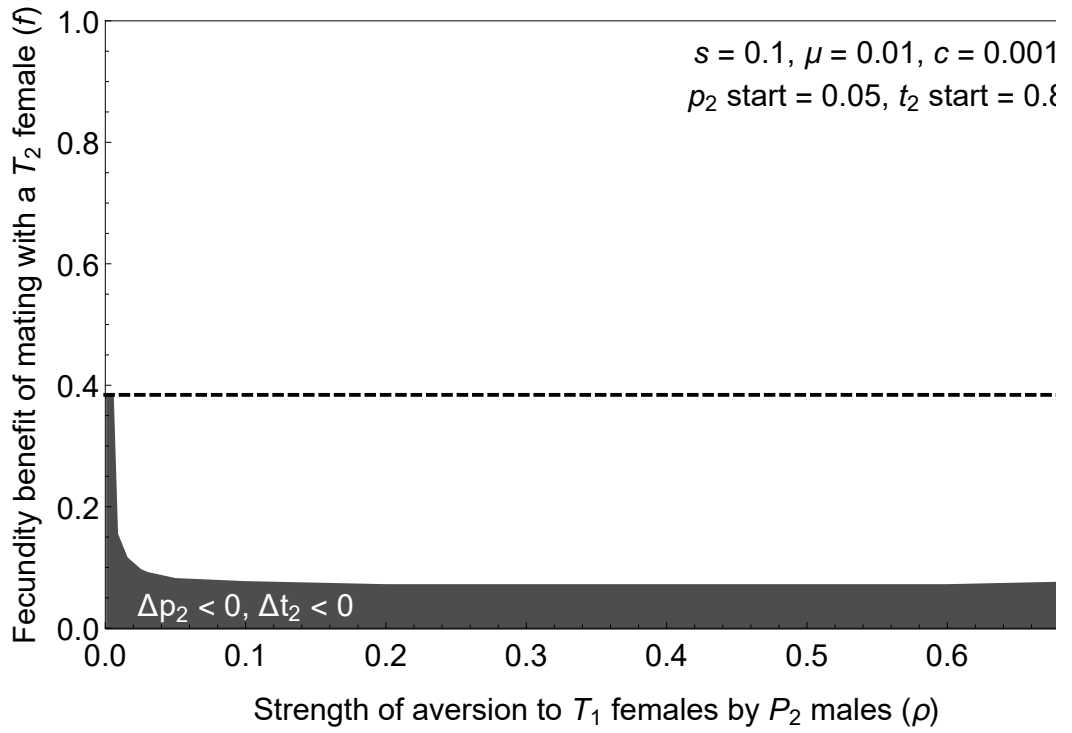
First line: highest value at which p_2 is decreasing and t_2 is increasing
 Second line: lowest value at which p_2 is decreasing and t_2 is increasing

```
A2p2dt2i = Show[
  ListPlot[{{{0.001, 0.99}, {0.01, 0.99}, {0.015, 0.99}, {0.02, 0.99}},
    {{0.001, 0.385}, {0.005, 0.385}, {0.012, 0.385}, {0.015, 0.45},
    {0.02, 0.65}}}, Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
    "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {GrayLevel[0.5]}, FrameStyle → Black,
  Filling → {{1 → {2}, GrayLevel[0.5]}}},
  Graphics[Text[" $\Delta p_2 < 0$ ", {0.07, 0.6}]],
  Graphics[Text[" $\Delta t_2 > 0$ ", {0.065, 0.53}]],
  Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.001$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```



First line: highest value at which p_2 and t_2 are both decreasing
 Second line: lowest value at which p_2 and t_2 are both decreasing

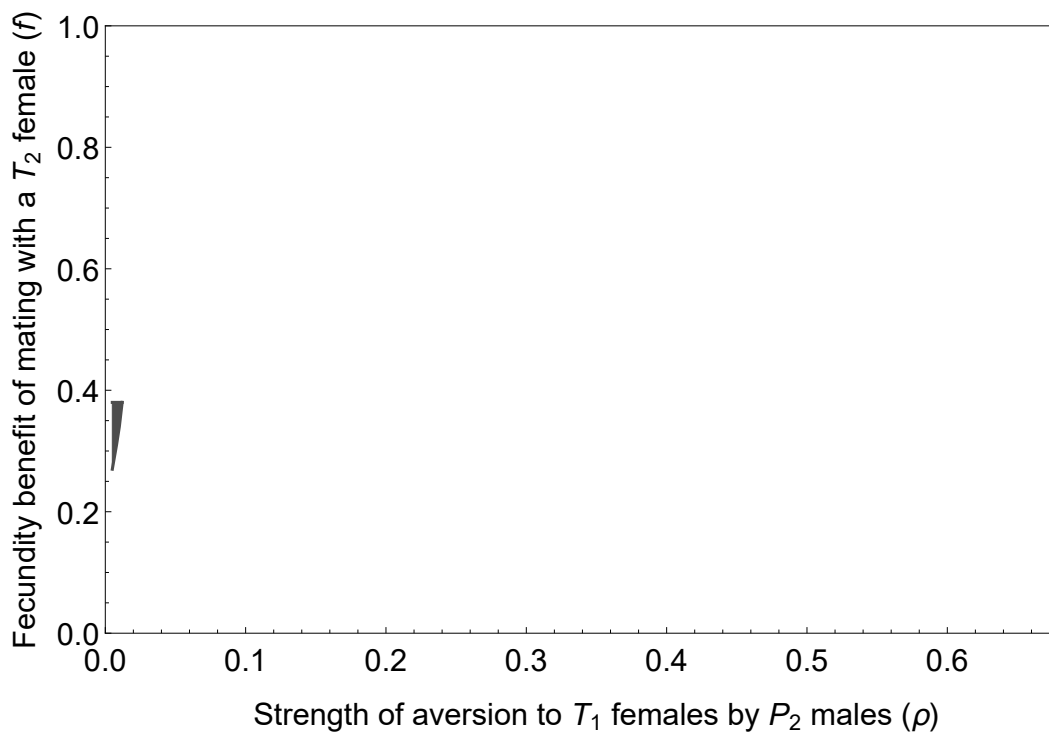
```
A2p2dt2d = Show[ListPlot[
  {{{0.001, 0.38}, {0.005, 0.38}, {0.008, 0.155}, {0.015, 0.115},
    {0.025, 0.095}, {0.03, 0.09}, {0.04, 0.085}, {0.05, 0.08},
    {0.1, 0.075}, {0.2, 0.07}, {0.3, 0.07}, {0.4, 0.07},
    {0.5, 0.07}, {0.6, 0.07}, {0.7, 0.075}}},
  {{{0.001, 0.0}, {0.01, 0.0}, {0.1, 0.0}, {0.2, 0.0},
    {0.3, 0.0}, {0.4, 0.0}, {0.5, 0.0}, {0.6, 0.0}, {0.7, 0.0}}},
  {{{0, 0.38419}, {1, 0.38419}}}], Joined → True,
PlotRange → {{0.0, 0.7}, {0.0, 1.0}}, Frame → True,
FrameTicks → {{True, False}, {True, False}},
FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
ImageSize → Large, BaseStyle → {FontSize → 16}, PlotStyle →
  {GrayLevel[0.3], GrayLevel[0.3], {Black, Thick, Dashed}},
FrameStyle → Black, Filling → {1 → {{2}, GrayLevel[0.3]}}],
Graphics[Text[Style[" $\Delta p_2 < 0, \Delta t_2 < 0$ ", White], {0.1, 0.04}]],
Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.001$ ", {0.55, 0.95}]],
Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```



```

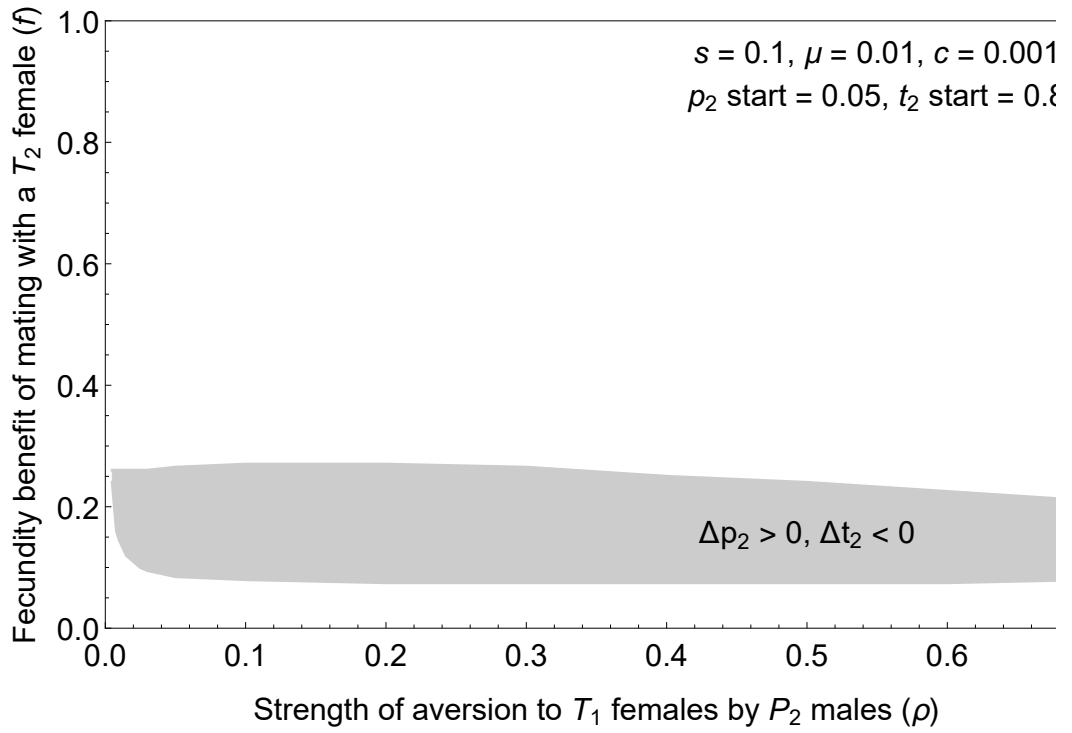
A2p2dt2dextra =
Show[ListPlot[{{0.005, 0.38}, {0.008, 0.38}, {0.01, 0.38},
  {0.011, 0.38}, {0.012, 0.38}},
  {{0.005, 0.27}, {0.008, 0.31}, {0.01, 0.34}, {0.012, 0.38}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16}, PlotStyle →
  {GrayLevel[0.3], GrayLevel[0.3], {Black, Thick, Dashed}},
  FrameStyle → Black, Filling → {1 → {{2}, GrayLevel[0.3]}}},
  Graphics[Text[Style[" $\Delta p_2 < 0, \Delta t_2 < 0$ ", White], {0.1, 0.04}]],
  Graphics[Text[Style[" $s = 0.1, \mu = 0.01, c = 0.001$ ", White],
  {0.55, 0.95}]], Graphics[Text[
  Style[" $p_2$  start = 0.05,  $t_2$  start = 0.8", White], {0.55, 0.88}]]]

```

First line: highest value at which p_2 is increasing and t_2 is decreasing
 Second line: lowest value at which p_2 is increasing and t_2 is decreasing

```
A2p2it2d = Show[
  ListPlot[{{{0.005, 0.26}, {0.03, 0.26}, {0.05, 0.265}, {0.1, 0.27},
    {0.2, 0.27}, {0.3, 0.265}, {0.4, 0.25}, {0.5, 0.24}, {0.6, 0.225},
    {0.7, 0.21}}, {{0.005, 0.24}, {0.008, 0.16}, {0.01, 0.145},
    {0.015, 0.12}, {0.025, 0.1}, {0.03, 0.095}, {0.04, 0.09},
    {0.05, 0.085}, {0.1, 0.08}, {0.2, 0.075}, {0.3, 0.075},
    {0.4, 0.075}, {0.5, 0.075}, {0.6, 0.075}, {0.7, 0.08}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
    "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {GrayLevel[0.8]}, FrameStyle → Black,
  Filling → {{1 → {{2}, GrayLevel[0.8]}}}],
  Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 < 0$ ", {0.5, 0.16}]],
  Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.001$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```



First line: highest value at which p_2 and t_2 are both increasing

Second line: lowest value at which p_2 and t_2 are both increasing

```
A2p2it2i =
Show[ListPlot[{{0.015, 0.4}, {0.02, 0.6}, {0.025, 0.99}, {0.05, 0.99},
  {0.1, 0.99}, {0.2, 0.99}, {0.3, 0.99}, {0.4, 0.99},
  {0.5, 0.99}, {0.6, 0.99}, {0.7, 0.99}},
  {{0.013, 0.39}, {0.015, 0.39}, {0.02, 0.39}, {0.025, 0.4},
  {0.05, 0.27}, {0.1, 0.275}, {0.2, 0.275}, {0.3, 0.27},
  {0.4, 0.255}, {0.5, 0.245}, {0.6, 0.23}, {0.7, 0.215}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {{White}}, FrameStyle → Black,
  Filling → {{1 → {{2}, GrayLevel[1.0]}}}],
Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 > 0$ ", {0.5, 0.5}]],
Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.001$ ", {0.55, 0.95}]],
Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```

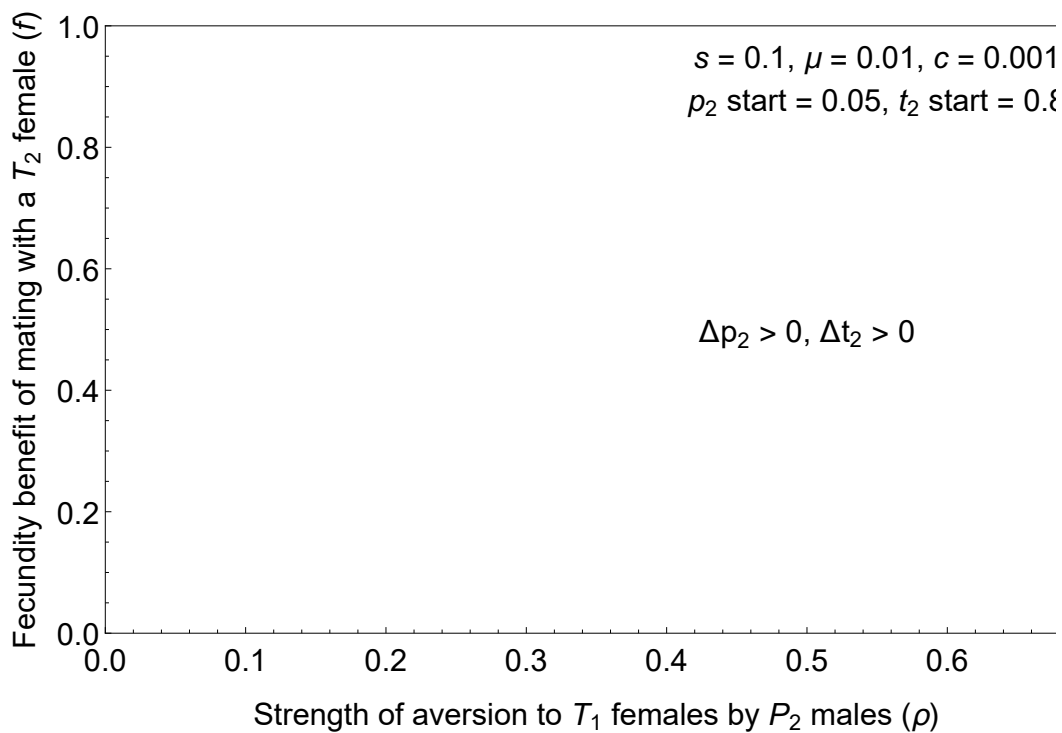
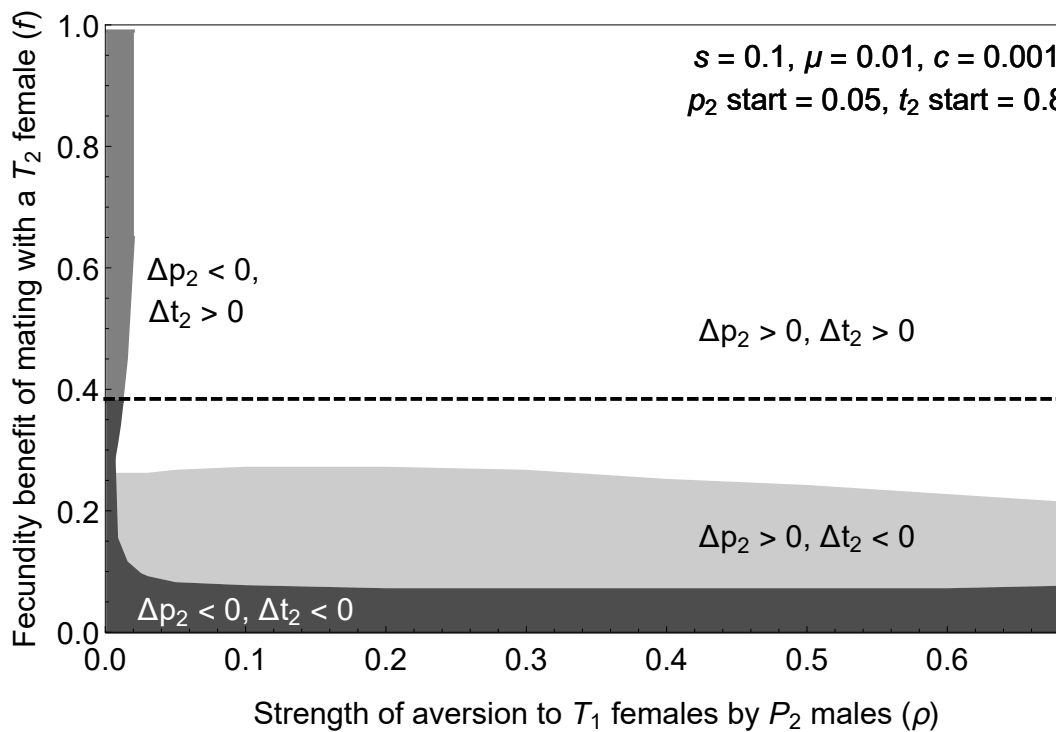
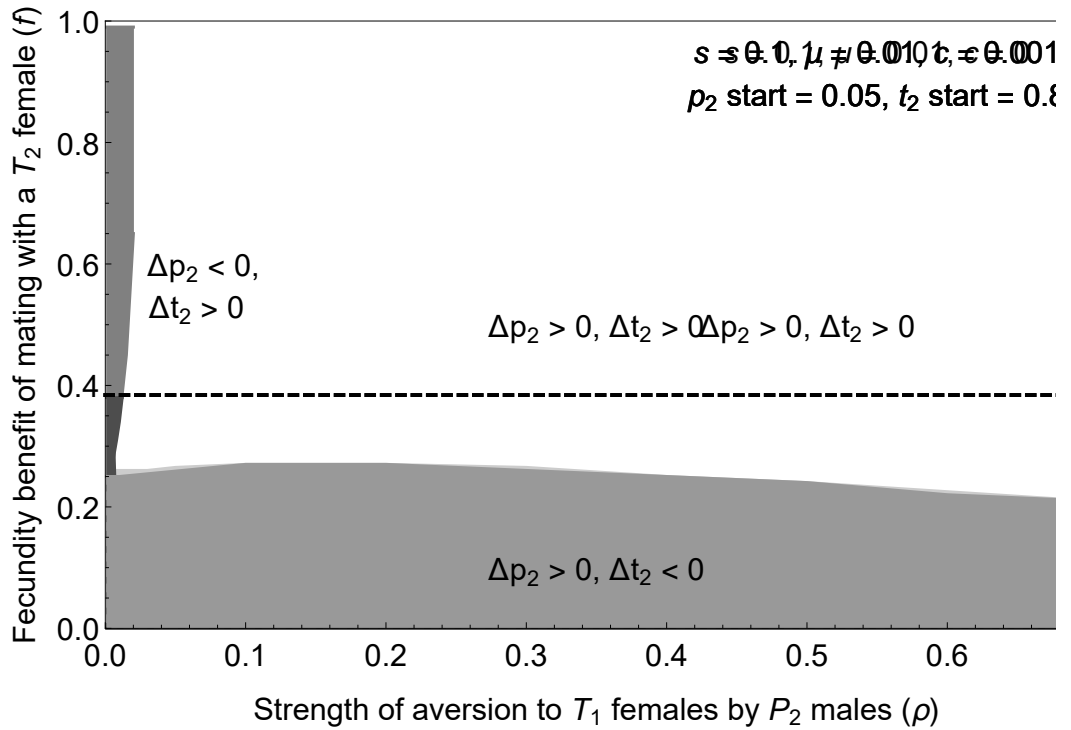


Figure2A = Show[A2p2it2i, A2p2dt2i, A2p2dt2dextra, A2p2it2d, A2p2dt2d]



How much do Figures 1A and 2A overlap?

Show[Figure2A, Figure1A]



Show[Figure1A, Figure2A]

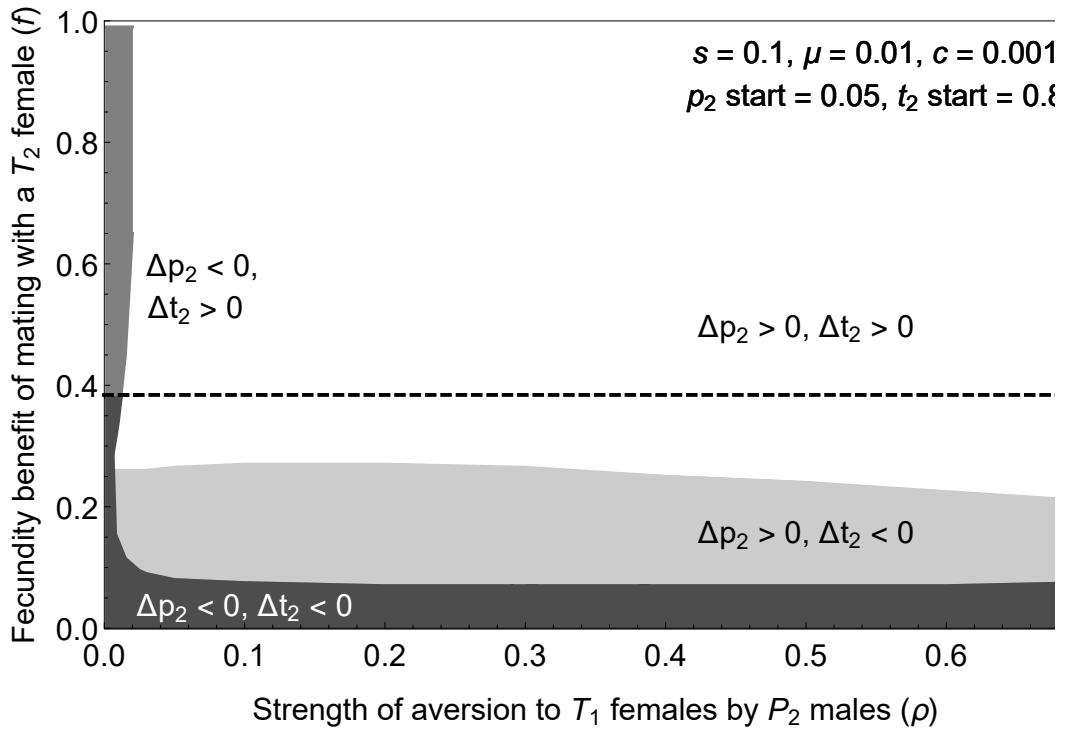


Figure 2B: effect of f and ρ on the sign of Δp_2 and Δt_2 when $\mu = 0.01$ and $c = 0.01$

$\{\rho, f\}$

First line: highest value at which p_2 is decreasing and t_2 is increasing

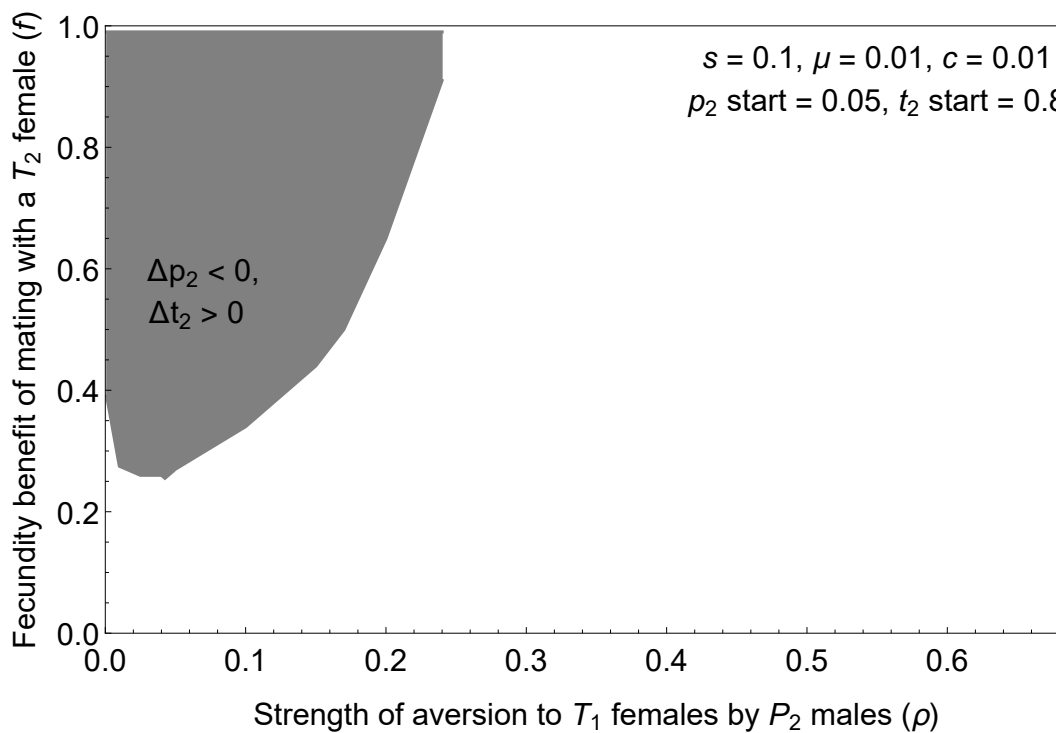
Second line: lowest value at which p_2 is decreasing and t_2 is

increasing

```

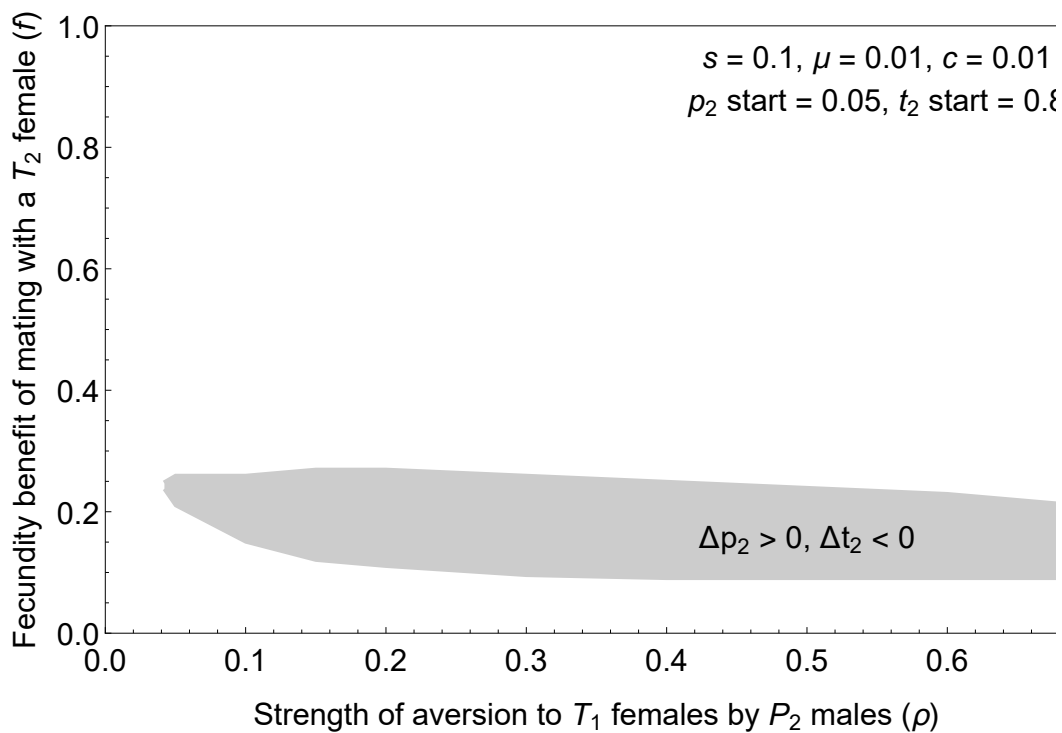
B2p2dt2i = Show[ListPlot[{{0.001, 0.99}, {0.1, 0.99}, {0.24, 0.99}},
  {{0.001, 0.39}, {0.01, 0.275}, {0.025, 0.26}, {0.03, 0.26},
  {0.04, 0.26}, {0.0425, 0.255}, {0.05, 0.27}, {0.1, 0.34},
  {0.15, 0.44}, {0.17, 0.5}, {0.2, 0.65}, {0.24, 0.91}}],
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {GrayLevel[0.5]}, FrameStyle → Black,
  Filling → {{1 → {{2}, GrayLevel[0.5]}}}],
Graphics[Text[" $\Delta p_2 < 0$ ", {0.07, 0.6}]],
Graphics[Text[" $\Delta t_2 > 0$ ", {0.065, 0.53}]],
Graphics[Text[" $s = 0.1$ ,  $\mu = 0.01$ ,  $c = 0.01$ ", {0.55, 0.95}]],
Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]

```



First line: highest value at which p_2 is increasing and t_2 is decreasing
 Second line: lowest value at which p_2 is increasing and t_2 is decreasing

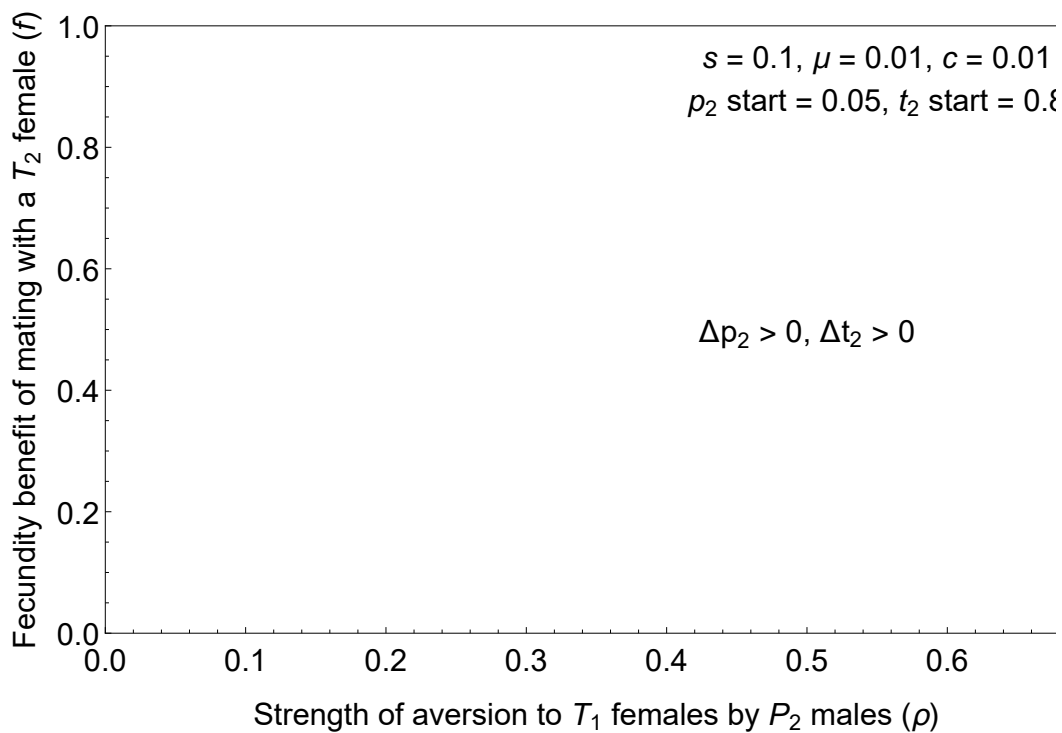
```
B2p2it2d = Show[
  ListPlot[{{0.0425, 0.25}, {0.05, 0.26}, {0.1, 0.26}, {0.15, 0.27},
    {0.2, 0.27}, {0.3, 0.26}, {0.4, 0.25}, {0.5, 0.24},
    {0.6, 0.23}, {0.7, 0.21}}, {{0.0425, 0.235}, {0.05, 0.21},
    {0.1, 0.15}, {0.15, 0.12}, {0.2, 0.11}, {0.3, 0.095},
    {0.4, 0.09}, {0.5, 0.09}, {0.6, 0.09}, {0.7, 0.09}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
    "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16},
  PlotStyle → {GrayLevel[0.8]}, FrameStyle → Black,
  Filling → {{1 → {2}, GrayLevel[0.8]}}},
  Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 < 0$ ", {0.5, 0.16}]],
  Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.01$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```



First line: highest value at which p_2 and t_2 are both increasing

Second line: lowest value at which p_2 and t_2 are both increasing

```
B2p2it2i =
Show[ListPlot[{{0.1, 0.33}, {0.2, 0.6}, {0.3, 0.99}, {0.4, 0.99},
  {0.5, 0.99}, {0.6, 0.99}, {0.7, 0.99}}, {{0.1, 0.27}, {0.2, 0.28},
  {0.3, 0.27}, {0.4, 0.26}, {0.5, 0.25}, {0.6, 0.24}, {0.7, 0.22}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16}, PlotStyle → {{White}},
  FrameStyle → Black, Filling → {{1 → {{2}, GrayLevel[1.0]}}}],
Graphics[Text[" $\Delta p_2 > 0, \Delta t_2 > 0$ ", {0.5, 0.5}]],
Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.01$ ", {0.55, 0.95}]],
Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```



First line: highest value at which p_2 and t_2 are both decreasing
 Second line: lowest value at which p_2 and t_2 are both decreasing

```
B2p2dt2d = Show[
  ListPlot[{{0.001, 0.38}, {0.01, 0.27}, {0.025, 0.255}, {0.03, 0.255},
    {0.04, 0.255}, {0.0425, 0.23}, {0.05, 0.205},
    {0.1, 0.145}, {0.15, 0.115}, {0.2, 0.105}, {0.3, 0.09},
    {0.4, 0.085}, {0.5, 0.085}, {0.6, 0.085}, {0.7, 0.085}},
    {{0.001, 0}, {0.1, 0}, {0.2, 0}, {0.3, 0}, {0.4, 0}, {0.5, 0},
    {0.6, 0}, {0.7, 0}}, {{0, 0.38419}, {1, 0.38419}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
    "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16}, PlotStyle →
    {GrayLevel[0.3], GrayLevel[0.3], {Black, Thick, Dashed}},
  FrameStyle → Black, Filling → {1 → {{2}, GrayLevel[0.3]}},
  Graphics[Text[Style[" $\Delta p_2 < 0, \Delta t_2 < 0$ ", White], {0.1, 0.05}]],
  Graphics[Text[" $s = 0.1, \mu = 0.01, c = 0.01$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.8", {0.55, 0.88}]]]
```

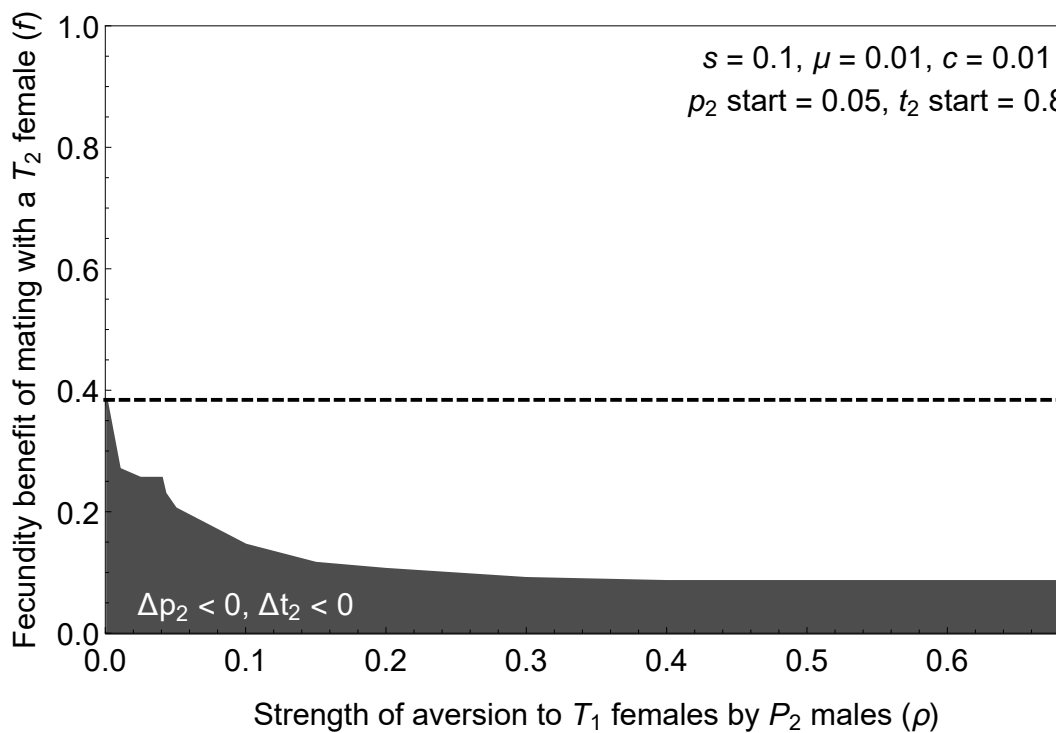
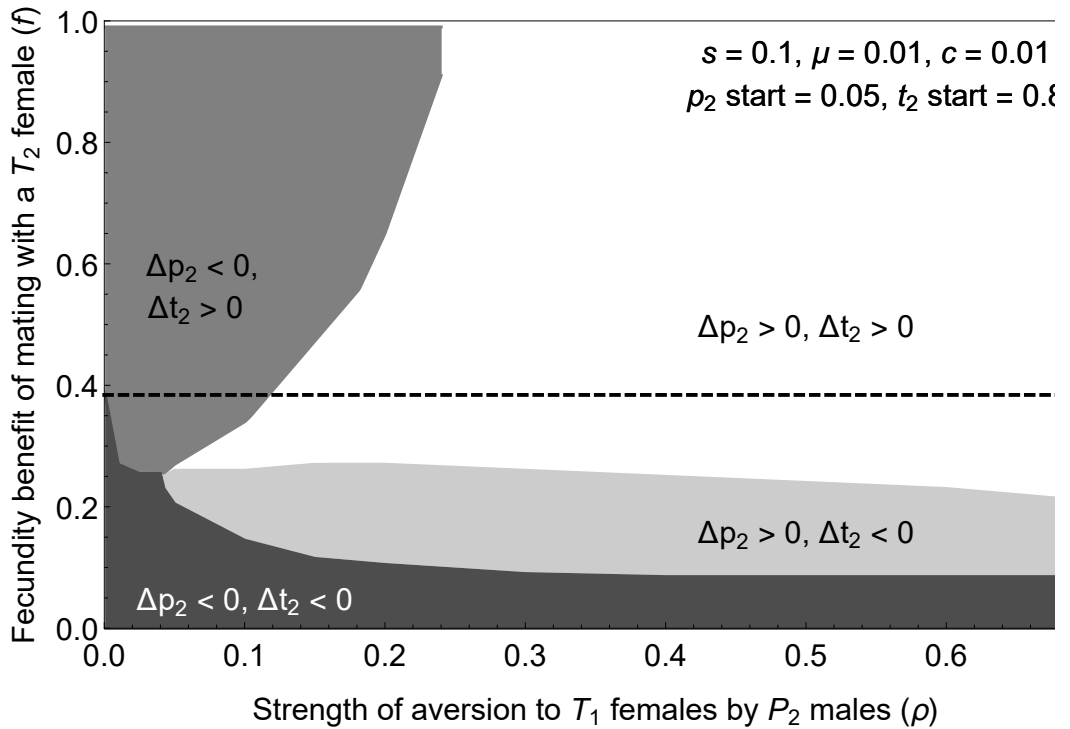


Figure2B = Show[B2p2dt2i, B2p2it2d, B2p2it2i, B2p2dt2d]



2. Supplementary Figures

Results when the starting frequency of the ornamental trait is low

Figure SI: effect of f and ρ on the sign of Δp_2 and

Δt_2 when $\mu = 0.01$, $c=0$, $s=0.1$, and t_2 start = 0.1
 $\{\rho, f\}$

Compare with figure 1A in the main text.

Fix this: the dashed line indicates the value that f must exceed in order for Δt_2 to be positive in the absence of p_2 ($p_2=0$).

```
In[132]:= Show[ListPlot[{{0.001, 0.24}, {0.1, 0.24}, {0.2, 0.24},
  {0.3, 0.23}, {0.4, 0.22}, {0.5, 0.21}, {0.6, 0.2}, {0.7, 0.19}},
  {{0.001, 0.06}, {0.1, 0.06}, {0.2, 0.06}, {0.3, 0.06},
  {0.4, 0.06}, {0.5, 0.06}, {0.6, 0.07}, {0.7, 0.07}},
  {{0, 0.24994265290988632}, {1, 0.24994265290988632}}],
  Joined -> True, PlotRange -> {{0.0, 0.7}, {0.0, 1.0}},
  Frame -> True, FrameTicks -> {{True, False}, {True, False}},
  FrameLabel -> {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize -> Large, BaseStyle -> {FontSize -> 16}, PlotStyle ->
  {{GrayLevel[0.6]}, {GrayLevel[0.3]}, {Black, Thick, Dashed}},
  FrameStyle -> Black, Filling ->
  {{1 -> {{2}, GrayLevel[0.6]}}, {2 -> {Axis, GrayLevel[0.3]}}},
  Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 > 0$ ", {0.35, 0.44}]],
  Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 < 0$ ", {0.35, 0.15}]],
  Graphics[Text[" $s = 0.1$ ,  $\mu = 0.01$ ,  $c = 0$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.1", {0.55, 0.88}]],
  Graphics[Text[Style[" $\Delta p_2 < 0$ ,  $\Delta t_2 < 0$ ", White], {0.35, 0.03}]]]
```

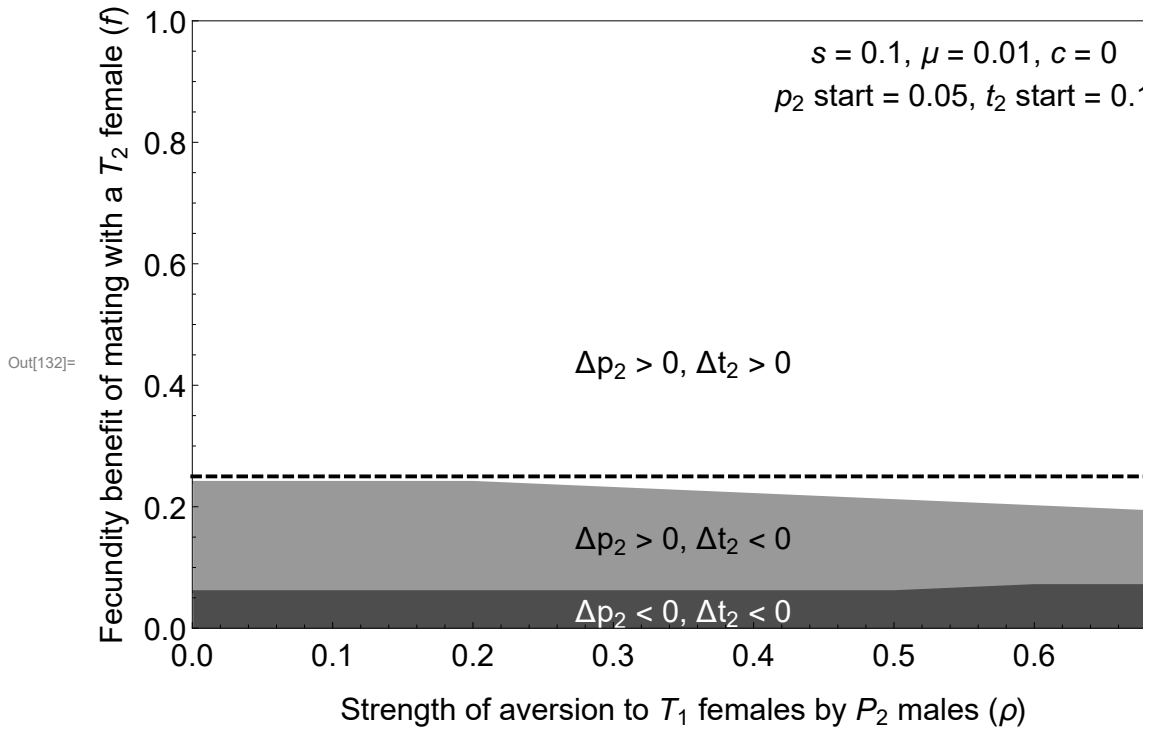


Figure S2: effect of f and ρ on the sign of Δp_2 and Δt_2 when $\mu = 0.01$, $c=0$, and $s=0.2$

$\{\rho, f\}$

Compare with figure 1B in the main text.

Fix this: the dashed line indicates the value that f must exceed in order for Δt_2 to be positive in the absence of p_2 ($p_2=0$).

```
In[142]:= Show[ListPlot[{{0.001, 0.53}, {0.1, 0.53}, {0.2, 0.52},
  {0.3, 0.51}, {0.4, 0.49}, {0.5, 0.47}, {0.6, 0.45}, {0.7, 0.43}},
  {{0.001, 0.13}, {0.1, 0.13}, {0.2, 0.13}, {0.3, 0.13},
  {0.4, 0.13}, {0.5, 0.13}, {0.6, 0.13}, {0.7, 0.13}},
  {{0, 0.5395692703536477}, {1, 0.5395692703536477}}},
  Joined → True, PlotRange → {{0.0, 0.7}, {0.0, 1.0}},
  Frame → True, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Strength of aversion to T1 females by P2 males ( $\rho$ )",
  "Fecundity benefit of mating with a T2 female ( $f$ )"},
  ImageSize → Large, BaseStyle → {FontSize → 16}, PlotStyle →
  {{GrayLevel[0.6]}, {GrayLevel[0.3]}, {Black, Thick, Dashed}},
  FrameStyle → Black, Filling →
  {{1 → {2}, GrayLevel[0.6]}}, {2 → {Axis, GrayLevel[0.3]}}}],
  Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 > 0$ ", {0.35, 0.64}]],
  Graphics[Text[" $\Delta p_2 > 0$ ,  $\Delta t_2 < 0$ ", {0.35, 0.25}]],
  Graphics[Text[" $s = 0.2$ ,  $\mu = 0.01$ ,  $c = 0$ ", {0.55, 0.95}]],
  Graphics[Text[" $p_2$  start = 0.05,  $t_2$  start = 0.1", {0.55, 0.88}]],
  Graphics[Text[Style[" $\Delta p_2 < 0$ ,  $\Delta t_2 < 0$ ", White], {0.35, 0.05}]]]
```

